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Cooperation in knowledge-intensive firms

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Cooperation in knowledge-intensive firms

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Abstract

The extent to which a knowledge-intensive firm should induce cooperation between its employees is analyzed in a model of relational contracting between a firm (principal) and its employees (two agents). The agents can cooperate by helping each other, i.e. provide effort that increases the performance of their peer without affecting their own performance. We extend the existing literature on agent-cooperation by analyzing the implications of incomplete contracts and agent hold-up. A main result is that if the agents’ hold-up power is sufficiently high, then it is suboptimal for the principal to implement cooperation, even if helping effort is productive per se. This implies, contrary to many property rights models, that social surplus may suffer if the investing parties (here the agents) are residual claimants. The model also shows that long-term relationships facilitate cooperation even if the agents cannot monitor or punish each others effort choices.

1 Introduction

There seems to be a consensus among scholars in human resource management (HRM) that teamwork or cooperation is particularly important in knowledge-intensive organizations. It is argued that teams are essential for knowledge sharing and innovation (see e.g. Cano and Cano, 2006), and that knowledge-intensive firms should therefore adopt compensation plans that reward cooperation (see e.g. Balkin and Banister, 1993). In this paper we argue that, although it may well be the case that teamwork is important

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in human-capital-intensive firms, one should also expect a positive relationship between a firm’s human-capital-intensity and the costs of implementing cooperation.

It is a well known result from the theory of task allocation that cooperation between employees is favorable if there are complementarities between their efforts, see e.g. Drago and Turnbull (1988, 1991), Itoh (1991, 1992), Holmström and Milgrom (1990), Ramakrishnan, and Thakor (1991), Macho-Stadler and Perez-Castrillo, (1993), for static relationships, and Che and Yoo (2001) for the case of repeated peer-monitoring. But these results are deduced from models assuming that the employees can commit to contracts inducing any kind of cooperative behavior. In this paper we show that if instead employees posses some form of hold-up power, then it may be suboptimal to implement cooperation, i.e. induce the employees to help each other, even if cooperation is productive per se.

An employee has hold-up power if he is able to prevent his employer from realizing his value added. In order to be in such a position, the employee must possess some kind of ownership rights to critical assets. According to the standard view of ownership, it is the owner of an asset who has residual control rights; that is “the right to decide all usages of the asset in any way not inconsistent with a prior contract, custom or law” (Hart, 1995). An employee is typically considered to be an agent without residual control rights, and hence not in a position to exercise hold-up power. It is the firm that holds critical assets, not its workers.

However, in knowledge-intensive firms the allocation of ownership rights is blurred. The main assets involved there are often the employees’ minds and knowledge. Their human capital can make them indispensable, and they can threaten to walk away with ideas, clients or new production technologies. When ownership is not associated with clear rights to control critical assets, the firm runs the risk of being expropriated or held-up by its own employees (see e.g. Rajan and Zingales, 2001).

In this paper we show that when employees (agents) are in a position to hold-up their employer (principal), it is costly to implement cooperation between the agents. The intuition is as follows: In order to induce cooperation, the principal must implement some form of group-based incentive

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1Based on Lavoie and Roy (1998), OECD measures the knowledge intensity of an industry by quantifying an industry’s knowledge base from its R&D and human capital characteristics. The latter refers to the share of workers in certain knowledge based occupations. Knowledge based occupations are defined as those that “mainly involve the production of knowledge and/or the provision of expert opinion.” (OECD 2001: 16), and are classified into five subcategories; applied science, pure science, engineering, computer science and social sciences and humanities.
schemes that makes it profitable for the agents to help each other, i.e. to provide costly effort that increases the performance of their peers without affecting their own performance. But group-based pay is susceptible to agent hold-up since an agent who performs well in a given period, is tempted to hold-up output and renegotiate his pay if his peers’ performances are poor that period. He thereby obstructs the incentive scheme necessary for implementing cooperation. The parties can mitigate this hold-up problem through repeated interaction, i.e. through self-enforcing relational contracting where contract breach is punished, not by the court, but by the parties themselves who can refuse to engage in relational contracting after a deviation. Since a hold-up will be regarded as a deviation from such a relational contract, the self-enforcing range of the contract is limited by the hold-up problem. If the agents’ hold-up power is sufficiently high, it may therefore be more costly to implement a relational contract inducing helping effort, than to just implement individualized incentives that trigger non-cooperative effort.

Interestingly, it follows from the analysis that not only the principal’s profit, but also the social surplus may decrease if the agents’ hold-up power is sufficiently high. This is at variance with the established idea from the property rights approach that the investing parties, the agents in our model, should be residual claimants (Grossman and Hart, 1986, and Hart and Moore, 1990). In our model, residual control rights in the hands of the agents trigger own efforts, but obstruct the principal from implementing socially efficient cooperation.

A secondary result from our analysis is that long-term relationships facilitate agent-cooperation even if the agents cannot monitor or punish each other’s effort decisions. This result complements the existing literature on team incentives in repeated settings, such as Che and Yoo (2001) where repeated peer-monitoring makes cooperation easier to sustain. In our model, a higher discount factor eases the implementation of relational contracts, making it less costly for the principal to implement cooperation even if there is no peer sanctioning.

To our knowledge, this paper is the first to consider the problem of im-

\footnote{Influential models of relational contracts include Klein and Leffler (1981), Shapiro and Stiglitz (1984), Bull (1987), Baker, Gibbons and Murphy (2002), MacLeod and Malcolmson (1989) generalize the case of symmetric information, while Levin (2003) makes a general treatment of relational contracts with asymmetric information, allowing for incentive problems due to moral hazard and hidden information.}

\footnote{Radner (1986), Weitzman and Kruse (1990), and FitzRoy and Kraft (1995) have all pointed out that the folk theorem of repeated games provides a possible answer to the free rider critique of group incentives. But Che and Yoo (2001) is the first to demonstrate this in a repeated game between the agents. See also Ishida (2006).}
plementing agent cooperation in a relational contracting model. It is also
the first paper to consider the effect of agent hold-up on helping effort. The
paper is related to our companion paper (Kvaløy and Olsen, 2007), where we
investigate the problem of implementing peer-dependent incentives schemes
when agents are ex post indispensable.\textsuperscript{4} But that paper does not consider a
multitask situation where the agents are allowed to help each other, which is
the main feature of the model presented here. In spirit, the paper is related
to Auriol and Friebel (2002) who show how limited principal commitment
in a two period model of career concerns can reduce the agents’ incentives
to help each other, since the agents expect that their \textit{relative} productivity
in period one will determine their fixed salary in period two. In our model,
their are no internal career concerns, i.e. productivity and expected wage remain
the same in all periods. What drives the results is the agents’ potential
exploitation of ex post outside opportunities.

Broadly speaking, a contribution of the paper, together with our com-
panion paper (Kvaløy and Olsen, 2007), is to consider the effect of residual
control rights in a multiagent moral hazard model. In the vast literature
on multiagent moral hazard it is (implicitly) assumed that residual control
rights are exclusively in the hands of the principal. And in the growing liter-
ature dealing with optimal allocation of control rights, the multiagent moral
hazard problem is not considered. (This literature begins with Grossman and
Hart, 1986; and Hart and More,\textsuperscript{1990,}\textsuperscript{5} who analyze static relationships. Re-
peated relationships are analyzed in particular by Halonen, 2002; and Baker,
Gibbons and Murphy, 2002). A contribution of the paper is thus to consider
the effect of workers possessing residual control rights when the firm faces a
multiagent moral hazard problem.

\section{The Model}

There are basically two kinds of agent-cooperation. One is where agents
cooperate performing a common task, a second is where agents help each
other performing each others’ tasks. In this paper we focus on the latter
since it represents the purest form of cooperative behavior. In particular
we assume that an agent who helps his peer does not increase his chance to
succeed on his own task, \textit{cet par}.

\textsuperscript{4}Kvaløy and Olsen (2006) analyze peer-monitoring and collusion in a relational con-
tracting model with no agent-hold-up.

\textsuperscript{5}Although Hart and More (1990) analyze a model with many agents, they do not
consider the classical moral hazard problem that we address, where a principal can only
observe a noisy measure of the agents’ effort.
We consider a relationship between a principal and two agents \((i = 1, 2)\), who each period can either succeed or fail when performing a task for their principal. Success yields high value \(Q_H\), while failure yields low value \(Q_L\). The agents can exert effort in order to increase the probability of success on their own task. In addition they can help each other and thereby increase the probability of success for their peer. Let \(e_i\) denote agent \(i\)'s own effort and \(a_i\) denote helping effort. Efforts can be either high (1) or low (0), where high effort has a cost \(c\) for own effort and \(c_A\) for helping effort. Low effort is costless. The probabilities for success is then \(\Pr(\text{success}) = p_i(e_i, a_j)\) for agent \(i\).6

Our restrictions on effort levels make it impossible for an agent who exert high effort on his own project, to trade-off helping effort with 'even higher' own effort. This is done for tractability reasons, and is not necessary for our main results to go through. However, it is not entirely unrealistic to assume that there is a limit on how much valuable effort an agent can exert on a given project. If the agent has more time to spend before starting on tomorrow's project, he can spend it on helping others. Proof-reading papers can serve as an example. There is a limit on how many times you can read your own paper, and still find new errors. Reading your colleague's paper, and make him read yours, may though be valuable.

We assume that the principal can only observe the realization of the agents' output, not the level of effort they choose. Similarly, agent \(i\) can only observe agent \(j\)'s output, not his effort level. Whether or not the agents can observe each others effort choices is not decisive for the analysis presented. However, by assuming that effort is unobservable among the agents, we get stronger results, since we do not need to rely on repeated peer monitoring and peer-sanctions.

We assume that if the parties engage in an incentive contract, agent \(i\) receives a bonus vector \(\beta \equiv (\beta_{HH}, \beta_{HL}, \beta_{LH}, \beta_{LL})\) where the subscripts refer to respectively agent \(i\) and agent \(j\)'s realization of \(Q_k\) and \(Q_l\), \(k, l \in \{L, H\}\).

Agent \(i\)'s expected wage is then

\[
\omega^i = p_i [p_j \beta_{HH} + (1 - p_j) \beta_{HL}] + (1 - p_i) [p_j \beta_{LH} + (1 - p_j) \beta_{LL}]
\]

\[= p_i [p_j (\beta_{HH} - \beta_{LH}) + (1 - p_j) (\beta_{HL} - \beta_{LL})] + p_j (\beta_{LH} - \beta_{LL}) + \beta_{LL} \quad (1)\]

It is assumed that all parties are risk neutral, but that the agents are subject

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6The basic set-up is a simple version of the more general model analyzed by Hideshi Itoh in his seminal 1991-paper. For tractability reasons, our relational contracting extension makes it necessary to simplify Itoh’s set up.
to limited liability: the principal cannot impose negative wages. Ex ante outside options are normalized to zero.

2.1 Optimal contract when output is verifiable

We first consider the least cost incentive contract when output is verifiable. The principal will minimize wages subject to the constraint that the agents must be induced to yield the desired levels of effort and help. Let the probability levels for each agent be denoted:

\[ p_i(e_i, a_j) = \begin{cases} q_{11} & \text{if both } e_i, a_j \text{ high (} e_i = a_j = 1) \\ q_{10} & \text{if high effort } e_i, \text{ but no help (} e_i = 1, a_j = 0) \\ q_{01} & \text{if low effort, but help (} e_i = 0, a_j = 1) \\ q_{00} & \text{if neither effort nor help (} e_i = a_j = 0) \end{cases} \]

Suppose the principal wants to implement high effort and help from both agents. The incentive compatibility constraints (IC) for each agent can then be written as follows (see the appendix for details). IC for not shirking own effort, but maintain help:

\[ q_{11} (\beta_{HH} - \beta_{LH}) + (1 - q_{11}) (\beta_{HL} - \beta_{LL}) \geq \frac{c}{(q_{11} - q_{01})} \]  

(ICe)

IC for not shirking help, but maintain own effort.

\[ q_{11} (\beta_{HH} - \beta_{HL}) + (1 - q_{11}) (\beta_{LH} - \beta_{LL}) \geq \frac{c_A}{(q_{11} - q_{10})} \]  

(ICa)

The left hand side (LHS) of ICe is the expected gain from obtaining high rather than low own output; the RHS is the cost per unit increase in the probability of success that follows when effort is increased. Condition ICa admits a similar interpretation, where the LHS is the expected gain to the agent when his partner realizes high rather than low output. In addition to these two constraints there is an IC constraint for not shirking both effort and help. We show in the appendix that this constraint is satisfied when the former two both hold.

As seems reasonable, we will assume here that helping effort is less productive than own effort, in the sense that the cost per unit increase in the

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7 Limited liability may arise from liquidity constraints or from laws that prohibit firms from extracting payments from workers.

8 With "shirking", we mean "low effort." With "effort and help" we mean both high own effort and high helping effort.
probability of success is highest for helping effort. For reasons that will become clear below, we will further invoke the technical assumption that this cost exceeds the sum \((c_A + c)/q_{11}\), so that we have

\[
\frac{c_A}{(q_{11} - q_{10})} > \max \left\{ \frac{c}{(q_{11} - q_{01})}, \frac{c_A + c}{q_{11}} \right\} \tag{A1}
\]

Now, using ICe in the expression (1) for the expected wage cost \(\omega^1\) for agent 1 we obtain (since \(p_1 = p_2 = q_{11}\))

\[
\omega^1 \geq p_1 \left( \frac{c}{q_{11} - q_{01}} \right) + p_2 (\beta_{LH} - \beta_{LL}) + \beta_{LL} = q_{11} \left( \frac{c}{q_{11} - q_{01}} \right) + q_{11} \beta_{LH} + (1 - q_{11}) \beta_{LL} \tag{2}
\]

Similarly, using ICa in the expression (1) for \(\omega^1\) yields

\[
\omega^1 \geq q_{11} \frac{c_A}{(q_{11} - q_{10})} + q_{11} \beta_{HL} + (1 - q_{11}) \beta_{LL} \tag{3}
\]

Due to limited liability \((\beta_{ij} \geq 0)\) and assumption A1 we then have

\[
\omega^1 \geq q_{11} \frac{c_A}{(q_{11} - q_{10})} \geq \omega_V(c_A, q)
\]

where \(\omega_V(c_A, q)\) is defined by the equality. Also, this lower bound for the wage cost can be attained by setting \(\beta_{LL} = \beta_{HL} = 0\), and by A1 it will exceed the total effort cost \(c + c_A\). In the appendix it is shown that this scheme also satisfies the IC condition for not shirking both effort and help. Thus we have:

**Lemma 1** Given assumption A1, then if output is verifiable the minimal wage cost (per agent) to implement effort & help is given by \(\omega_V(c_A, q)\). This minimal cost is attained for \(\beta_{LL} = \beta_{HL} = 0\) and ICa binding.

Note that a cost minimizing scheme has \(\beta_{LL} = 0\) and \(\beta_{HL} = 0\), hence it has the feature that an agent never gets a bonus if his partner has a bad outcome. This stimulates cooperation, and is the least costly way to do so when help is less productive than own effort. The bonus scheme has ICa binding (so \(q_{11} \beta_{HH} + (1 - q_{11}) \beta_{LH} = \frac{c_A}{q_{11} - q_{10}}\)) and must satisfy ICe (so \(q_{11} (\beta_{HH} - \beta_{LH}) \geq \frac{c}{q_{11} - q_{01}}\)). The latter naturally requires that an agent’s bonus when both he and his peer succeed must exceed his bonus when he
himself fails but his partner succeeds. We see from the IC conditions that these two bonuses are not completely pinned down, but that one feasible choice is to set $\beta_{LH} = 0$ and $q_{11} \beta_{HH} = \frac{c_A}{q_{11} - q_{10}}$.

**Case: additive probabilities.** It will be instructive to consider an additive structure where we have

$$q_{ij} = r_i + s_j, \quad \text{with} \quad r_1 - r_0 = r > 0 \quad \text{and} \quad s_1 - s_0 = s > 0 \quad (4)$$

This specification implies that the marginal productivity of help ($(q_{11} - q_{01})(Q_H - Q_L)$) is independent of the level of effort and vice versa. In this case assumption A1 is equivalent to assuming $\frac{c_A}{r} > \frac{s}{s_0}$, i.e. assuming that helping effort is less productive than own effort. This holds because we here have $q_{11} = r + s + q_00$ and thus $q_{11} \max \{ \frac{c_A}{r}, \frac{s}{s_0} \} \geq c + c_A$.

We will focus on cases where it is optimal for the firm to implement both effort and help when output is verifiable. The lemma shows that the profit generated by doing so is

$$\Pi_{11} = 2Q_L + 2 \left[ q_{11} \Delta Q - \omega_V(c_A, q) \right],$$

where $\Delta Q = Q_H - Q_L$. For this to be optimal the last term must be positive, and this profit must dominate the profit generated by just implementing effort without help; i.e.\(^9\)

$$\Pi_{11} \geq \Pi_{10} = 2Q_L + 2q_{10} \left[ \Delta Q - \frac{c}{(q_{10} - q_{00})} \right]$$

It must also dominate the profit generated by just implementing help without own effort, i.e.

$$\Pi_{11} \geq \Pi_{01} = 2Q_L + 2q_{01} \left[ \Delta Q - \frac{c_A}{(q_{01} - q_{00})} \right]$$

All this will hold if $\Delta Q$ is sufficiently large, or if $(q_{01} - q_{00})$ and $(q_{10} - q_{00})$ are both 'small' and $(q_{11} - q_{01})$ and $(q_{11} - q_{10})$ are both 'large', i.e. if effort and help are very productive together but not so productive in isolation. More formally we have:

**Lemma 2** For verifiable output, and given assumption A1, it is optimal to

\(^9\)An argument similar to that leading to (2) shows that the minimal cost to implement effort without help is $q_{10}c/(q_{10} - q_{00})$. 

8
implement effort & help when \( q_{11} \Delta Q > \omega_V(c_A, q) \), and in addition

\[
\Delta Q \geq \max \left\{ \frac{1}{q_{11} - q_{10}} \left( \omega_V - \frac{q_{10}c}{q_{10} - q_{00}} \right), \frac{1}{q_{11} - q_{01}} \left( \omega_V - \frac{q_{01}c_A}{q_{01} - q_{00}} \right) \right\}.
\]

For the additive model these conditions are equivalent to

\[
\Delta Q \geq \frac{c_A}{s} + \frac{r + q_{00}}{s} \left( \frac{c_A}{s} - \frac{c}{r} \right).
\]

2.2 Relational contracting

Assume now that output is non-verifiable. The incentive contract must then be self-enforcing, and thus ‘relational’ by definition. We consider a multilateral punishment structure where any deviation by the principal triggers low effort from both agents. The principal honors the contract only if both agents honored the contract in all previous periods. The agents honor the contract only if the principal honored the contract with both agents in all previous periods. A natural explanation for this is that the agents interpret a unilateral contract breach (i.e. the principal deviates from the contract with only one of the agents) as evidence that the principal is not trustworthy (see Bewley, 1999, and Levin, 2002).\(^{10}\)

The relational incentive contract is self-enforcing if, for all parties, the present value of honoring is greater than the present value of reneging. Ex post realizations of values, the principal can renege on the contract by refusing to pay the promised wage, while the agents can renege by refusing to accept the promised wage, and instead hold-up values and renegotiate what we can call a spot contract. The spot price is denoted \( \eta Q_k \). If values accrue directly to the principal, then \( \eta = 0 \). But if the agent is able to hold-up values ex-post, then \( \eta \) is determined by bargaining power, ex post outside options and the ability to hold-up values.\(^{11}\) Assume that there exists an alternative market for the agents’ output, and that the agents are able to independently realize values \( \theta Q_k, \theta \in (0, 1) \) ex post. If we assume Nash bargaining between principal and agents, each agent will then receive \( \theta Q_k \) plus a share \( \gamma \) from the surplus from trade i.e. \( \theta Q_k + \gamma(Q_k - \theta Q_k) = \eta Q_k \) where \( \eta = \gamma + \theta(1 - \gamma) \).\(^{12}\)

\(^{10}\)Modelling multilateral punishments is also done for convenience. Bilateral punishments will not alter our results qualitatively.

\(^{11}\)We take \( \eta \) as an exogenous parameter. In Kvaløy and Olsen (2008) we endogenize the agents’ hold-up power in a single-task model where relative performance evaluation is optimal.

\(^{12}\)It should be noted that the ability to hold-up values rests on the assumption that agents become indispensable in the process of production (as in e.g. Halonen, 2002).
We will assume that effort is not implementable in a spot contract, which is the case if \((q_{10} - q_{00})\eta\Delta Q < c\), i.e.

\[
\eta < \frac{c}{(q_{10} - q_{00})\Delta Q} \equiv \eta_s \tag{5}
\]

This implies that the agent’s surplus in the spot contract equals the spot price and is given by

\[
u_s = S = \eta Q_L + q_{00} \eta \Delta Q \tag{6}
\]

As in e.g. Baker, Gibbons and Murphy (2002), we analyze trigger strategy equilibria in which the parties enter into spot contracting forever after one party reneges. I.e. if the principal reneges on the relational contract, both agents insist on spot contracting forever after. And vice versa: if one of the agents (or both) renege, the principal insists on spot contracting forever after.

### 2.2.1 Relational contract constraints

Consider now the conditions for the incentive contract to be self-enforcing, i.e. the conditions for implementing a relational incentive contract. The relational incentive contract is self-enforcing if all parties honor the contract for all possible values of \(Q_k\) and \(Q_l\), \(k, l \in \{L, H\}\). The parties decide whether or not to honor the incentive contract ex-post realization of output, but ex-ante bonus payments. Agents are treated symmetrically, and thus receive the same contract (\(\beta\)) and obtain the same expected wage (\(\omega\)). The principal will honor the contract if

\[-\beta_{kl} - \beta_{lk} + \frac{\delta}{1 - \delta} \Pi^R \geq -\eta (Q_k + Q_l) + \frac{2\delta}{1 - \delta} [Q_L + q_{00} \Delta Q - S], \quad \text{all } k, l \in \{L, H\}, \tag{EP}\]

where \(\delta\) is the discount factor and \(\Pi^R\) is the principal’s profit in the relational contract. The LHS of the inequality shows the principal’s expected present value from honoring the contract, while the RHS shows the expected present value from reneging.

Each agent will honor the contract if

do not analyze the incentives to invest in firm-specific human capital (as in e.g Kessler and Lülfesmann, 2006). Rather, we just assume that agents become indispensable ex-post, and then focus on how this affects the multiagent moral hazard problem. We thus follow the relational contracting literature, and abstract from human capital accumulation. The expected output realization is therefore assumed to be constant each period. This allows us to concentrate on stationary relational contracts where the principal promises the same contingent compensation in each period.
\[ \beta_{kl} + \frac{\delta}{1 - \delta} (\omega - c - c_A) \geq \eta Q_k + \frac{\delta}{1 - \delta} u_s, \quad \text{all } k, l \in \{L, H\}, \quad \text{(EA)} \]

where similarly the LHS shows the agent’s expected present value from honoring the contract, while the RHS shows the expected present value from reneging.

In addition to fulfilling these enforceability conditions, the contract must also fulfill the IC conditions (ICe-a) for implementing help and effort.

2.2.2 Cost minimization

We will now consider the implications of the enforceability conditions above for the costs of implementing help and effort in a relational contract. As a first step we consider the minimal cost subject to EA, IC and limited liability \((\beta_{kl} \geq 0)\). We will show that this minimal cost as a function of the agents’ hold-up parameter \(\eta\) is a piecewise linear, continuous and increasing function. This shape reflects an increased tightening of the EA constraints as the agent’s hold-up power increases.

Using first \(\beta_{LL} \geq 0\) and EA for the bonus \(\beta_{HL}\) in (3) (with \(\omega^1 = \omega^2 = \omega\)) we get

\[ \omega \geq q_{11} \frac{c_A}{q_{11} - q_{10}} + q_{11} \left( \eta Q_H + \frac{\delta}{1 - \delta} [u_s - \omega + c + c_A] \right) \quad \text{(7)} \]

and hence, collecting terms involving \(\omega\):

\[ \omega \geq \left( \frac{q_{11}c_A}{q_{11} - q_{10}} + q_{11} \left( \eta Q_H + \frac{\delta}{1 - \delta} [u_s + c + c_A] \right) \right) \frac{1 - \delta}{1 - \delta + q_{11} \delta} \equiv \omega_m(\delta, \eta) \quad \text{(8)} \]

We see that \(\omega_m(\delta, \eta)\) defined here is a lower bound for the cost (per agent), and will be attained if the two constraints \(\beta_{LL} \geq 0\) and EA for the bonus \(\beta_{HL}\) both bind.

Next, using EA for bonuses \(\beta_{HL}\) and \(\beta_{LL}\) in (3) we obtain

\[ \omega \geq q_{11} \frac{c_A}{q_{11} - q_{10}} + q_{11} \eta \Delta Q + q_{11} \eta Q_L + \frac{\delta}{1 - \delta} [u_s - \omega + c + c_A] \quad \text{(9)} \]

and hence

\[ \omega \geq (1 - \delta) \left[ \frac{q_{11}c_A}{q_{11} - q_{10}} + q_{11} \eta \Delta Q + q_{11} \eta Q_L \right] + \delta [u_s + c + c_A] \equiv \omega_A(\delta, \eta) \quad \text{(10)} \]
The expression $\omega_A(\delta, \eta)$ defined here is also a lower bound for the cost, and will be attained if the EA constraints for the bonuses $\beta_{HL}$ and $\beta_{LL}$ both bind.

We have thus obtained two lower bounds for the wage payments that are necessary in order to induce a worker to exert effort on his own task as well as help to his colleague. Note that $\omega_A(\delta, \eta)$ and $\omega_m(\delta, \eta)$ are both increasing in $\eta$ (the outside value $u_s$ is also increasing in $\eta$), reflecting the effect that it generally becomes more costly to induce this behavior when the workers’ ex post hold-up power increases.

The cost $\omega_V(c, c_A, q)$ defined for the verifiable case is of course also a lower bound for wage costs in the present case. (This cost is derived from the IC and limited liability conditions, which must hold also in the present case.) So we must have $\omega \geq \max \{ \omega_V, \omega_m(\delta, \eta), \omega_A(\delta, \eta) \}$. We can show that the cost defined by this expression is indeed the minimal cost to induce effort and help, subject to IC and EA (and limited liability).

**Lemma 3** Given assumption A1, the minimal cost to implement effort and help, subject to IC and EA (and limited liability) is

$$\min_{IC,EA} \omega = \max \{ \omega_V, \omega_m(\delta, \eta), \omega_A(\delta, \eta) \} \equiv \omega_{11}(\delta, \eta)$$

With agent spot surplus $u_s = \eta Q_L + q_{00} \eta \Delta Q$ we have the following: For $\delta \in (0,1]$ there exists $\eta_a(\delta) > \eta_m(\delta) > 0$ such that

$$\omega_{11}(\delta, \eta) = \begin{cases} 
\omega_V = \frac{q_1 c_A}{\eta_1 - \eta_0} & \text{for } 0 \leq \eta \leq \eta_m(\delta) \\
\omega_m(\delta, \eta) & \text{for } \eta_m(\delta) < \eta \leq \eta_a(\delta) \\
\omega_A(\delta, \eta) & \text{for } \eta_a(\delta) < \eta
\end{cases}$$

(11)

Moreover, $\eta_a(\delta), \eta_m(\delta)$ are increasing in $\delta$ and satisfy: (i) $\eta_a(\delta), \eta_m(\delta) \to 0$ as $\delta \to 0$, and (ii) $\eta_a(1) < \eta_s$ if and only if

$$\eta_s Q_L > \eta_s (q_{11} - q_{00}) \Delta Q + [\omega_V - c - c_A]$$

(12)

The lemma confirms that the cost function is piecewise linear, continuous and increasing in $\eta$, and the reasoning preceding the lemma shows that this shape reflects increased tightening of the EA constraints as the agent’s hold-up power increases. For small $\eta$ ($\eta < \eta_m$) the cost minimizing bonus scheme for verifiable output does not violate any EA constraint, and neither of these constraints are therefore binding. Each agent gets a rent (since $\omega_V > c + c_A$),
and their spot surplus is so low that they are not tempted to renegotiate. This is the case even for the outcome pair \( Q_H, Q_L \), where the agent’s own output is high, but his bonus is \( \beta_{HL} = 0 \). But for \( \eta = \eta_m \) the EA constraint for this bonus just starts to bind. The principal is thus forced to modify the initial scheme, where an agent never gets a bonus if his partner fails, into a scheme where an agent gets a bonus if his partner fails, but the agent himself does well (\( \beta_{HL} > 0 \)).

The EA constraint for the bonus \( \beta_{HL} \) continues to bind for larger \( \eta \), and this implies increased wage costs for the principal, but it is the only binding EA constraint for \( \eta < \eta_a \). At this point the constraints start binding also for the outcomes where the agent’s own output is low. For \( \eta > \eta_a \) the EA constraints for the bonuses (\( \beta_{LH}, \beta_{LL} \)) associated with these outcomes are also binding, implying even higher wage costs.

Figure 1 gives a partial illustration of how the wage cost increases with increasing \( \eta \) for the case \( \eta > \eta_a \). As we have seen, in that case the EA constraints for the bonuses (\( \beta_{LH}, \beta_{LL} \)) are both binding, implying that these bonuses are equal (\( \beta_{LH} = \beta_{LL} \)). This implies (i) that the IC constraints can be written as functions of the two bonus differences \( \beta_{HH} - \beta_{LH} \) and \( \beta_{HL} - \beta_{LL} \), (ii) that the EA constraints then simply require that each of these bonus differences must exceed \( \eta \Delta Q \), and (iii) that the cost function (see (1)) also can be written as a function of these bonus differences plus a ‘fixed term’ involving \( \beta_{LL} \). These properties allow us to draw lines representing IC constraints, EA constraints and isocost curves as indicated in Figure 1.

Regarding the IC constraints, note that while the bonus \( \beta_{HH} \) stimulates
own as well as helping effort, the bonus $\beta_{HL}$ stimulates own effort, but discourages help. This implies that the lines representing ICE and ICa have negative and positive slopes, respectively. The intercept of ICa is larger due to the assumption that help is less productive than own effort. The bonuses that satisfy both IC constraints are then those on or above ICa.

By property (ii) the EA constraints here reduce to $\beta_{Hj} - \beta_{Lj} \geq \eta \Delta Q$, $j = H, L$, and can thus be represented by the L-shaped curve in the figure. The EA constraints here impose the intuitively reasonable requirement that an agent’s additional bonus for realizing a high rather than a low own output cannot be lower than the difference between the spot prices of these outputs. As $\eta$ increases, the EA curve will move outwards in the figure.

By property (iii) isocost lines (for fixed $\eta$) can be drawn as indicated in the figure; parallel to the ICE constraint. If there were no EA constraints, the lowest cost would be attained for the bonuses represented by point J, which corresponds (with $\beta_{LH} = \beta_{LL} = 0$) to the optimal solution for the verifiable case (see Lemma 1). The EA constraints imply that this solution is no longer feasible, and that the minimal cost will be realized at point A. This point represents a higher cost partly because the agent’s ‘fixed wage’ is higher ($\beta_{LH} = \beta_{LL} > 0$) and partly because the additional bonuses for high output are higher at point A than at point J. Both of these cost elements will increase with increasing $\eta$.

The cost characterized in Lemma 3 will be attainable for the principal if the associated bonuses also satisfy the EP conditions, so that the principal is not tempted to renegotiate ex post. These conditions are more easily satisfied, the larger is $\delta$. The minimal cost given in the lemma will therefore generally be attainable only if $\delta$ exceeds some critical level. We will return to this issue below.

### 2.3 Optimal relational contract

We now turn to optimal contracts, with a particular focus on the question of whether the optimal contract induces the agents to exert both own effort and helping effort, or just own effort. A contract inducing just own effort from the agents is less costly, and may be easier to implement. But such a contract will of course generate less gross value.

Given that the contract inducing help & effort can be implemented, the profit associated with this contract will be

$$\Pi^R_{11}(\delta, \eta) = 2Q_L + 2[q_{11}\Delta Q - \omega_{11}(\delta, \eta)]$$

Since the wage cost increases with $\eta$, the profit decreases with $\eta$. For $\eta = 0$
the EA constraints do not bind (we have \( S = u_s = 0 \) in this case), and the profit for the relational contract is then equal to the profit for the verifiable case (provided implementation, i.e. EP, is feasible). Thus we have

\[
\Pi_{11}^R(\delta, \eta) \leq \Pi_{11}, \quad \Pi_{11}^R(\delta, 0) = \Pi_{11} = 2Q_L + 2q_{11} \left[ \Delta Q - \frac{c_A}{(q_{11} - q_{10})} \right].
\]

The next-to-last equality here presumes that \( \delta \) is sufficiently large to make \( \omega_1 \) implementable, i.e. to make the associated bonuses compatible with EP.

Alternatively, the principal could seek to implement a contract with effort but no help. We can show (see the appendix) that the wage cost per agent for this contract is given by

\[
\omega_{10}(\delta, \eta) = \max \left\{ \frac{q_{10} c}{q_{10} - q_{00}}, \omega_0(\delta, \eta) \right\}
\]

where \( \frac{q_{10} c}{q_{10} - q_{00}} \) is the cost to implement effort (and no help) in the verifiable case, and

\[
\omega_0(\delta, \eta) = (1 - \delta) \left[ \frac{q_{10} c}{q_{10} - q_{00}} + \eta Q_L \right] + \delta [u_s + c],
\]

This holds provided that \( \delta \) is sufficiently large to make the associated bonuses implementable, i.e. compatible with EP. Given these provisions, the profit associated with this contract is

\[
\Pi_{10}^R(\delta, \eta) = 2Q_L + 2 \left[ q_{10} \Delta Q - \omega_{10}(\delta, \eta) \right]
\]

As above the profit decreases with \( \eta \) (because the cost \( \omega_0(\delta, \eta) \) is increasing in \( \eta \)), and we have (again provided implementation, i.e. EP is feasible):

\[
\Pi_{10}^R(\delta, \eta) \leq \Pi_{10}, \quad \Pi_{10}^R(\delta, 0) = \Pi_{10} = 2Q_L + 2q_{10} \left[ \Delta Q - \frac{c}{(q_{10} - q_{00})} \right]
\]

We will now investigate the conjecture that a contract inducing effort\&help is optimal for small \( \eta \), while a contract inducing only effort is optimal for large \( \eta \). This amounts to the following:

\[
\Pi_{11}^R(\delta, \eta) > \Pi_{10}^R(\delta, \eta) \quad \text{for 'small' } \eta \text{ (and } \Pi_{11}^R \text{ implementable)}
\]

\[
\Pi_{11}^R(\delta, \eta) < \Pi_{10}^R(\delta, \eta) \quad \text{for 'large' } \eta \text{ (or } \Pi_{11}^R \text{ not implementable)}
\]

A weak interpretation of the conjecture is to say that there are discount
factors where the first claim holds, and there are (possibly different) discount factors for which the second claim holds. We first provide two results that confirm this version of the conjecture. A stronger interpretation is to say that there is a set of discount factors such that both claims hold for each $\delta$ in this set. Our last result will confirm this stronger and more interesting version of the conjecture.

Consider first the case of small $\eta$. If the effort&help contract is implementable for $\eta = 0$ (or $\eta$ close to 0), then it is more profitable than the contract inducing effort alone if we just have

$$\Pi_{11} > \Pi_{10} \quad \text{i.e.} \quad q_{11} \left[ \Delta Q - \frac{c_A}{(q_{11} - q_{10})} \right] > q_{10} \left[ \Delta Q - \frac{c}{(q_{10} - q_{00})} \right]$$

(This inequality is implied by assumption A0.) We can now prove the following result.

**Proposition 1** Given $\Pi_{11} > \Pi_{10}$, then for all $\eta$ sufficiently small there is $\delta_0 < 1$ such that a contract inducing effort & help is implementable and optimal ($\Pi^R_{11}(\delta, \eta) > \Pi^R_{10}(\delta, \eta)$) for $\delta > \delta_0$.

This result confirms that a contract inducing effort and help dominates a contract inducing only own effort when $\eta$ is small, provided the parties are sufficiently patient. It is worth noting that the proposition also shows that high discount factors, which supports long-term relationships, facilitate agent-cooperation even when the agents cannot monitor or punish each other’s effort choices.

Proposition 1 is formulated for small $\eta$, but will in fact hold for any $\eta$, in the sense that for each $\eta$ one can find a critical discount factor such that the contract inducing effort and help is optimal for all $\delta$ exceeding this critical level. This can be seen by noting that in the limit as $\delta \to 1$, the wage cost for the effort&help contract converges to $u_s + c + c_A$ (see (8-10)), while the cost for the effort alone contract converges to $u_s + c$ (see (14)). The profit difference thus converges to $2 \left[ (q_{11} - q_{10}) \Delta Q - c_A \right]$, which is positive by assumption. Moreover, for $\delta$ sufficiently close to 1 the relational contract constraints (EA,EP) will be satisfied, and hence both contracts can be implemented. For large $\delta$, where implementation of a relational contract is not particularly challenging, the contract inducing effort and help thus remains optimal, also when the agents’ hold up power is large.

Having noted this, we move on to the case of small $\delta$, where implementation of a relational contract is more of a challenge. The smaller is $\delta$, the harder it generally is to implement a relational contract. We will show that
for large $\eta$ it becomes relatively harder to implement a contract inducing both effort and help than a contract inducing effort alone when $\delta$ becomes small.

Consider first the contract inducing own effort but no help. It follows from the analysis in Kvaløy and Olsen (2007) that this contract gets easier to implement as $\eta$ increases. In fact, the critical discount factor for implementation goes to zero as $\eta \to \eta_s$. This is quite intuitive, given that $\eta_s$ was defined as the minimal level of $\eta$ for which the spot market yields sufficient incentives for own effort, see (5). Bonuses equal to spot prices ($\beta_{kl} = \eta Q_k$) can be implemented for arbitrarily small $\delta$; they satisfy the EA and EP conditions for $\delta = 0$. And when these bonuses are sufficient to induce effort, which they are for $\eta = \eta_s$, then effort can be implemented at spot market cost for any $\delta \geq 0$.

Implementing effort and help, however, requires bonuses that deviate from spot prices. While spot prices here can give sufficient incentives to induce own effort, they give no incentives at all for helping effort. But when bonuses must deviate from spot prices, a minimal $\delta_c > 0$ is required in order to implement these bonuses in a relational contract. If this was not the case, implementable such bonuses would exist for arbitrarily small $\delta$, and hence in the limit ($\delta \to 0$) satisfy $\beta_{kl} + \beta_{lk} \leq \eta(Q_k + Q_l)$ from EP and $\beta_{kl} \geq \eta Q_k$ from EA, implying $\beta_{kl} = \eta Q_k$, i.e. bonuses equal to spot prices for all $k,l$. Hence bonuses different from spot prices cannot be implemented for $\delta < \delta_c$, for some critical discount factor $\delta_c > 0$.

These arguments show that the critical discount factor for implementing effort and help is bounded away from zero for all $\eta \leq \eta_s$, while the critical factor for implementing effort alone goes to zero as as $\eta \to \eta_s$. This means that there is some interval $(\eta_c, \eta_s)$ of 'large' $\eta$ where the critical discount factor for implementing effort alone is smaller than the critical factor for implementing both effort and help. Thus we have

**Proposition 2** There is $\eta_c < \eta_s$ such that for $\eta \in (\eta_c, \eta_s)$ effort alone can be implemented for $\delta \geq \delta_c(\eta)$ while effort & help can only be implemented for $\delta \geq \delta_c(\eta) > \delta(\eta)$

The proposition shows that for large $\eta$ there will be an interval of discount factors where only the contract inducing own effort can be implemented, and hence where this contract is optimal. Together with Proposition 1 this result confirms the weak version of our conjecture, saying that if the agents’ hold-up power $\eta$ is small there are discount factors for which effort&help is optimal, while if this power is large there are discount factors for which effort alone is optimal.
We now turn to the stronger and more interesting version of the conjecture, dealing with comparisons of the contracts for a fixed discount factor $\delta$. To this end we consider the profit difference $\Pi_{11}(\delta, \eta) - \Pi_{10}(\delta, \eta)$, and show that there are parameters such that for a set of discount factors this difference is positive when $\eta$ is small and negative when $\eta$ is large. From Proposition 1 and the following discussion it is clear that this can occur only for relatively small discount factors. For this reason we consider first the limiting case $\delta \to 0$.

Note that for $\delta$ small, the relevant cost functions for a given $\eta$ are $\omega_A(\delta, \eta)$ for the effort&help contract, and $\omega_0(\delta, \eta)$ for the effort alone contract. (This follows from Lemma 3 by noting that $\eta_s(\delta) \to 0$ as $\delta \to 0$, and from (14) by noting that $\omega_0(\delta, \eta) > \frac{q_{10}c}{q_{10} - q_{00}}$ for $\delta$ small.) For these cost functions we obtain, from (10) and (14);

$$\omega_A(\delta, \eta) - \omega_0(\delta, \eta) \to \left( \frac{q_{11}c_A}{q_{11} - q_{10}} + q_{11}\eta \Delta Q \right) - \frac{q_{10}c}{q_{10} - q_{00}} \quad \text{as} \quad \delta \to 0$$

Consider now situations where the agents’ hold up power, as represented by $\eta$, is large. So consider $\eta$ close to the upper bound $\eta_s$ introduced above, see (5). Noting that the definition of $\eta_s$ implies $q_{11}\eta_s \Delta Q = \frac{q_{11}c}{q_{10} - q_{00}}$, we see that for $\eta = \eta_s$ we have

$$\omega_A(\delta, \eta_s) - \omega_0(\delta, \eta_s) \to \frac{q_{11}c_A}{q_{11} - q_{10}} + \frac{(q_{11} - q_{10})c}{q_{10} - q_{00}} \quad \text{as} \quad \delta \to 0$$

and consequently

$$\left( \Pi_{11}(\delta, \eta_s) - \Pi_{10}(\delta, \eta_s) \right) \frac{1}{2} \to (q_{11} - q_{10}) \Delta Q - \left( \frac{q_{11}c_A}{q_{11} - q_{10}} + \frac{(q_{11} - q_{10})c}{q_{10} - q_{00}} \right) = D_0$$

(15)

We see that, for given probability and cost parameters, this profit difference is positive for $\Delta Q$ large, but negative otherwise. A large $\Delta Q$ will in this model imply that help as well as effort are quite productive. We have previously seen (Lemma 2) that a contract inducing effort and help is optimal in the verifiable case only if $\Delta Q$ is not too small, i.e. only if both effort and help are sufficiently productive. The interesting question now is therefore whether there is a range of intermediate $\Delta Q$’s such that effort and help is optimal in the verifiable case, but not optimal in the non-verifiable case, and in particular such that the profit difference is negative ($D_0 < 0$) while the assumptions of Lemma 2 still hold.

To examine this issue, consider first the additive specification (4), for
which we obtain
\[
\frac{D_0}{q_{11} - q_{10}} = \Delta Q - \left( \frac{1}{s} \left( s + r + q_{00} \right) c_A + \frac{c}{r} \right) = \Delta Q - \left( \frac{1 + r + q_{00}}{s} \right) \left( \frac{c_A}{s} + \frac{c}{r} \right)
\]

Comparing with the conditions in Lemma 2, we see that there is indeed a range of \(\Delta Q\)'s such that these conditions hold and yet \(D_0 < 0\). (Assumption A1 implies here \(\frac{c_A}{s} > \frac{c}{r}\), and the range is then defined by \(\frac{r + q_{00}}{s} \left( \frac{c_A}{s} - \frac{c}{r} \right) < \Delta Q - \frac{c}{r} < \frac{r + q_{00}}{s} \left( \frac{c_A}{s} + \frac{c}{r} \right).\) There is thus a range of intermediate \(\Delta Q\)'s for which a contract inducing effort and help is optimal when output is verifiable, but not necessarily so when output is non-verifiable.

We have so far not considered the implementability conditions EP for the principal. Now, in the discussion leading up to Proposition 2 we saw that for large \(\eta\) (close to \(\eta_s\)) implementation of effort alone is indeed feasible even for \(\delta\) very small. Based on this we can therefore conclude the following.

**Lemma 4** When
\[
\Delta Q < \frac{q_{11} c_A}{(q_{11} - q_{10})^2} + \frac{c}{(q_{10} - q_{00})}
\]
and (A0, A1) and (12) hold, there exists a \(\eta_1 < \eta_s\) such that for every \(\eta \in (\eta_1, \eta_s)\) there is an interval \((\delta(\eta), \delta(\eta))\) such that for \(\delta \in (\delta(\eta), \delta(\eta))\) we have \(\Pi_{R11}(\delta, \eta) < \Pi_{R10}(\delta, \eta)\).

The conditions in this lemma are not particularly strict. A0 and A1 are plausible assumptions, and (12) is compatible with the other conditions in the proposition, since it is the only condition involving \(Q_L\). This condition holds in addition to the other ones if \(Q_L\) is sufficiently large.

The lemma shows that if the agents’ hold up power \(\eta\) is high, then there are discount factor intervals where effort & help is dominated by effort alone. Using this lemma in combination with Proposition 1 we can verify our initial conjecture, and show that for a given discount factor, it is optimal to induce cooperation when \(\eta\) is small, but not so if \(\eta\) is large.

**Proposition 3** There is a set of parameters satisfying (A0, A1) and (12), and for which the following is true. There is an interval \((\delta_1, \delta_0)\) such that for \(\delta\) in this interval the contract inducing effort & help is optimal for \(\eta\) sufficiently small (\(\eta\) close to 0), while the contract inducing only own effort and no help is optimal for \(\eta\) sufficiently large (\(\eta\) close to \(\eta_s\)).
This proposition has an interesting corollary. Since own effort without help yields a lower social surplus than own effort and help together, a higher $\eta$ may reduce the social surplus:

**Corollary:** There is a set of parameters satisfying $(A0, A1)$ and $(12)$, and for which the following is true. There is an interval $(\delta_1, \delta_0)$ such that for $\delta$ in this interval the social surplus is smaller when $\eta$ is large ($\eta$ close to $\eta_s$) than when $\eta$ is small ($\eta$ close to 0).

This result is not in line with the established idea from the property rights approach that the investing parties should be the residual claimants. In our model - where the principal does not make any investment decisions - this principle would indicate that the social surplus should increase when the agents’ ex post share of value added ($\eta$) increases. But we see that the opposite happens here: If $\eta$ is sufficiently high, then social surplus suffers since the principal cannot implement efficient cooperation (helping effort).

If we interpret $\eta$ as proxy for asset ownership, where a high $\eta$ implies that the agents own assets, then the corollary has implications for the theory of the firm: It implies that if cooperation is valuable (and output is non-verifiable), then the firm and not the agents should own the assets (at least for some parameter configurations). The result is thus related to Holmström’s (1999) claim - building on Alchian and Demsetz (1972) - that firms will arise in situations where it is important to mitigate individual incentives and foster cooperative behavior.

The model also implies that when cooperation is important, the firm should be designed to dilute the agents’ hold-up power. This perspective complements the literature on human capital and the problems of expropriation, which focuses on how organizational design and incentive structure can affect the firm’s ability to protect strategic assets (see e.g. Liebeskind 2000; Rebitzer and Taylor 2007; and Rajan and Zingales 1998, 2001).

### 3 Concluding remarks

In so-called knowledge-intensive industries we often hear managers stress the importance of cooperation, team-work and knowledge sharing. And these claims are not only accompanied by dry complementarity arguments. The updated HR-manager would say that cooperation and helping-on-the job increase job satisfaction, and she will even find scientific support for her claim (Heywood et al. 2005). In contrast to these observations, empirical findings suggest that the use individual incentives, as opposed to team incentives, is higher in knowledge-intensive firms (see e.g. Long and Shields, 2005, and
Barth et al. 2006), and some empirical findings also suggest that people with more education are less satisfied with their job than people with lower levels of education (Clark and Oswald, 1996).\footnote{And the layman reads magazines about stress, burning-out and pushy behaviour in the high-skilled workforce.}

Our paper responds to these findings by showing that cooperation can be more costly to implement in human capital-intensive industries. The reason is that human capital blurs the allocation of ownership rights. As noted by Liebeskind (2000), if human-capital intensive firms are unable to establish intellectual property rights with respect to the ideas generated by their employees, they run the risk of being expropriated or held-up by their own employees. Our point is that this hold-up problem increases if the firm encourages cooperation between its employees, since the incentive regimes that are necessary to encourage cooperation are susceptible to employee hold-up.

As noted, a higher hold-up power, $\eta$, decreases not only the firm’s surplus, but also social surplus if it prevents the agents from helping each other. This contrasts with the standard property rights argument that the investing party (the agents in our paper) should own assets. We thus present a cost of providing agents with ownership rights that can be explored further within the modelling framework presented in this paper.

Finally, an interesting corollary that follows from the model is that long-term relationships foster cooperation between agents even if the agents cannot monitor or punish colleagues who free-ride, or refuse to cooperate. That is; a higher discount factor eases implementation of relational contracts, making it less costly for the principal to implement cooperation. This adds to the literature, since peer-monitoring has been more or less the "folk explanation" of why repeated interaction foster cooperation at the workplace.

Appendix

Proof of Lemma 1

We first demonstrate that ICe-a are the relevant IC conditions. From (1) and the definitions of the probabilities $q_{ij}$ the condition for not shirking own effort is:

$$q_{11} [q_{11} (\beta_{HH} - \beta_{LH}) + (1 - q_{11}) (\beta_{HL} - \beta_{LL})] + q_{11} (\beta_{LH} - \beta_{LL}) - c - c_A$$

$$\geq q_{01} [q_{11} (\beta_{HH} - \beta_{LH}) + (1 - q_{11}) (\beta_{HL} - \beta_{LL})] + q_{11} (\beta_{LH} - \beta_{LL}) - c_A$$

A little algebra shows that this is equivalent to (ICe). Similarly, the condition
for not shirking help;

\[ q_{11} [q_{11} (\beta_{HH} - \beta_{LL}) + (1 - q_{11}) (\beta_{HL} - \beta_{LL})] + q_{11} (\beta_{LL} - \beta_{HH}) - c - c_A \]

\[ \geq q_{11} [q_{10} (\beta_{HH} - \beta_{LL}) + (1 - q_{10}) (\beta_{HL} - \beta_{LL})] + q_{10} (\beta_{LL} - \beta_{HH}) - c, \]

is seen to be equivalent to ICa.

We next show that a joint deviation, i.e. shirking both own effort and helping effort, is not profitable for the agent. This holds if

\[ q_{11} [q_{11} (\beta_{HH} - \beta_{LL}) + (1 - q_{11}) (\beta_{HL} - \beta_{LL})] \] (IC-ae)

\[ + q_{11} (\beta_{LL} - \beta_{HH}) - c - c_A \]

\[ \geq q_{10} [q_{10} (\beta_{HH} - \beta_{LL}) + (1 - q_{10}) (\beta_{HL} - \beta_{LL})] + q_{10} (\beta_{LL} - \beta_{HH}) \]

We have from first ICa and then ICe above:

\[ q_{11} [q_{11} (\beta_{HH} - \beta_{LL}) + (1 - q_{11}) (\beta_{HL} - \beta_{LL})] + q_{11} (\beta_{LL} - \beta_{HH}) - c - c_A \]

\[ \geq q_{11} [q_{10} (\beta_{HH} - \beta_{LL}) + (1 - q_{10}) (\beta_{HL} - \beta_{LL})] + q_{10} (\beta_{LL} - \beta_{HH}) - c \]

\[ \geq (q_{11} - q_{10}) [q_{10} (\beta_{HH} - \beta_{LL}) + (1 - q_{10}) (\beta_{HL} - \beta_{LL})] + q_{10} (\beta_{LL} - \beta_{HH}) \]

\[ - (q_{11} - q_{10}) [q_{11} (\beta_{HH} - \beta_{LL}) + (1 - q_{11}) (\beta_{HL} - \beta_{LL})] \]

\[ = (q_{11} - q_{10}) (q_{10} - q_{11}) [(\beta_{HH} - \beta_{LL}) - (\beta_{HL} - \beta_{LL})] \]

\[ + q_{11} [q_{10} (\beta_{HH} - \beta_{LL}) + (1 - q_{10}) (\beta_{HL} - \beta_{LL})] + q_{10} (\beta_{LL} - \beta_{HH}) \]

Since \((q_{11} - q_{10}) (q_{10} - q_{11}) < 0\) we see that IC-ae will indeed hold if \((\beta_{HH} - \beta_{LL}) - (\beta_{HL} - \beta_{LL}) \geq 0\). Now, the cost-minimizing bonuses satisfy this condition since they satisfy \(\beta_{LL} = \beta_{HL} = 0\) and from ICe \(\beta_{HH} - \beta_{LL} > 0\). This proves that IC-ae holds.

Assumption A1 further assures participation, since it implies \(\omega_V > c + c_A\). This proves the lemma.

**Proof of Lemma 2.**

Condition A0 is just a different way of writing \(\Pi_{11} \geq \Pi_{10}\) and \(\Pi_{11} \geq \Pi_{01}\). For the additive model (4), the assumption \(\frac{c_A}{q_{11} - q_{10}} > \frac{c}{q_{11} - q_{01}}\) is equivalent to \(\frac{c_A}{s} > \frac{c}{t}\). Then we have \(\omega_V = \frac{c_A}{s}q_{11}\), and the condition \(q_{11}\Delta Q > \omega_V\) is then equivalent to \(\Delta Q > \frac{c_A}{s}q_{11}\). Condition A0 is now

\[ \Delta Q \geq \max \left\{ \frac{1}{s} \left( \frac{q_{11} c_A}{s} - \frac{q_{10} c}{r} \right), \frac{1}{r} \left( \frac{q_{11} c_A}{s} - \frac{q_{01} c_A}{s} \right) \right\} \]
Using \( q_{11} = s + q_{10} = r + q_{00} \), this is equivalent to

\[
\Delta Q \geq \max \left\{ \frac{1}{s} \left( c_A + q_{10} \left( \frac{c_A}{s} - \frac{c}{r} \right) \right), \frac{1}{r} \left( r c_A \right) \right\} = \frac{c_A}{s} + \frac{q_{10} \delta}{s} \left( \frac{c_A}{s} - \frac{c}{r} \right)
\]

which coincides with the condition stated in the lemma, since \( q_{10} = r + q_{00} \).

**Proof of Lemma 3.**

The proof entails showing that the asserted minimum cost can be attained by nonnegative bonuses that satisfy IC and EA. We first prove (11).

By construction of the functions \( \omega_m(\delta, \eta) \) and \( \omega_A(\delta, \eta) \) they satisfy, respectively, (7) and (9) with equalities, thus;

\[
\omega_m(\delta, \eta) = \omega_V + q_{11} \left[ \eta \Delta Q + \eta Q_L + \frac{\delta}{1-\delta} [u_s - \omega_m(\delta, \eta) + c + c_A] \right], \quad (16)
\]

\[
\omega_A(\delta, \eta) = \omega_V + q_{11} \eta \Delta Q + \eta Q_L + \frac{\delta}{1-\delta} [u_s - \omega_A(\delta, \eta) + c + c_A]. \quad (17)
\]

Hence we have \( \omega_m(\delta, \eta) = \omega_V \) for \( \eta = \eta_m > 0 \) that solves

\[
\eta \Delta Q + \eta Q_L + \frac{\delta}{1-\delta} [u_s(\eta) - \omega_V + c + c_A] = 0 \quad (18)
\]

Substituting for \( u_s(\eta) = \eta Q_L + q_{00} \eta \Delta Q \) this yields

\[
\eta_m = \frac{\delta [\omega_V - c - c_A]}{Q_L + (1-\delta) \Delta Q + \delta q_{00} \Delta Q} > 0
\]

Since \( \omega_m(\delta, \eta) \) is increasing (linearly) in \( \eta \), we have \( \omega_m(\delta, \eta) > \omega_V \) iff \( \eta > \eta_m \).

Similarly we see from (17) that we have \( \omega_A(\delta, \eta) = \omega_V \) for \( \eta = \eta_a \) given by \( q_{11} \eta \Delta Q + \eta Q_L + \frac{\delta}{1-\delta} [u_s(\eta) - \omega_V + c + c_A] = 0 \). Comparing with (18) we see that, since \( q_{11} < 1 \), this yields \( \eta_a > \eta_m \), and hence \( \omega_A(\delta, \eta) < \omega_V \) for \( \eta < \eta_m \).

We now claim that \( \omega_A(\delta, \eta) = \omega_m(\delta, \eta) \) for the unique \( \eta = \eta_a \) that solves \( \omega_A(\delta, \eta) = \omega_V + q_{11} \eta \Delta Q \), i.e. for \( \eta = \eta_a \) that solves (see (17))

\[
\eta Q_L + \frac{\delta}{1-\delta} [u_s(\eta) - (\omega_V + q_{11} \eta \Delta Q) + c + c_A] = 0 \quad (19)
\]

The claim is verified by noting from (16) that this \( \eta \) also solves \( \omega_m(\delta, \eta) = \omega_V + q_{11} \eta \Delta Q \), and hence solves \( \omega_m(\delta, \eta) = \omega_A(\delta, \eta) \). Substituting for \( u_s(\eta) = \)
\( \eta Q_L + q_{00} \eta \Delta Q \) in (19) we obtain
\[
\eta_a = \frac{\delta [w_V - c - c_A]}{Q_L - \delta (q_{11} - q_{00}) \Delta Q} \quad \text{(for } Q_L - \delta (q_{11} - q_{00}) \Delta Q > 0)\]

We have here tacitly assumed \( Q_L - \delta (q_{11} - q_{00}) \Delta Q > 0 \); otherwise we will have \( \omega_m(\delta, \eta) > \omega_A(\delta, \eta) \) for all \( \eta > 0 \).

We see that \( \eta_a > \eta_m \), that \( \eta_a \) and \( \eta_m \) are both increasing in \( \delta \), and that \( \eta_a < \eta_s \) for \( \delta = 1 \) iff (12) holds. This proves (11) and the ensuing statement in the lemma.

Now we will show that the asserted minimum cost can be attained by nonnegative bonuses that satisfy IC and EA.

First, for \( \eta \leq \eta_m \) let the bonuses \( \beta_{kl} \) be given by the optimal scheme for verifiable output. This scheme satisfies IC and yields wage cost \( \omega_V = \frac{q_{11} c_A}{q_{11} - q_{10}} > 0 \). The scheme has nonnegative bonuses \( \beta_{HH} > \beta_{LH} \geq \beta_{HL} = \beta_{LL} = 0 \) and satisfies EA, since we for \( \eta \leq \eta_m \) by definition of \( \eta_m \) (see (18)) have
\[
\eta \Delta Q + \eta Q_L + \frac{\delta}{1 - \delta} [u_s(\eta) - \omega_V + c + c_A] \leq 0 \leq \beta_{kl}
\]
This shows that for \( \eta \leq \eta_m \) the lower bound \( \omega_V \) is attainable.

For \( \eta > \eta_m \) the EA constraint is violated if \( \beta_{HL} = 0 \), hence the above scheme is no longer feasible. Note that by definition of \( \omega_m(\delta, \eta) \), a set of bonuses will yield wage cost \( \omega = \omega_m(\delta, \eta) \) if (i) ICa is binding, which yields equality in (3), and (ii) \( \beta_{LL} = 0 \) and EA binds for \( \beta_{HL} \), which yields equality in (8). Define such a set of bonuses, specifically; let \( \beta_{LH} = \beta_{LL} = 0 \), and let \( \beta_{HL}, \beta_{HH} \) be given by EA and ICa; thus
\[
\beta_{HL} = \eta \Delta Q + \eta Q_L + \frac{\delta}{1 - \delta} [u_s(\eta) - \omega_m(\delta, \eta) + c + c_A] \quad \text{(EA_m)}
\]
\[
q_{11} (\beta_{HH} - \beta_{HL}) = \frac{c_A}{(q_{11} - q_{10})} \quad \text{(ICA)}
\]
These bonuses then yield cost \( \omega = \omega_m(\delta, \eta) \). The bonus \( \beta_{HL} \) satisfies EA by construction, and since \( \beta_{HH} > \beta_{HL} \), so does \( \beta_{HH} \). From the definition of \( \beta_{HL} \) and (16) we see that \( \omega_m(\delta, \eta) - \omega_V = q_{11} \beta_{HL} \), and hence that \( \beta_{HL} > 0 \),
since \( \eta > \eta_m \). Moreover, the bonuses satisfy ICe, since we have

\[
q_{11} (\beta_{HH} - \beta_{ LH}) + (1 - q_{11}) (\beta_{HL} - \beta_{LL}) = q_{11} \left( \beta_{HL} + \frac{c_A}{(q_{11} - q_{10})} q_{11} \right) + (1 - q_{11}) \beta_{HL} > \frac{c_A}{(q_{11} - q_{10})}
\]

and \( \frac{c_A}{q_{11} - q_{10}} > \frac{c_E}{q_{11} - q_{01}} \) by assumption A1.

It remains to verify that \( \beta_{LH} = \beta_{LL} = 0 \) satisfy EA. We show that this is the case for \( \eta \leq \eta_a \). Recall that \( \omega_m(\delta, \eta) \leq \omega_V + q_{11} \eta \Delta Q \) for \( \eta \leq \eta_a \), and hence that (16) then implies

\[
\eta Q_L + \frac{\delta}{1 - \delta} [u_s - \omega_m(\delta, \eta) + c + c_A] \leq 0
\]

This shows that \( \beta_{LH} = \beta_{LL} = 0 \) satisfy EA for \( \eta \leq \eta_a \). Hence we have shown that for \( \eta_m < \eta \leq \eta_a \) there is a set of non-negative bonuses that satisfies EA and IC, and which yields wage costs \( \omega = \omega_m(\delta, \eta) \) on this interval.

Finally consider \( \eta > \eta_a \). We now derive a set of bonuses that yield the cost \( \omega_A(\delta, \eta) \), satisfy EA for all outcomes, and satisfy IC. The first requirement follows by definition of \( \omega_A(\delta, \eta) \) once ICa is binding and EA binds for the bonuses \( \beta_{LL} \) and \( \beta_{HL} \). So define the bonuses as follows:

\[
\beta_{LL} = \beta_{LH} = \eta Q_L + \frac{\delta}{1 - \delta} [u_s - \omega_A(\delta, \eta) + c + c_A], \quad \beta_{HL} = \eta \Delta Q + \beta_{LL}
\]

(\( \text{EA}_A \))

\[
q_{11} (\beta_{HH} - \beta_{HL}) + (1 - q_{11}) (\beta_{HL} - \beta_{LL}) = \frac{c_A}{(q_{11} - q_{10})}
\]

(\( \text{IC}_A \))

This yields \( \beta_{HH} = \beta_{HL} + \frac{c_A}{q_{11} - q_{01}} > \beta_{HL} = \eta \Delta Q + \beta_{LL} \), and shows that \( \beta_{HH} \) also satisfies EA.

To verify that the bonuses are positive, note from the definition of \( \beta_{LL} \) and (17) that we have \( \omega_A(\delta, \eta) = \omega_V + q_{11} \eta \Delta Q + \beta_{LL} \). This shows that \( \beta_{LL} > 0 \), since we have \( \omega_A(\delta, \eta) > \omega_V + q_{11} \eta \Delta Q \) for \( \eta > \eta_a \). (We showed that \( \omega_A(\delta, \eta) = \omega_V + q_{11} \eta \Delta Q \) for \( \eta = \eta_a \), and the inequality then follows from linearity and \( \omega_A(\delta, \eta) < \omega_V \) for \( \eta \) small.)

It then only remains to verify that the given bonuses satisfy ICe. We
have now

$$q_{11} (\beta_{HH} - \beta_{LH}) + (1 - q_{11}) (\beta_{HL} - \beta_{LL})$$

\[= q_{11} \left( \beta_{HL} + \frac{c_A}{(q_{11} - q_{10}) q_{11}} - \beta_{LH} \right) + (1 - q_{11}) \eta \Delta Q \]

\[= \frac{c_A}{q_{11} - q_{10}} + \eta \Delta Q \]

which exceeds $$\frac{c}{q_{11} - q_{01}}$$ according to assumption A1. The given bonuses thus satisfy IC, they are positive and satisfy EA, and they yield the cost $$\omega_A(\delta, \eta)$$ for $$\eta > \eta_a$$. This completes the proof.

**Proof of (13).**

By an argument similar to that leading to (2) one sees that the cost to implement effort alone (with no help) must satisfy

$$\omega_{10} \geq \frac{q_{10} c}{q_{10} - q_{00}} + q_{11} \beta_{LH} + (1 - q_{11}) \beta_{LL}$$

(20)

Limited liability ($$\beta_{kl} \geq 0$$) shows that $$\omega_{10} \geq \frac{q_{10} c}{q_{10} - q_{00}}$$. Substituting next from the EA constraints for the bonuses $$\beta_{LH}$$ and $$\beta_{LL}$$ (with $$\omega = \omega_{10}$$ and $$c_A = 0$$) in (20) we obtain

$$\omega_{10} \geq \frac{q_{10} c}{q_{10} - q_{00}} + \eta Q_L + \frac{\delta}{1 - \delta} (u_s - \omega_{10} + c)$$

Collecting terms involving $$\omega_{10}$$ yields the inequality $$\omega_{10} \geq \omega_0(\delta, \eta)$$ with $$\omega_0(\delta, \eta)$$ defined in (14). This proves (13).

**Proof of Proposition 1.**

Consider the limiting case $$\eta = 0$$. Then the EA constraints do not bind, and the optimal bonuses for the verifiable case can be implemented if they satisfy EP. From Lemma 1 and assumption A1 these bonuses satisfy $$\beta_{LL} = \beta_{HL} = 0$$ and ICa binding, hence we have $$q_{11} \beta_{HH} + (1 - q_{11}) \beta_{LH} = \frac{c_A}{q_{11} - q_{01}}$$. In addition ICe holds, i.e. $$q_{11} (\beta_{HH} - \beta_{LH}) \geq \frac{c}{q_{11} - q_{01}}$$.

These bonuses are easiest to implement when $$\beta_{HH}$$ is minimal, which is obtained when ICe binds. This yields

$$\beta_{HH} - \beta_{LH} = \frac{c}{q_{11} (q_{11} - q_{01})}, \quad \beta_{LH} = \frac{c_A}{(q_{11} - q_{10})} - \frac{c}{(q_{11} - q_{01})}$$

(21)
For η = 0 EP takes the following form

\[ \beta_{ki} + \beta_{lk} \leq \frac{2\delta}{1 - \delta} \left[ \frac{1}{2} \Pi_{11} - QL - q_{00} \Delta Q \right] \]

(By assumption the latter square bracket, which equals \( q_{11} \left[ \Delta Q - \frac{c_{11}}{(q_{11} - q_{00})} \right] - q_{00} \Delta Q \) is positive.) For the bonuses given above we have \( \beta_{HH} > \beta_{LH} > \beta_{LL} = 0 \). Hence EP for outcome HH is the critical condition for implementation, thus we must have

\[ 2\beta_{HH} \leq \frac{2\delta}{1 - \delta} \left[ \frac{1}{2} \Pi_{11} - QL - q_{00} \Delta Q \right] \] (22)

Substituting for \( \beta_{HH} \), we see that there is a critical \( \delta_0 < 1 \) such that this condition holds for all \( \delta > \delta_0 \). The other EP conditions are then also satisfied, hence we have shown that the bonuses that yield wage costs \( \omega_V \) and profits \( \Pi_{11} \) are implementable for \( \delta > \delta_0 \). This proves the proposition for \( \eta = 0 \). By continuity the result will also hold for \( \eta > 0 \) sufficiently close to zero.

**Proof of Lemma 4.**

First note that condition (12) ensures that for \( \eta \) close to \( \eta_s \) the cost function for the contract inducing effort and help is given by \( \omega_A(\delta, \eta) \) for all \( \delta \leq 1 \). (More precisely; taking also EP into account, that effort and help can not be implemented at a cost lower than \( \omega_A(\delta, \eta) \).) This can be seen from Lemma 3, since (12) implies that \( \eta_a(\delta) \leq \eta_a(1) < \eta_s \), and hence that for \( \eta_a(1) < \eta < \eta_s \) the minimal cost is given by \( \omega_A(\delta, \eta) \) for all \( \delta < 1 \).

Now consider the comparison of the two contracts. Regarding the effort-alone contract, it follows from the analysis in Kvaløy and Olsen (2007) that the following statement holds true. There is \( \eta_0 < \eta_s \) such that for every \( \eta \in (\eta_0, \eta_s) \) there is a critical \( \bar{\delta}(\eta) \) such that effort-alone can be implemented at minimal cost \( \omega_0(\delta, \eta) \) for \( \delta \geq \bar{\delta}(\eta) \), and moreover that \( \bar{\delta}(\eta) \to 0 \) as \( \eta \to \eta_s \).

For completeness we verify this statement below.

Taking the statement for granted, consider \( \eta > \max\{\eta_0, \eta_a(1)\} \), where the relevant costs are \( \omega_A(\delta, \eta) \) and \( \omega_0(\delta, \eta) \), respectively. Define

\[ D(\eta) \equiv \Pi_{11}^R(\bar{\delta}(\eta), \eta) - \Pi_{10}^R(\bar{\delta}(\eta), \eta). \]

Since \( \bar{\delta}(\eta) \to 0 \) as \( \eta \to \eta_s \), it follows from (15) that \( D(\eta) \to D_0 < 0 \) as \( \eta \to \eta_s \). Hence by continuity there is \( \eta_1 < \eta_s \) such that for every \( \eta \in (\eta_1, \eta_s) \) we have \( D(\eta) < 0 \). For such a \( \eta \), we thus have \( \Pi_{11}^R(\delta, \eta) < \Pi_{10}^R(\delta, \eta) \) for \( \delta = \bar{\delta}(\eta) \).

Hence by continuity there is \( \bar{\delta}(\eta) > \bar{\delta}(\eta) \) such that \( \Pi_{11}^R(\delta, \eta) < \Pi_{10}^R(\delta, \eta) \) for \( \delta \in (\bar{\delta}(\eta), \bar{\delta}(\eta)) \). This verifies the statement in Lemma 4.
We finally verify the claim regarding the effort-alone contract stated above. As noted in the text, the minimal wage cost, subject to EA and IC for the effort-alone contract is \( \omega_0(\delta, \eta) \) defined in (14), provided that this cost \( \omega_0(\delta, \eta) \) exceeds \( \frac{q_{10} - q_{00}}{c_{10} - c_{00}} \), which is the cost to implement effort (and no help) in the verifiable case. Substituting \( u_s = \eta(Q_L + q_{00}\Delta Q) \) in (14) we obtain

\[
\omega_0(\delta, \eta) - \frac{q_{10}c}{q_{10} - q_{00}} = \eta Q_L - \delta \left( \frac{c}{q_{10} - q_{00}} - \eta \Delta Q \right) q_{00}
\]

This expression is positive for all \( \delta < 1 \) if \( \eta > \eta_0 = \frac{c}{q_{10} - q_{00}} \frac{1}{\Delta Q} \). We see that \( \eta_0 < \eta_s = \frac{c}{q_{10} - q_{00}} \Delta Q \). Hence for \( \eta \in (\eta_0, \eta_s) \) we have \( \omega_0(\delta, \eta) > \frac{q_{10} - q_{00}}{c_{10} - c_{00}} \) for all \( \delta < 1 \), and \( \omega_0(\delta, \eta) \) is then indeed the minimal cost to implement effort-alone, subject to the relevant IC and EA constraints. Moreover, the discussion leading up to Proposition 2 showed that the critical discount factor \( \delta(\eta) \) for implementing the effort-alone contract converges to zero when \( \eta \to \eta_s \). (Kvaløy and Olsen (2007) shows that this critical factor is given by \( \frac{1}{\delta-1} = k/\left[c - \eta \Delta Q (q_{10} - q_{00}) \right] \) where \( k \) is independent of \( \eta \).) This verifies the claim, and thus completes the proof.

**Proof of Proposition 3.**

From Proposition 1 we know that there is a critical \( \delta(\eta) < 1 \) such that for \( \eta \) sufficiently small and \( \delta > \delta(\eta) \) the contract inducing effort and help is implementable and optimal;

\[
\Pi_{11}^R(\delta, \eta) - \Pi_{10}^R(\delta, \eta) > 0 \quad \text{for} \quad 0 \leq \eta < \eta_1 \text{ and } \delta > \delta(\eta).
\] (23)

Letting \( \eta \to 0 \), then by continuity \( \delta(\eta) \to \delta_0 \), where \( \delta_0 \) is the critical factor corresponding to \( \eta = 0 \) defined in the proof of Proposition 1. From that proof (see (22)) we have \( \delta_0 \) defined by

\[
\beta_{HH} \delta_0 \left[ 1 - \frac{1}{2} \Pi_{11} - Q_L - q_{00} \Delta Q \right] = \delta_0 \left[ (q_{11} - q_{00}) \Delta Q - \frac{q_{11} c_A}{q_{11} - q_{10}} \right]
\]

where we have substituted for \( \Pi_{11} \), and where \( \beta_{HH} \) is given by (see (21));

\[
\beta_{HH} = \frac{c}{q_{11} (q_{11} - q_{01})} + \beta_{LH} = \frac{c}{q_{11} (q_{11} - q_{01})} + \frac{c_A}{(q_{11} - q_{10})} - \frac{c}{(q_{11} - q_{01})}
\]

\[
= \frac{(1 - q_{11})c}{q_{11} (q_{11} - q_{01})} + \frac{c_A}{(q_{11} - q_{10})}
\]

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The critical factor $\delta_0$ is thus given by the following equation

\[
\left[ \frac{(1-q_{11})c}{q_{11}(q_{11}-q_{01})} + \frac{c_A}{(q_{11}-q_{10})} \right] = \frac{\delta_0}{1-\delta_0} \left[ (q_{11}-q_{00})\Delta Q - \frac{q_{11}c_A}{(q_{11}-q_{10})} \right]
\]

(d0)

From Lemma 4 we know that, under the stated assumptions the contract inducing effort only is implementable and optimal for $\eta$ sufficiently large, and for $\delta \in (\bar{\delta}(\eta), \tilde{\delta}(\eta))$:

\[
\Pi_{11}^R(\delta, \eta) - \Pi_{10}^R(\delta, \eta) < 0 \quad \text{for} \quad \eta_2 < \eta < \eta_s \quad \text{and} \quad \delta \in (\bar{\delta}(\eta), \tilde{\delta}(\eta)).
\]

(24)

Here $\bar{\delta}(\eta)$ is the critical factor for implementing the 'only effort' contract, and we know that $\bar{\delta}(\eta) \to 0$ as $\eta \to \eta_s$. Since $\Pi_{11}^R(\delta, \eta) - \Pi_{10}^R(\delta, \eta)$ is linear in $\delta$, and positive for $\delta = 1$, it must be the case that (the largest) $\bar{\delta}(\eta)$ is defined by $\Pi_{11}^R(\delta, \eta) - \Pi_{10}^R(\delta, \eta) = 0$ for $\delta = \bar{\delta}(\eta)$. Hence, letting $\eta \to \eta_s$, then by continuity $\bar{\delta}(\eta) \to \bar{\delta}_1$ defined by $\Pi_{11}^R(\bar{\delta}_1, \eta_s) - \Pi_{10}^R(\bar{\delta}_1, \eta_s) = 0$, i.e. by

\[
(q_{11} - q_{10})\Delta Q - (\omega_A(\bar{\delta}_1, \eta_s) - \omega_0(\bar{\delta}_1, \eta_s)) = 0
\]

(d1)

The proof is then complete if we show that (for a set of parameters) $\bar{\delta}_1 > \delta_0$, because for given $\delta \in (\delta_0, \bar{\delta}_1)$ we can then by continuity find $\eta_1 > 0$ and $\eta_2 < \eta_s$ such that both (23) and (24) hold for the given $\delta$.

Consider the equation (d1) defining $\bar{\delta}_1$. Using first (10) and (14), and then $\eta_s \Delta Q = -\frac{c}{q_{10} - q_{00}}$ we obtain

\[
\omega_A(\delta, \eta_s) - \omega_0(\delta, \eta_s) = (1-\delta) \left[ \frac{q_{11}c_A}{q_{11} - q_{10}} + q_{11}\eta_s\Delta Q - \frac{q_{10}c}{q_{10} - q_{00}} \right] + \delta [c_A]
\]

\[
= (1-\delta) \left[ \frac{q_{11}c_A}{q_{11} - q_{10}} + (1-q_{11})\frac{(q_{11} - q_{10})c}{q_{10} - q_{00}} \right] + \delta [c_A]
\]

Substituting this in the equation (d1) defining $\bar{\delta}_1$, this equation becomes

\[
(q_{11} - q_{10})\Delta Q - \left( \frac{q_{11}c_A}{q_{11} - q_{10}} + \frac{(q_{11} - q_{10})c}{q_{10} - q_{00}} \right) + \delta_1 \left[ \frac{(q_{11} - q_{10})c}{q_{10} - q_{00}} + \frac{q_{10}c_A}{(q_{11} - q_{10})} \right] = 0
\]

(d1)

We will consider the additive model (4). The equations defining $\delta_0$ and

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\( \delta_1 \) then take the following form

\[
\left[ \frac{1 - q_{11} c}{q_{11} r} + \frac{c_A}{s} \right] = \frac{\delta_0}{1 - \delta_0} \left[ (r + s) \Delta Q - \frac{q_{11} c_A}{s} \right]
\]

(d0)

\[
s\Delta Q - \left( \frac{q_{11} c_A}{s} + \frac{sc}{r} \right) + \delta_1 \left[ \frac{sc}{r} + \frac{q_{10} c_A}{s} \right] = 0
\]

(d1)

where \( q_{11} = r + s + q_{00} \) and \( q_{10} = r + q_{00} \), and the assumptions A0 and A1 entail

\[
\frac{c_A}{s} > \frac{c}{r} \quad \text{and} \quad \frac{r + q_{00}}{s} \left( \frac{c_A}{s} - \frac{c}{r} \right) < \Delta Q - \frac{c_A}{s} < \frac{r + q_{00} c_A}{s} + \frac{c}{r}
\]

(A)

Condition (12) involves \( Q_L \), and can be fulfilled independently of the other conditions. For the additive model we have \( \eta_s = \frac{c}{r} \Delta Q \), and we see that condition (12) is then (for \( \frac{c_A}{s} > \frac{c}{r} \)) equivalent to

\[
Q_L > (r + s) \Delta Q + \left[ q_{11} \frac{c_A}{s} - c - c_A \right] / \eta_s = (r + s) \Delta Q + \left[ (r + q_{00}) \frac{c_A}{s} / \frac{c}{r} - r \right] \Delta Q
\]

(25)

Define

\[
\gamma = \frac{\Delta Q}{c_A / s} > 1, \quad \alpha = \frac{c/r}{c_A / s} < 1
\]

and note that \( q_{10} = r + q_{00} = q_{11} - s \). Then the conditions defining \( \delta_0 \) and \( \delta_1 \) above are

\[
s\gamma - (q_{11} + sa) \+ \delta_1 [s\alpha + (q_{11} - s)] = 0
\]

(d1)

\[
\left[ \frac{1 - q_{11}}{q_{11}} \alpha + 1 \right] = \frac{\delta_0}{1 - \delta_0} [(r + s)\gamma - q_{11}]
\]

(d0)

where

\[
1 > \alpha \quad \text{and} \quad \frac{q_{11} - s}{s} (1 - \alpha) < \gamma - 1 < \frac{q_{11} - s}{s} + \alpha
\]

(A)

We will now show that, keeping \( q_{11}, \gamma \) and \( r \) fixed, then for \( s \) sufficiently small there is \( \alpha \) close to 1 such that (A) holds and \( 0 < \delta_0 < \delta_1 \). To see this, let \( \alpha \to 1 \) and \( s \to 0 \) such that \( \frac{\alpha}{s} \leq k \), where \( q_{11} k < \gamma - 1 \), and \( \gamma > q_{11}/r \). Then we obtain

\[
\delta_1 = \left( \frac{q_{11} + sa}{s\alpha + q_{11} - s} \right) \to 1
\]

\[
\delta_0 = \left[ \frac{\frac{1 - q_{11}}{q_{11}} \alpha + 1}{\frac{1 - q_{11}}{q_{11}} \alpha + 1} + [(r + s)\gamma - q_{11}] \right] \to 1 + [(r\gamma - q_{11}) q_{11}]
\]

Moreover, condition (A) will clearly hold for \( s \) small and \( \alpha \) close to 1 since
\[
\frac{q_{11}-s}{s}(1-\alpha) \leq q_{11}k + (1-\alpha) < \gamma - 1 \text{ and } \frac{q_{11}-s}{s} + \alpha \to \infty.
\]
This completes the proof.

Remark. The following is an example of parameters that yield \(\delta_0 < \delta_1\):

\[
q_{11} = 0.9, \quad r = 0.45, \quad s = 0.1, \quad \gamma = \frac{\Delta Q}{c_A/s} = 3, \quad \alpha = \frac{c/r}{c_A/s} = 0.9
\]

For these parameters we find \(\delta_0 = 0.746\) and \(\delta_1 = 0.775\). Checking condition (A), we see that \(\frac{q_{11}-s}{s}(1-\alpha) = 0.8 < \gamma - 1 < \frac{q_{11}-s}{s} + \alpha = 8.9\), and hence this condition is satisfied.

References


