Price-dependent Profit-Sharing as a Channel Coordination Device

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Abstract: We show how an upstream firm by using a price-dependent profit-sharing rule can prevent destructive competition between downstream firms that produce relatively close substitutes. With this rule the upstream firm induces the retailers to behave as if demand has become less price elastic. As a result, competing downstream firms will maximize aggregate total channel profit. When downstream firms are better informed about demand conditions than the upstream firm, the same outcome cannot be achieved by alternative vertical restraints such as resale price maintenance (RPM). Price-dependent profit-sharing may also ensure that the downstream firms undertake market expanding investments. The model is consistent with observations from the market for content commodities distributed by mobile networks.

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1 Introduction

The Bertrand paradox may provide a plausible explanation why the majority of the content commodities on the internet are offered for free (marginal costs). The rival is "one click away", and competing content providers have strong incentives to undercut each other as long as there are positive profit margins.

In recent years mobile phone operators have allowed downstream content providers to sell content commodities like ringtones, football goal alerts and jokes to the mobile subscribers. Similar to the internet, the entry barriers for providers of content commodities are low, and the rival is just "one click away" also for mobile content commodities. However, in contrast to what we have observed on the internet, mobile content commodities are not offered for free. End-user prices are well above marginal costs.

One potential explanation why the Bertrand paradox is not observed for such goods, is the price-dependent profit-sharing rule used by some upstream mobile providers. With this rule each content provider decides the end-user price for the good he sells, but he has to pay a share of the end-user price to the upstream firms in order to get access to the customers on the mobile networks. The crucial feature of the rule is that it is progressive, in the sense that the share maintained by the content provider is increasing in the end-user price. Table 1 shows the profit-sharing rule used by the dominant Norwegian mobile operator Telenor. If a content provider sells his good for NOK 3, say, he receives 62% of the revenue, while he only receives 45% of the revenue if he reduces the price to NOK 1.

<table>
<thead>
<tr>
<th>End-user price (NOK)</th>
<th>1.0</th>
<th>1.5</th>
<th>3</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share to the content provider</td>
<td>45%</td>
<td>54%</td>
<td>62%</td>
<td>66%</td>
<td>68%</td>
<td>70%</td>
<td>80%</td>
</tr>
</tbody>
</table>

Table 1: A price-dependent profit-sharing rule used for content messages downloaded by mobile phones.

A progressive profit-sharing rule implies that the opportunity cost of setting a low end-user price is relatively high, and therefore reduces the incentives to engage in fierce price competition. More specifically, in the formal model we show how an
upstream firm can use such a rule to reduce the content providers' undercutting incentives by lowering their perceived elasticity of demand. Thereby the upstream firm can prevent destructive price competition. Even more interestingly, we show that a progressive profit-sharing rule achieves higher aggregate channel profit than a market structure where the upstream firm directly dictates the end-user prices (e.g. through retail price maintenance, RPM). This is true if we make the realistic assumption that the content providers are better informed about the demand for their goods than is the upstream firm (asymmetric information).

There exists a sizeable literature on how vertical restraints can help solve channel coordination problems. Our model is related to this literature in that we consider a context where one manufacturer sells an input to several downstream retailers, and where some of the variables cannot be controlled and observed by the manufacturer. Such lack of control may give rise to horizontal and vertical externalities. If the manufacturer cannot control retailers’ sales effort, this may give rise to under- or overprovision of effort compared to the level which maximizes channel profit (e.g. Telser, 1960). Retailers’ undercutting incentives may lead to too low retail prices if the manufacturer lacks control of retail prices. One strand of literature focuses on how to find the minimum number of vertical restraints sufficient to maximize the total channel profit. Mathewson and Winter (1984) show how a combination of a two-part tariff and RPM may be used to achieve the integrated channel outcome where retailers undertake market expanding sales effort with potential spillovers. Lal (1990) shows that revenue-sharing may be used as an additional instrument to a two-part tariff in a context where both the manufacturer and the retailer undertake non-contractible sales efforts (see also Rao and Srinavasan,1995).

Rey and Tirole (1986) emphasize that both the private and social desirability of a given vertical restraint depend on the underlying delegation problem. They compare RPM and exclusive territories (ET) under uncertainty about demand or cost. Our starting point, too, is the underlying delegation problem; the retailers have more accurate demand information than the manufacturer. We also follow Rey and Tirole (op cit) in that we do not search for the minimum sufficient number of vertical restraints inducing the same profit outcome as under channel integration. Rather
we show how the price-dependent profit sharing rule may be used to suppress the competing retailers undercutting incentives, and, furthermore, that this restraint may be superior to alternatives such as RPM.

In contrast to our approach, Lal (1990) and recent papers like Cachon and Lariviere (2005), Dana and Spier (2001), and Mortimer (2006) consider a revenue sharing scheme that specifies fixed rather than price-dependent shares to the manufacturer and the retailer (e.g. 60% to the manufacturer and 40% to the retailer). Like our paper, Cachon and Lariviere (2005), Dana and Spier (2001), and Mortimer (2006) are motivated by observed contracts. These papers focus on revenue-sharing contracts implemented in the video rental industry, and show how revenue-sharing schemes may be used to solve channel coordinating problems related to inventory choices.

In the next section we present a case study of how the price-dependent profit-sharing rule has been used in practise, and in Section 3 we set up a formal model to show how an optimal profit-sharing rule may induce competing content providers to choose end-user prices that maximize aggregate channel profit. In Section 4 we extend the model to allow each downstream firm to undertake non-contractible market expanding investments (e.g. marketing) with potential spillovers, and Section 5 concludes.

2 A price-dependent profit-sharing rule - used in practice

Despite an awkward user interface, text-messaging has been an overwhelming success in Europe and Asia.2 The average usage per month by customers in several European countries exceeded sixty messages in 2004.3 In several markets, person-to-person

\footnote{By typing 7777 44 2 555 555 0 9 33 0 4 666 0 666 88 8 0 333 666 777 0 2 0 3 777 444 66 55 1111 on your Nokia mobile phone, you would be sending a text-message asking your friend “Shall we go out for a drink?”.

2There is a striking discrepancy between Europe and the United States with respect to the take up of text messaging. “No text please, we’re American” is the headline in The Economist (2003) when focusing on this feature.

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messaging has been followed by a successful deployment of content messaging, which enables the mobile users to buy different types of content such as ringtones, music, logos, alerts (e.g. goal alerts), jokes, quizzes and games, directory enquiries and so forth.

In 1997, in the infancy of the market, the two Norwegian mobile providers Telenor and NetCom introduced content messaging services like news, stock quotes and weather forecasts. The mobile access providers themselves decided which types of services that should be offered and they also took care of end-user pricing. However, this model of vertical integration did not seem to work very well; the services generated limited revenues and profit.

In 2000, the two mobile providers voluntarily shifted strategy from in-house development and production of content to one of vertical separation. With this business model independent content providers behave as downstream firms ("retailers") responsible for sales effort, marketing, and end-user pricing, while the mobile providers act as upstream firms providing access to the customers (the mobile subscribers) as an input. The mobile providers offer take-it-or-leave-it wholesale contracts, specifying a menu of end-user prices among which the content providers may choose (ranging from NOK 1 to NOK 60). Moreover, the wholesale contract specifies the revenue split between the mobile provider and the content provider, where the share to the content provider increases with the end-user price (c.f. Table 1 above).

Note that there is no competition between the mobile providers in the upstream market for content messaging. In order to gain access to Telenor’s customers, a provider of content message services needs an agreement with Telenor, and, similarly, the content provider needs an agreement with NetCom in order to reach NetCom’s customers. We have observed a high degree of cooperation between NetCom and Telenor.\textsuperscript{4} In April 2000 the two mobile network providers launched what to a

\textsuperscript{4}One example is the introduction of common shortcodes. It is important for the content provider to have the same number from all the mobile operators to facilitate marketing to all users. One of the most important content message has been TV-related text-messaging where viewers vote and send comments. For such services it is important that the providers offer common shortcodes.
large extent was a common wholesale concept towards content message providers. The outcome is that every mobile phone subscriber may access the same content messaging services at the same price independent of which provider they subscribe to. In the formal model below, we consequently assume that there is an upstream monopoly selling access to a large number of independent retailers.

Content messaging became a success, and in 2004 the mobile customers on average bought 15 content messages per month in Norway, and the total revenue generated from content messaging (NOK 1 billion) was approximately 15% of the revenues from mobile voice traffic. Vertical separation through delegation of retail activities such as retail pricing and marketing has been considered as a key feature behind the success. The Norwegian business model with delegation of content provision to independent firms is now widely adopted in Europe and Asia (Strand, 2004).

The motivation behind the mobile providers’ delegation of retail pricing and marketing was that small and independent content providers appeared to have superior hands-on market knowledge (Nielsen and Aanestad, 2006). Consequently, there is a potential gain from delegation since decisions on marketing, retail pricing and introduction of new services may be based on more accurate demand information when undertaken by independent content providers rather than the mobile providers themselves. In the formal model below, we thus assume that the source of the delegation problem is that independent downstream firms have more accurate demand information than has the upstream firm.

By providing a standard interface and allowing for free entry for content providers, the mobile environment resembles what we have observed in the internet. As Shapiro and Varian (1998) put it: “Any idiot can establish a Web presence – and lots of them have”. In 2004, approximately 50 different companies were active in providing content messaging services in Norway (Nielsen and Aanestad, 2006). Due to

\[\text{(four-digit numbers) for all subscribers. NetCom and Telenor offered common shortcodes from 2000, while common shortcodes were not offered before 2002 in the majority of other European countries. Common shortcodes have probably been the most important factor for the take-off of TV-related text messages (Economist, 2002).}\]
low entry barriers and the fact that the services may easily be replicated by rivals, the vast majority of the content messaging services may be considered as commodities. However, a remarkable difference from the internet is that competition among providers of content messaging services has not driven prices down to marginal costs. In Figure 1 we have the monthly average prices for content messages in the period March 2000-July 2002.5

![Figure 1: Average prices for mobile content services. Source: Telenor](image)

It is interesting to note that income from content messaging in the Norwegian mobile networks in 2004 was twice as high as the revenues from internet ads.6 Since content commodities are offered for free in the internet, advertising is the only revenue source for the majority of internet content providers. Our conjecture is that

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5In this period, we have monthly data on the total revenue from content messaging and the number of content messages bought by Telenor’s customers. The average price is then just calculated from total revenue/number of messages. We have no data on content messages bought by the customers of the other Norwegian mobile provider NetCom. However, since the content providers charge the same end-user price independent of which of the two mobile providers the customer subscribes to, it seems reasonable to assume that the pattern in Figure 1 holds for the total market. Moreover, Telenor had a market share of approximately 70% in this period.

6Calculated from statistics from the Norwegian Post and Telecommunications Authority.
the gross willingness to pay is significantly higher for content commodities available on the internet than for mobile content commodities like ringtones and jokes. As total revenues are higher for mobile content commodities than for internet content commodities, this indicates that a significantly higher share of the potential channel profit is extracted from mobile content commodities than from internet content commodities.

3 The model

We consider an upstream firm selling access to distribution facilities to \( n \) downstream firms. The demand curve faced by downstream firm \( i = 1, \ldots, n \) is given by \( q_i = q_i(a, p) \), where \( a \) is a demand parameter and \( p \) is the vector of prices charged by the \( n \) downstream firms.\(^7\) The demand parameter \( a \) is known by the downstream firms when they set end-user prices. The upstream firm knows that \( a \) is distributed on the interval \([a, \pi]\), but does not observe the exact level of \( a \). Later on we shall let \( a^e \) denote the upstream firm’s expectation about the demand. We assume that the demand functions are well behaved and downward sloping in own price \((\partial q_i/\partial p_i < 0)\). The consumers perceive the goods sold by the downstream firms as imperfect substitutes \((\partial q_i/\partial p_j > 0)\).

Marginal costs both at the upstream and downstream levels are set equal to zero; this does, however, not matter for the qualitative results. Hence, we can write total operating profit in the industry as

\[
\Pi = \sum_{i=1}^{n} p_i q_i(a, p).
\]  

Below, we consider a two-stage game where the upstream firm at stage 1 determines the wholesale conditions, and where the downstream firms subsequently compete in prices. Later, we shall investigate the consequences of allowing the downstream firms to make market-expanding investments.

\(^7\)With linear demand curves \( a \) is simply the intercept with the price axis.
The upstream firm uses a profit-sharing rule where downstream firm $i$ keeps a share $\beta(p_i)$ of its operating profit, while the upstream firm gets the share $(1 - \beta(p_i))$. The literature conventionally assumes that the revenue share is a constant; i.e. $\beta' = 0$ (see e.g. Lal, 1990). However, below we show that it is optimal for the upstream firm to have $\beta' > 0$, which means that the share accruing to each downstream firm is increasing in its end-user price. We label this as a price-dependent profit-sharing rule.

**Stage 2**

The operating profit of downstream firm $i$ equals $\pi_i = \beta(p_i)q_i$, and at the last stage each firm solves $p_i^* = \text{arg max} \pi_i$. This yields the FOCs

$$
\left[ q_i^* + p_i^* \frac{\partial q_i}{\partial p_i} \right] + \frac{\beta'(p_i^*)}{\beta(p_i^*)} p_i^* q_i^* = 0.
$$

(2)

The second term in (2) would vanish if $\beta$ were constant ($\beta' = 0$), in which case we would get the standard result that a profit maximizing price $\hat{p}_i$ satisfies $[\hat{q}_i + \hat{p}_i \frac{\partial q_i}{\partial p_i}] = 0$. With $\beta' > 0$ the second term on the left-hand side of equation (2) is positive, implying that the marginal profit at any given price is higher than if $\beta' = 0$. This induces each of the downstream firms to behave less aggressively, and we can state:

**Proposition 1:** The downstream firms’ profit-maximizing prices are higher for $\beta'(p_i) > 0$ compared to $\beta'(p_i) = 0$.

By defining $\varepsilon_{ii} \equiv \frac{p_i}{q_i} \frac{\partial q_i}{\partial p_i}$ as the price elasticity of demand for good $i$, and $\eta \equiv \frac{\beta'(p_i)}{\beta(p_i)} p_i$ as the elasticity of the profit share with respect to downstream firm $i$’s price, we can rewrite (2) as

$$
\varepsilon^*_{ii} + \eta^*_{i} = -1
$$

(3)

Equation (3) characterizes the profit-maximizing equilibrium price for firm $i$. It is well known that revenue - and thus profit for a firm facing zero marginal costs - other things equal, is maximized by choosing a price for which the elasticity is equal to minus one. However, since $\eta_i > 0$ for $\beta' > 0$, we see from (3) that the given profit
sharing rule induces the downstream service provider to behave as if the demand has become less price elastic. Hence, profit maximizing prices will be higher if $\beta' > 0$.

**Proposition 2:** A price-dependent profit-sharing rule $\beta'(p_i) > 0$ reduces the perceived elasticity of demand for the downstream firms, making them behave less aggressively.

In the sequel we assume an isoelastic sharing rule so that $\beta(p_i) = \theta p_i^\lambda$, where $\lambda$ is the elasticity parameter determined by the upstream firm at stage 1. If $\lambda = 0$, we have the conventional constant sharing rule where $\beta = \theta$ and $\beta' = 0$. As long as $\lambda > 0$ we have a price-dependent rule where $\beta'(p_i) > 0$. In absence of any fixed costs the downstream firms’ participation constraints are satisfied for any $\theta \geq 0$. With this specification we can reformulate (2) and (3) as

$$
(1 + \lambda) q^*_i + p^*_i \frac{\partial q_i}{\partial p_i} = 0 \quad (4)
$$

$$
\varepsilon^*_i + \lambda = -1 \quad (5)
$$

**Stage 1**

The upstream firm will use $\lambda$ to induce the downstream firms to set prices that maximize total channel profit. To find the optimal value of $\lambda$ we first derive the hypothetical equilibrium with vertical integration (VI) and complete information about the demand parameter $a$. Solving $p_i = \arg\max \Pi(p)$ yields the FOCs

$$
\left[ q_i + p_i \frac{\partial q_i}{\partial p_i} \right] + \sum_{j \neq i} p_j \frac{\partial q_j}{\partial p_i} = 0 \quad (i = 1, ..., n). \quad (6)
$$

The term in the square bracket of (6) measures the marginal profit on good $i$ and is analogous to the term in the square bracket of (2). The second term of (6) internalizes the horizontal pecuniary externality when products are imperfect substitutes; other things equal, each downstream firm has incentives to set a relatively low end-user price in order to steal business from its competitors. Since the size of this business-stealing effect is larger the less differentiated the downstream goods are, we shall now introduce $\omega^p_{ji}$ as a measure of the degree of substitutability between services offered by the downstream firms:
\[ \omega_{ji} = -\frac{\partial q_j}{\partial p_i} / \frac{\partial q_i}{\partial p_i} \tag{7} \]

Hence, \( \omega_{ji} \) measures the increased demand for good \( j \) per unit reduction in the demand for good \( i \) when \( p_i \) increases. The larger this ratio, the higher \( p_i \) should be set in order to maximize aggregate channel profit. The challenge for the upstream firm in a vertically separated market structure is to set access conditions that induce the downstream firms to internalize this effect at stage 2.

Inserting for \( \omega_{ji} \) into (6) we can now characterize industry optimum as

\[ q_i + \left[ p_i - \sum_{j \neq i} p_j \omega_{ji} \right] \frac{\partial q_i}{\partial p_i} = 0. \quad (i = 1, ..., n). \]

By imposing symmetry this expression can be reformulated as (with subscript \( VI \) for vertical integration)

\[ q_{VI} + p_{VI} \left[ 1 - (n - 1) \omega_{ji} \right] \frac{\partial q_i}{\partial p_i} = 0. \tag{8} \]

The optimal value of \( \lambda \) ensures that aggregate profit is the same in the vertically separated market structure as in the hypothetical equilibrium with vertical integration. This value can be found by using equations (4) and (8) and setting \( q_i^*/p_i^* = q_{VI}/p_{VI} \). We then have

\[ \lambda = \lambda^* \equiv -1 + \frac{1}{1 - (n - 1) \omega_{ji}}. \tag{9} \]

Thus, the upstream firm only needs information about the degree of substitutability, measured by (7), and not about the accurate level of \( a \). Inserting for (9) into (5) we further find

\[ \varepsilon_{ii}^* = -\frac{1}{1 - (n - 1) \omega_{ji}}. \]

At the industry optimum \( \frac{\partial \pi_i}{\partial p_i} < 0 \). Hence, there is a trade-off between reducing profit in firm \( i \) and increasing profits in the firms other than \( i \). The parameter \( \lambda \) given by (9) strikes the optimal balance.

\[ ^8 \text{Setting } q_i^*/p_i^* = q_{VI}/p_{VI} \text{ uniquely determines the prices, since } q_i/p_i \text{ is monotonically decreasing in } p_i \text{ when } \partial q_i/\partial p_i < 0. \]
If a price reduction of good $i$ does not affect demand for good $j$, we have $\frac{\partial q_j}{\partial p_i} = \omega_{ji}^p = 0$. The downstream firms thus choose prices such that $\epsilon_{ii}^* = -1$, which is optimal also from the industry’s point of view ($\lambda^* = 0$). However, if the goods are imperfect substitutes (such that $\frac{\partial q_j}{\partial p_i} > 0$), each downstream firm fully internalizes the effect its price has on the profit of the other firms when $\lambda = \lambda^* > 0$. Hence, the downstream firms will not engage in destructive price competition even if they produce close substitutes, and the Bertrand paradox is avoided:

**Proposition 3:** The profit-sharing rule $\beta(p_i) = \theta p_i^\lambda$ with $\lambda = \lambda^*$ solves the Bertrand paradox, and induces downstream prices that maximize aggregate channel profit.

As long as the horizontal pecuniary externality is the only problem to solve, we see from (9) that $\theta$ has no impact on the outcome. Consequently, $\theta$ may be used as a profit distribution instrument. Alternatively, the upstream firm could require a franchising fee. However, $\theta$ may be a superior way of redistributing profit. If the franchising fee must be specified at stage 1 by the upstream firm, then the size of the franchising fee must be based on $a^e$. In contrast, if $\theta$ specifies the share to the downstream firm, the upstream firm does not need to know $a$ at stage 1. It is sufficient that it is able to monitor the revenues.

What about other types of vertical restraints? The source of the problem is that the downstream firms know $a$, while the upstream firm only has an expectation $a^e$ about demand. The novelty of the price-dependent profit-sharing rule is its ability to ensure that competing downstream firms individually choose end-user prices which maximize total channel profit. The combination of delegation of retail pricing and the price-dependent profit-sharing rule is thus more effective than alternatives that do not imply delegation of end-user pricing, such as RPM. The present proposal is also more effective than several other alternatives even if these entail delegation of retail pricing. The most obvious example is one where the upstream firms set a unit wholesale price that may deviate from the marginal costs. By increasing the unit wholesale price above the marginal costs (which are zero in the present model), the downstream firms will increase end-user prices. However, analogous to RPM,
the upstream firm must use the expected value \( a^e \) instead of the true value of \( a \) in calculating the unit wholesale price, and the outcome achieved by the current sharing rule cannot be achieved. Thus, we have the following result:

**Proposition 4:** Assume that only the downstream firms know the accurate level of \( a \). The profit-sharing rule \( \beta(p_i) = \theta p_i^2 \) is then superior to vertical restraints (such as RPM) that require the upstream firm to know \( a \) in order to achieve maximum channel profit.

It should be noted that an efficient implementation of exclusive clauses (exclusive dealing or exclusive territory) may resemble the current outcome. However, in many markets it is difficult to enforce exclusive contracts, and such exclusive contracts imply that the upstream firm picks the firms/services that will be allowed to enter the retail market. Such restrictions on entry will in many circumstances have significant disadvantages. In fact, in the case of content messaging discussed above, one of the key features behind the success seems to be that there are no such restrictions on entry. The strategy of letting a thousand flowers bloom has ensured a wide variety of services which has made the system attractive for the consumers and profitable for the industry.

To clearly see the intuition behind the result in Proposition 4, we now use the following Shubik-Levitan (1980) utility function:

\[
U(q_1, \ldots, q_i, \ldots, q_n) = a \sum_{i=1}^{n} q_i - \frac{n}{2} \left( (1 - s) \sum_{i=1}^{n} q_i^2 + \frac{s}{n} \left( \sum_{i=1}^{n} q_i \right)^2 \right). \tag{10}
\]

The parameter \( a > 0 \) in equation (10) is a measure of the market potential, \( q_i \) is the quantity from retailer \( i \), and \( n \geq 2 \) the number of retailers. The parameter \( s \in [0, 1) \) is a measure of how differentiated the services are; from the consumers’ point of view they are closer substitutes the higher is \( s \). The merit of using this particular utility function is that the size of the market does not vary with \( s \).

Solving \( \partial U/\partial q_i - p_i = 0 \) for \( i = 1, \ldots, n \), we find

\[9\] Others using the Shubik-Levitan framework to analyze vertical restraints include Shafer (1991) and Motta (2004).
\[ q_i = \frac{1}{n} \left( a - \frac{p_i}{1 - s} + \frac{s}{(1 - s) n} \sum_{j=1}^{n} p_j \right). \]  \hspace{1cm} (11)

When marginal costs are zero, it is straightforward to show that the price that maximizes total channel profits equals \( p_i = a/2 \) for \( i = 1, \ldots, n \). Thus, in order to set the optimal price, the upstream firm needs global information about the demand as given by the parameter \( a \). Absent perfect information the best the upstream firm can do if it uses RPM or other alternatives which do not delegate end-user pricing to the downstream firms, is consequently to set \( p_i = a^e/2 \). However, with the sharing rule \( \beta_i(p_i) = \theta p_i^\lambda \), it follows from (7) and (9) that in order to induce the optimal end-user price \( p_i = a/2 \) the upstream firm only needs local information about demand as given by \( \omega^p_{ji} \), the degree of substitutability. By using (7) and (11), we find that \( \omega^p_{ji} = s/(n - s) \). The optimal level of \( \lambda \) then becomes \( \lambda^* = s(n - 1)/n(1 - s) \) from equation (9).

4 Market-expanding investments

We now extend the model to allow each downstream firm at the second stage to undertake non-contractible market-expanding (or quality-enhancing) investments with potential spillovers.\(^{10}\) At the outset, it is not clear how one firm’s investments affect sales and profits of the other firms. The investing firm’s product will typically become relatively more attractive than those of the rivals. Thereby the latter could be harmed. However, there might also be technological or marketing spillovers from an investment such that one firm’s investment is to the benefit of all the downstream firms. A given firm’s marketing of ringtones, for instance, is also likely to benefit other firms selling ring tone services. We thus open up for both positive and negative spillovers from investments, and let the downstream profit function of firm \( i \) be given by

\[ \pi_i = \beta(p_i) p_i q_i(a, p, x) - \varphi(x_i), \] \hspace{1cm} (12)

\(^{10}\)If we had considered contractible investments, it could be natural to assume that this activity takes place at stage 1. Non-contractible investments, on the other hand, should be modelled as taking place in the last stage, since it has no commitment value.
where the new variable $x$ denotes the vector of market-expanding investments undertaken by the $n$ downstream firms, and $\varphi(x_i)$ is the investment cost function. The more a firm invests, the higher is the demand it faces; $\partial q_i / \partial x_i > 0$. Investments thus increase the size of the market beyond the initial exogenous market size $a$. We assume that $\varphi'(x_i) > 0$, and that it is sufficiently convex to satisfy all second-order conditions for a profit maximum.

The upstream firm determines the access conditions at stage 1, and at stage 2 the downstream firms decide non-cooperatively on end-user prices and investment levels. As above, we consider the isoelastic sharing-rule $\beta(p_i) = \theta p_i^\lambda$, where $\theta \geq 0$. At stage 2 the first-order condition $\partial \pi_i / \partial p_i = 0$ is still given by equation (4), which for convenience is repeated here:

$$ (1 + \lambda) q_i^* + p_i^* \frac{\partial q_i}{\partial p_i} = 0. $$

Since $\theta$ does not enter into this first-order condition, we argued in the previous section that it did not have any strategic value. Thus $\theta$ could be used as a pure profit distribution parameter, with no influence on industry performance. This is no longer true when the downstream firms can make market expanding investments, as we then have:

$$ \frac{\partial \pi_i}{\partial x_i} = \theta (p_i^*)^{\lambda + 1} \frac{\partial q_i}{\partial x_i} - \varphi'(x_i^*). \quad (13) $$

Downstream firm $i$’s marginal profitability of investing is thus strictly increasing in $\theta$. If the upstream firm sets $\theta \approx 0$ it will capture nearly the entire channel profit, but the downstream firms will have very small incentives to make market expanding investments. By marginally increasing $\theta$, the size of the pie increases, but the upstream firm will capture a smaller share of it. This does not mean that the size of the pie is uniformly increasing in $\theta$; if the firms invest too much the marginal benefit will be smaller than the marginal costs. In general, we might therefore expect aggregate channel profit to be a hump-shaped function of $\theta$, so that there is a unique value of $\theta$ maximizing total industry profit.

As for now, let us abstract from uncertainty and let us further assume that the upstream firm can use lump-sum transfers (fixed fees) to redistribute profits. It will
then choose $\lambda$ and $\theta$ to maximize aggregate channel profit, which is now given by

$$\Pi = \sum_{i=1}^{n} [p_i q_i(a, p, x) - \varphi(x_i)]. \quad (14)$$

To find the optimal value of $\theta$, we note that solving $\partial \Pi / \partial x_i = 0$ yields

$$p_i \frac{\partial q_i}{\partial x_i} + \sum_{j \neq i} p_j \frac{\partial q_j}{\partial x_i} = \varphi'(x_i) \quad (i = 1, ..., n). \quad (15)$$

If there are no investment spillovers the term $\partial q_j / \partial x_i$ is negative, and more so the closer horizontal substitutes are the goods. This effect is not taken into account by independent downstream firms, and tends to generate overinvestments in a decentralized market structure. However, if one firm’s investments increase demand for its rivals as well, we have $\partial q_i / \partial x_i > 0$. This is more likely to be the case the poorer horizontal substitutes the goods are and the stronger the investment spillovers.

Analogous to our procedure above, we define $\omega_{ji}^x = \frac{\partial q_j}{\partial x_i} / \frac{\partial q_i}{\partial x_i}$. The variable $\omega_{ji}^x$ measures the increase in demand for good $j$ per unit change in the demand for good $i$ resulting from a higher investment by downstream firm $i$. With perfect spillovers an investment by firm $i$ benefits all firms equally ($\partial q_i / \partial x_i = \partial q_j / \partial x_i > 0$), and we then have $\omega_{ji}^x = 1$. Otherwise, $\omega_{ji}^x < 1$ (and $\omega_{ji}^x$ is negative if $\partial q_j / \partial x_i < 0 \forall i$).

Imposing symmetry, we can now reformulate (15) as

$$p_{VI} [1 + (n - 1) \omega_{ji}^x] \frac{\partial q_i}{\partial x_i} = \varphi'(x_i). \quad (16)$$

The first-order condition $\partial \Pi / \partial p_i = 0$ is still given by equation (8), so that $\lambda^*$ depends on the substitutability between the goods. Clearly, aggregate profit is maximized also in the decentralized market structure if it yields the same prices and investment levels as under vertical integration. We can therefore use equations (13) and (16) to find that the upstream firm at stage 1 should set

$$\theta = \theta^* = \frac{1 + (n - 1) \omega_{ji}^x}{p_{VI}^\lambda}. \quad (17)$$

The intuition for equation (17) is as follows. Suppose that investments primarily have business-stealing effects. Then the extra sales firm $i$ gains when it invests is
approximately countered by correspondingly lower sales by the other downstream firms \((\partial q_i / \partial x_i \approx -(n-1) \partial q_j / \partial x_i)\). Thus, investments are a waste of resources from the industry’s point of view, and the upstream firm should set \(\theta^*\) close to zero. However, the more beneficial (less negative) one firm’s investment is for its rivals, the higher \(\theta\) should be set if the aim is to maximize aggregate channel profit. This explains why \(\partial \theta^* / \partial \omega_{ji} > 0\).

It also follows from (17) that \(\partial \theta^* / \partial p_{VI} < 0\). This reflects the fact that a higher end-user price increases the downstream firms’ marginal profitability. This in turn reduces the necessity of setting a high value of \(\theta\) in order to ensure that the downstream firms have sufficiently strong investment incentives.

We can state:

**Proposition 5:** Assume that there is no demand uncertainty. The profit-sharing rule \(\beta_i(p_i) = \theta p_i^\lambda\) with \(\lambda = \lambda^*\) and \(\theta = \theta^*\) gives downstream pricing and investment incentives conducive to maximum total channel profit.

In the absence of uncertainty there is actually no need for the upstream firm to delegate retail pricing to the downstream firms. Abstracting from any legal considerations, the upstream firm could for instance use RPM. By using the simple profit scheme \(\pi_i = \theta_{RPM} p_i q_i - \varphi(x_i)\) gross of any fixed fees, the upstream firm can ensure correct investments by offering each downstream firm an appropriate profit share \(\theta_{RPM}^*\) (see Appendix). That would be a perfect substitute for using the price-dependent profit-sharing rule. However, once we introduce uncertainty, RPM may have negative impacts both on pricing and investment decisions compared to the profit-sharing rule. To see this, we shall in the remaining part of the paper return to our basic assumption that the upstream firm does not know the exact value of \(a\).

For most well-behaved demand functions, the end-user price which maximizes total channel profit is higher the larger the exogenous size of the market \((a)\). This has two important implications. First, under the profit-sharing rule, it implies that \(d\theta^* / da = (\partial \theta^* / \partial p_{VI}) (\partial p_{VI} / da) < 0\). This is quite intuitive; the larger the size of the market, the higher the end-user price will be, and the smaller is the optimal size of \(\theta\). Second, under RPM, it is important to note that the upstream firm’s choice of \(p_{RPM} \)
has a decisive effect on the downstream firms’ investment levels, since the marginal profitability of investing in market expansion is increasing with \( p_{RPM} \) (under RPM we have \( \frac{\partial \pi_i}{\partial x_i} = \theta_{RPM} \frac{\partial q_i}{\partial x_i} - \varphi'(x_i) \)).\(^{11}\) If the realization \( \hat{a} \) is higher than the upstream firm expected (\( \hat{a} > a^e \)), it will therefore typically be the case that \( p_{RPM} < \hat{p} \) and \( x_{RPM} < \hat{x} \), where \( \hat{p} \) and \( \hat{x} \) are the optimal values relative to the market size \( \hat{a} \). Likewise, if \( \hat{a} < a^e \) we typically have \( p_{RPM} > \hat{p} \) and \( x_{RPM} > \hat{x} \). Put differently, RPM tends to yield too low prices and investment levels when demand is higher than the upstream firm expected, and vice versa.

The basic problem with RPM is that the pricing decision is made by the upstream firm instead of by the firms with hands-on market information. This is in sharp contrast to what is the case under the profit-sharing rule, where the inherent delegation-principle ensures that the downstream firms choose correct prices for any given market size. The only distorting factor with this rule is that the upstream firm must choose \( \theta \) in order to maximize expected profit. As argued above, \( \theta \) should be set at a lower value the larger the exogenous market size \( \frac{d\theta^*}{da} < 0 \). When the upstream firm has to set \( \theta \) based on the expectation of market demand, the rule therefore tends to yield too high investments if the actual value \( \hat{a} > a^e \) and too low investments if \( \hat{a} < a^e \). Contrary to what is the case under RPM, the likelihood of too high investments is therefore increasing with the size of the market under profit-sharing. However, the crucial feature of the profit-sharing rule is that for any given realized market size, the end-user price will be correct from the industry’s point of view.\(^{12}\) Particularly when the exogenous market size differs significantly from its expected value, the profit-sharing rule is therefore superior to RPM. To illustrate this, we now turn to a simple example.

Demand uncertainty; RPM versus profit-sharing. An example.

To allow the firms to make market-expanding investments, we modify the utility function in equation (10) to

\(^{11}\)With linear demand curves the RPM-price completely determines the investment levels; see Appendix.

\(^{12}\)The realized market size is the sum of the exogenous market size and the expansion caused by investments.
where \( a_i = a + x_i \). Each downstream firm can increase the size of its market by \( x_i \) units by investing in e.g. marketing. The cost of doing so is given by \( \varphi(x_i) = (\phi/2) x_i^2 \), where \( \phi \) is sufficiently large to ensure that all stability and second-order conditions are satisfied. To make it simple we assume that there are only two firms (\( n = 2 \)) and that \( s = 2/3 \). We further assume that the upstream firm believes that \( a = 2 \) with 70 % probability, \( a = 1 \) with 10 % probability, and \( a = 3 \) with 20 % probability. The expected size of the market is thus equal to \( a^e = 2.1 \).

The aim of this paper is to demonstrate the efficiency of the profit-sharing rule in delegating pricing decisions to informed market players. Table 2 therefore shows the loss of profit relative to what could have been achieved if also the upstream firm knew the size of the market (labelled potential profit). Column 2 in the table compares actual to potential profit under RPM, while column 3 makes the same comparison under the profit-sharing rule (see Appendix for calculations).

<table>
<thead>
<tr>
<th>Actual exogenous market size</th>
<th>Actual profit relative to potential</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RPM</td>
</tr>
<tr>
<td>( a = 1 )</td>
<td>Not operative</td>
</tr>
<tr>
<td>( a = 2 )</td>
<td>-1.2 %</td>
</tr>
<tr>
<td>( a = 3 )</td>
<td>-6.7 %</td>
</tr>
<tr>
<td></td>
<td>-5.4 %</td>
</tr>
</tbody>
</table>

**Table 2: Profitability performance**

The first thing to note from Table 2 is that RPM fails completely when the actual market size is small. The reason is that the value of \( \beta^{RPM} \) which maximizes expected profits is so high that demand is equal to zero if \( a = 1 \). Thus, the industry will not be
operative at all, and the loss of profit relative to the case with no uncertainty is 100\%. The profit-sharing rule, on the other hand, fares relatively well; the profit is only 5.7\% lower than what would be achievable under certainty. Such differences in the ability to handle market uncertainty can clearly be decisive for whether emerging and potentially profitable industries take off.

When $a = 2$, the actual market size is close to its expected value. In this case RPM and the profit-sharing rule perform almost equally well. However, for $a = 3$ we again see that the profit-sharing rule performs significantly better than RPM. It actually performs relatively better than when $a = a^e$.\footnote{The reason for this is that industry profit increases with the square of the market size with linear demand curves (see appendix). Therefore the upstream firm will put relatively more weight on the possibility for high than for low demand when it maximizes expected profit under the profit-sharing rule. Under RPM, on the other hand, it is equivalent to maximize expected profit and profit of expected demand. Thus, it is possible to make examples where RPM outperforms the profit-sharing rule when $a = a^e$.}

Consistent with the discussion above, the firms underinvest compared to industry optimum under RPM when $a > a^e$, while they overinvest under the profit-sharing rule. However, the overinvestment in the latter case has a comparatively small impact on industry profitability, since the downstream firms can adjust the end-user price correspondingly. Indeed, unlike what is the case under RPM, the downstream firms make the correct investments under the profit-sharing rule for any given total market size ($a + x$ in our notation). This is why the last row in Table 2 shows that the expected profit loss under profit sharing in our example is as small as 0.9\%, compared to 5.4\% under RPM.

## 5 Concluding remarks

A major problem in many network industries is that firms may end up with destructive competition because they produce relatively close substitutes. This may prevent the firms from undertaking investments which could benefit the industry in aggregate. Such an outcome can be avoided by implementing a profit-shifting rule which reduces the downstream firms’ perceived elasticity of demand. Optimal
investment levels are ensured by giving the downstream firms an appropriate profit margin that depends on how one firm’s investments affect its rivals.

The market in the case at hand, content messaging like ringtones, may not be economically important as such. However, the underlying delegation problem in which independent content providers have better demand information than the platforms providing access to the customers, seems to be quite general in markets where content providers use digital distribution networks to reach customers. Moreover, when competing downstream firms have better demand information than the upstream firm, we also show that the vertical restraint examined here is superior to alternative instruments such as RPM when it comes to maximizing total channel profit.

Finally there may be some practical features that make the present rule appealing. In general, a limitation of revenue sharing is the costs of monitoring the retailer’s revenue (Cachon and Lariviere, 2005, and Dana and Spier, 2001). However, in the case at hand, this problem is rarely significant, since the upstream mobile provider collects the revenue from the end users. Another practical merit of profit sharing schemes in market with low marginal costs is that profit sharing in that case approaches revenue sharing. In most situations it is easier to monitor retail revenue than retail profit.

### 6 Appendix

#### 6.1 Calculation of potential profit and the corresponding profit-sharing rule

Using the utility function defined in equation (18) to solve \( \partial U / \partial q_i - p_i = 0 \) for \( n = 2 \), we find that consumer demand is given by

\[
q_i = \frac{1}{2(1-s)} \left( a_i - p_i + \frac{s}{2} ((p_1 - a_1) + (p_2 - a_2)) \right), \tag{19}
\]

where \( a_i = a + x_i \). We thus have

\[
\frac{\partial q_i}{\partial p_i} = -\frac{2-s}{4(1-s)}; \quad \frac{\partial q_j}{\partial p_i} = \frac{s}{4(1-s)} \quad \Rightarrow \quad \omega_{ij}^p = \frac{s}{2-s}. \tag{20}
\]
Equation (19) also implies that

\[
\frac{\partial q_i}{\partial x_i} = \frac{2 - s}{4(1 - s)}; \quad \frac{\partial q_j}{\partial x_i} = -\frac{s}{4(1 - s)} \Rightarrow \omega_{ij}^2 = -\frac{s}{2 - s}. \tag{21}
\]

The cost of market-expanding investments is equal to \(\varphi(x_i) = (\phi/2) x_i^2\). Assuming that \(\phi\) is sufficiently large to ensure a unique and symmetric equilibrium, it follows from (19) that \(q = (a + x - p)/2\), where we have omitted subscripts. We can thus rewrite first-order conditions (8) and (16) for a vertically integrated firm with full market information as \((a + x - p)/2 - p/2 = 0\) and \(p/2 - \phi x = 0\). Solving these two equations simultaneously implies that

\[
x^* = \frac{a}{4\phi - 1} \quad \text{and} \quad p^* = \frac{4\phi a}{4\phi - 1}. \tag{22}
\]

Industry profit is equal to

\[
\Pi^* = \frac{4\phi a^2}{2(4\phi - 1)}. \tag{23}
\]

Using equations (9), (17), (21) and (22) with \(s = 2/3\) we have \(\lambda^* = 1\) and

\[
\theta^* = \frac{4\phi - 1}{4\phi a}. \tag{14}
\]

The upstream firm thus ensures that the two competing downstream firms choose prices and investment levels that maximize aggregate profit by using the profit-sharing rule \(\pi_i = \frac{4\phi - 1}{4\phi a} p_i^2 q_i - \frac{\phi}{2} x_i^2\).

In order to calculate potential profit in Table 2, we have used (23) with \(\phi = 2\).

### 6.2 Calculation of the profit-sharing rule under uncertainty

At stage 2 the downstream firms know actual demand. Solving \(\partial \pi_i/\partial p_i = 0\) and \(\partial \pi_i/\partial x_i = 0\) for \(n = 2\) and then imposing symmetry we find respectively

\[
\theta (2 - s) p^{\lambda+1} - 4x\phi (1 - s) = 0 \quad \text{and} \quad 2(1 - s) (1 + \lambda)(a + x) - p(2\lambda (1 - s) + 4 + 3s) = 0.
\]

\[^{14}\text{Note that this generally implies that } \pi = |1 - |\omega_{12}^2|| p^* q^* - \frac{\phi}{2} (x^*)^2 \text{ and that the upstream firm makes a profit equal to } |\omega_{12} p^* q^*| \text{ net of any fixed fees.}\]
Setting $s = 2/3$ implies that $\lambda = 1$, and we can solve the first-order conditions to find explicit solutions for the price and investment level:

$$p(a) = \frac{\phi - \sqrt{\phi(\phi - \theta a)}}{\theta} \quad \text{and} \quad x(a) = \left[\frac{\phi - \sqrt{\phi(\phi - \theta a)}}{\theta \phi}\right]^2.$$  \hfill (24)

Let $v(k)$ denote the upstream firm’s probability that the exogenous demand parameter is equal to $a(k), k = 1, ..., m$.\textsuperscript{15} Evaluated at these probabilities expected profits give

$$E_v[\tilde{\Pi}] = 2 \sum_{k=1}^{m} v(k) \left( p(a(k))q(a(k)) - \frac{\phi}{2} x(a(k))^2 \right),$$  \hfill (25)

where $q(a(k))$ can be found by inserting for $p(a(k))$ and $x(a(k))$ into equation (19).

At stage 1 the upstream firm maximizes expected profit with respect to $\lambda$ and $\theta$. Since $\lambda = 1$ with $s = 2/3$, the remaining problem is to solve $\hat{\theta} = \text{arg max} E_v(\tilde{\Pi}) = \sum_{k=1}^{m} v(k)\Pi(k)$. With the example used in Table 2 this yields $\hat{\theta} \approx 0.339$, which can be used to calculate expected profits in equation (25). Actual profits in state $k$ can likewise be found by setting $\theta = \hat{\theta}$ and calculate $\Pi(k) = p(a(k))q(a(k)) - \frac{\phi}{2} x(a(k))^2$.

To calculate expected potential profit, we may imagine that we have a stage 0 where demand is uncertain, while actual demand is revealed at stage 1. The latter means that the upstream firm knows actual demand when it sets $\lambda$ and $\theta$. At stage 0, expected potential profits thereby equal $2 \left( \sum_{v=1}^{m} v(k)[p^*(k)q^*(k) - \frac{\phi}{2} x^*(k)^2] \right)$, where $q^*(k)$, $p^*(k)$ and $x^*(k)$ are given from equations (19) and (22) and are the profit maximizing values for each market size.

### 6.3 Calculation of RPM under uncertainty

Under RPM the profit level of downstream firm $i$ equals $\pi_i = \theta^{\text{RPM}} p_i^{\text{RPM}} q_i - (\phi/2) x_i^2$, gross of any fixed fees. At stage 2 the price level $p$ and the profit share $\theta$ (for notational simplicity we omit the superscript RPM from here on) have already been set by the upstream firm. Using equation (19) we have

$$\frac{\partial \pi_i}{\partial x_i} = \theta p_i \frac{\partial q_i}{\partial x_i} - \phi x_i.$$  \hfill (26)

\textsuperscript{15}In the example we have $v(1) = 1/10$, $v(2) = 7/10$, $v(3) = 2/10$ and $a(1) = 1$, $a(2) = 2$, $a(3) = 3.$
With \( n = 2 \) we know from equation (20) that \( \frac{\partial q_i}{\partial x_i} = \frac{2-s}{4(1-s)} \), and solving \( \partial \pi_i / \partial x_i = 0 \) we find

\[
x_i = \theta p_i \frac{2-s}{4\phi (1-s)}.
\]

(27)

The important lesson from equation (27) is that apart from the exogenous parameters \( s \) and \( \phi \), the marginal profitability of investing in market expansion is completely determined through the upstream firm’s choice of \( p_i \) and \( \theta \). The investment incentives are in particular independent of the market size \( a \), once \( p_i \) and \( \theta \) are determined. Thus, the upstream firm faces no uncertainty with respect to the downstream firms’ investment levels, and expected industry profit from the upstream firm’s point of view at stage 1 can thus be written as

\[
E_v[\Pi] = p_i \left( \sum_{k=1}^{m} v(k)q_i(a(k)) \right) - \phi x_i^2.
\]

As before, \( v(k) \) is the upstream firm’s perceived probability for \( a = a(k) \). Solving \( \{p_i, \theta\} = \arg \max E_v[\Pi] \), we find a symmetric solution

\[
p^{\text{RPM}} = \left( \sum_{k=1}^{m} v(k)p^*(a(k)) \right) \quad \text{and} \quad \theta^{\text{RPM}} = \frac{2(1-s)}{2-s},
\]

where \( p^*(a(k)) \) is the optimal price given that demand is equal to \( a(k) \).\(^{16}\) The upstream firm thus sets \( p^{\text{RPM}} \) such that it is equal to the expected monopoly price over all states, given by the sum of the state contingent profit maximizing prices, one for each \( k \), weighted by the likelihood that this state will occur.

Unlike what is the case under the profit-sharing rule - where the downstream firms can react to actual market demand - we see that \( \theta^{\text{RPM}} \) is independent of \( a \). This reflects the fact that \( \theta^{\text{RPM}} \) can only be used to adjust for the competitive pressure between the downstream firms.

Inserting for \( p^{\text{RPM}} \) and \( \theta^{\text{RPM}} \) we further find

\[
x^{\text{RPM}} = \frac{\sum_{k=1}^{m} v(k)a(k)}{4\phi - 1}.
\]

\(^{16}\)From equation (22) we know that this price is given by \( p^* = \frac{4\phi}{4\phi - 1} s \).
Investments are thus proportional to expected market size, instead of being dependent on the actual size of the market. All adjustments to actual demand \((a_{\text{act}})\) being different from expected demand will therefore take place through the quantities sold:

\[
q(a(k)) = \frac{a_{\text{act}}(4\phi - 1) - (\sum_{k=1}^{m} v(k)a(k)) (2\phi - 1)}{2(4\phi - 1)}.
\]

(28)

Expected profits are equal to

\[
E_v[\Pi] = \frac{\phi (\sum_{k=1}^{m} v(k)a(k))^2}{4\phi - 1},
\]

while actual industry profits in each state are equal to \(\Pi(a(k)) = p_{\text{RPM}} q(a(k)) - x_{\text{RPM}}\).

Some comments on the calculation of RPM in Table 2

Using the example in Table 2, we find that \(q < 0\) if \(a = 1\). The upstream firm will therefore at the outset be aware of the fact that the industry will be inoperative if \(a = 1\), and will use this information to obtain higher profits. More precisely, this means that the upstream firm solves \(p_{\text{RPM}} = \arg \max \left\{ p_i \left( \frac{v(2)q(2)}{v(2)+v(3)} + \frac{v(3)q(3)}{v(2)+v(3)} - \phi x_i^2 \right) \right\}\). Expected profits are equal to \(\frac{1}{10} * 0 + \frac{7}{10} \Pi(a(2)) + \frac{2}{10} \Pi(a(3))\).

7 Literature


\footnote{Note that \(p_{\text{RPM}}\) and thus \(x_{\text{RPM}}\) will be equal to first-best if \(a = a^e\), c.f. equation (22), in which case it will outperform the profit-sharing rule.}


