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The Danish Green Certificate System:
Some simple analytical results

by

Eirik S. Amundsen and Jørgen Birk Mortensen

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Eirik S. Amundsen¹ and Jørgen Birk Mortensen²

Abstract
We formulate a simple static equilibrium model for the electricity market taking account of both Green Certificates and CO2-emission permits. The objective is to investigate the relationship between these markets under the existence of upper and lower price-bounds on the Green certificates, both in the short and in the long run. We perform various comparative static changes of the parameters of the model and study how these affect the endogenous variables of the model as well as producers’ and consumers’ surplus. We also investigate the effects of imports and exports of electricity. Several comparative static results are derived and in particular it is shown that harsher CO2-emission constraints and increased import wholesale price both may lead to reduced capacity of “green electricity”.

JEL classification code: Q28, Q42, Q48

Keywords: Electricity, Green certificates, Emission permits.

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1. Introduction

In March 1999, the Danish Parliament adopted the principles for a reform of the hitherto extensively regulated Danish electricity market. Among the ideas and principles laid down was the idea to create a Green Certificate market to stimulate and foster the expansion of the capacity of “green electricity” stemming from renewable energy sources e.g. wind and bio-masses (The Danish Ministry of Environment and Energy, 1999 and 2000). It is the intention of the Danish Parliament that the Green Certificate market shall be fully effective by 2003. However, even though the main principles of this new market have been agreed upon, it seems fair to say that the general functioning of the Green Certificate market has not yet been fully investigated and that much work remains before the market can be designed in detail (see, however, Morthorst, 1999, Nielsen and Jeppesen, 1999, Voogt et al. 1999, Skytte, 2000).

In this paper we seek to formalise the essential characteristics of the proposed market for Green Certificates in Denmark and investigate how such a market may interact with a simultaneously functioning market for CO2-emission permits in the short and in the long run. We concentrate on the analytics of the market itself and do not in this setting address the question of whether such a system is economically sound as compared to other possible ways of stimulating the generation of “green electricity” (e.g. lump sum subsidies, Pigouvian taxes). Furthermore, we do not consider any uncertainty or any financial markets for forward or future trade in certificates, nor do we consider an international system of “Green Certificates”.

In short, the Green Certificates market consists of sellers and buyers of certificates. The sellers are the producers of electricity using renewable sources. These producers are each allowed to sell an amount of certificates corresponding to the electricity that they feed into the electricity network. The buyers of certificates are consumers/distribution companies that are required by the government to hold a certain percentage of certificates corresponding to total end-use deliveries and consumption of electricity. The Green Certificates are seen as permits for consuming electricity. The system implies that the producers using renewable energy sources receive both the wholesale price and a certificate price for each kWh fed into the electricity network.

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network. In this way the certificate system is supposed to induce new investment in “green electricity”.

In the following we formulate a simple static equilibrium model taking account of markets for Green Certificates and CO2-emission permits as well as the market for electricity. A basic assumption of the model is that the percentage requirement for the possession of certificates of consumers/distribution companies functions as a check on total electricity consumption. The percentage requirement really implies a limit on total consumption since the total number of certificates available are constrained by the total capacity of renewable technologies (due consideration taken to the stochastic elements of wind power). Hence, a requirement of 20% implies that total consumption can be no larger than five times the electricity produced from renewable sources, unless the price of certificates tend to increase above an upper price bound. In that case additional consumption may be allowed if consumers/distribution companies pay a fixed fine corresponding to the upper price bound per kWh of additional consumption.

The model presented in the following takes account of both an upper- and a lower price-bound of certificates. In addition to characterising equilibrium conditions for the various markets in the short and in the long run, we perform comparative static changes of the parameters of the model and study how these affect the endogenous variables of the model as well as producers’ and consumers’ surplus. In particular we investigate the effects of imports and exports. Finally we concentrate on a limited number of specific applications that we give a somewhat more detailed treatment.

2. The Model

The following model is designed to capture a short run situation for the Green Certificate system. Further below the model is extended to comprise the long run.

2.1. Legend and functional relationships

\[ p = \text{consumer price of electricity} \]
\[ b = \text{mark-up and excise taxes in distribution (fixed by regulation)} \]
\[ s = \text{price of Green Certificates} \]
\[ \bar{s} = \text{maximum price of Green Certificates} \]
\( s = \) minimum price of Green Certificates
\( q = \) domestic wholesale price of electricity
\( q_w = \) external wholesale price of electricity
\( r = \) price of emission permits
\( \bar{r} = \) maximum price of emission permits
\( x = \) total consumption of electricity
\( y = \) production of electricity from non-renewable energy sources
\( z = \) production of electricity from renewable energy sources
\( \bar{z} = \) capacity of electricity production from renewable energy sources
\( m = \) imports (positive) or exports (negative) of electricity
\( \alpha = \) percentage of electricity consumption from renewable energy sources
\( \beta = \) CO2-emission reduction (a choking factor)
\( g^d = \) demand of Green Certificates
\( g^s = \) supply of Green Certificates

We apply the following general functions
\[ p(x) : \text{demand function, with } (\partial p(x)/\partial x) = p' < 0 \]
\[ c = c(y; \beta) : \text{emission constrained cost function} \]
\( c = c(y; \beta) \) for electricity production using non renewable energy sources, with \( c_y > 0, c_{yy} \geq 0, c_{\beta} > 0, c_{y\beta} > 0 \). When \( \beta = 0 \), the cost function (i.e. \( c = c(y;0) \)) signifies the case where the emission constraint is not taken into account.

2.2. Objectives and first order conditions in the short run

We consider a short-run situation and assume perfect competition (efficiency) all around. We

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4 The emission constrained cost function is derived from a standard cost-minimisation problem with the additional constraint of a given CO2-emission. In particular this function takes into account that a given level of production may be attained even for harsher CO2-constraints by substituting to cleaner fuels and technologies. However, this kind of substitution implies increasing cost, wherefore the marginal cost function shifts upwards.
assume \( i, (i = 1, \ldots, I) \) distribution companies, \( j (j = 1, \ldots, J) \) identical\(^5\) producers of electricity using non renewable sources and \( k, (i = 1, \ldots, K) \) producers of electricity using renewable sources; all maximising profits \( \Pi \).

**Distribution company**

The maximisation problem for a distribution company is as follows

\[
Max \quad \Pi(x_i) = px_i - [b + \alpha s + q]x_i,
\]

The first order condition for an optimum is

\[
p - [b + \alpha s + q] \geq 0, \quad \forall i
\]

1)

**Producer of electricity using non renewable energy sources**

We assume that each producer is constrained by an equal share of the total CO2 - emission permits that corresponds to the given choking factor \( \beta \). Hence, each producer must reduce its emission by a factor equal to \( \beta_j = \beta / J \). The maximisation problem of this category of producers is as follows

\[
Max \quad \Pi(y_j) = qy_j - c(y_j; \beta_j),
\]

The first order condition for an optimum is:

\[
q = c'_y(y_j; \beta_j), \quad \forall j
\]

2)

**Producer of electricity from renewable energy sources**

Assuming that the short-run cost for renewable electricity is equal to zero, we have the following maximisation problem

---

\(^5\) The reason why we consider identical producers of electricity using non renewable sources is that we want to avoid the additional apparatus necessary for formulating trade in CO2-emission permits as this is not in the forefront of this analysis. What we need in the present analysis is an endogenous (shadow)-price of CO2-emission permits. This will be established assuming identical producers with an equal number of emission permits even though the producers will not trade when they are identical.
Max $\Pi(z_k) = [q + s]z_k$;

subject to:

$z_k \leq \bar{z}_k$, \hspace{1cm} 3)

The first order conditions for an optimum is: $z_k = \bar{z}_k \forall k$, (assuming, $[q + s] > 0$)

Market for Green Certificates

The demand for Green Certificates is given by

$g^d = \alpha x$ \hspace{1cm} 4)

The supply of Green Certificates is assumed measured in the same units as capacity and is given by

$$g^s = \begin{cases} (\bar{z}, \rightarrow) & \text{for } s = \bar{s} \\ \bar{z} & \text{for } s \in (\underline{s}, \bar{s}) \\ \bar{z} & \text{for } s = \underline{s} \end{cases} \hspace{1cm} 5)$$

Market for CO2-emission permits

The supply of CO2-emission permits is indirectly expressed by the “choking factor”, $\beta$. The larger is $\beta$ the smaller is the number of permits. It should be noted that in the Danish system, the permits are handed out free of charge to the producers of electricity using non-renewable sources. However, in the Danish system it is assumed that the producers can trade the permits among themselves. A net purchaser of permits is a producer for which the permit price is lower than the additional cost of expanding production without additional emission. For a net seller the opposite is true. (In the present model, however, no trade will take place since all producers using non-renewable sources are identical). The demand for CO2-emission permits is indirectly determined by the marginal abatement cost through emission preserving substitution. We return to the equilibrium conditions below.

2.3. Short run equilibrium

The short run equilibrium of the markets are illustrated in Fig. 2. for the case of $s \in (\underline{s}, \bar{s})$, in Fig. 3 for the case of $s = \bar{s}$ and in Fig. 4 for the case of $s = \underline{s}$.
**End-user market**

Equilibrium quantity: \( x(p^*) = y^* + \bar{z} + m^* \)  

Equilibrium price: \( p^* = b + \alpha s^* + q^* \); where \( x^* = \sum_i x_i^*, y^* = \sum_j y_j^*, z^* = \sum_k z_k^* \)

**Wholesale market**

Equilibrium quantity: \( x(q^*) = y(q^*) + \bar{z} + m^* \)

Equilibrium price: \( q^* = q_m = c_j(y_j^*; \beta_j), \forall j \)

Observe that since the external wholesale price is exogenously given the domestic wholesale price will - in equilibrium - be identical to this.

**Market for Green Certificates**

The equilibrium conditions for the market for Green Certificates are as follow

\[
g^* = \begin{cases} 
\alpha \alpha(b + \alpha s^* + q^*) \text{ for } s^* = \bar{s} \\
\alpha \alpha(b + \alpha s^* + q^*) = \bar{s} \text{ for } s^* \in (s, \bar{s}) \\
\alpha \alpha(b + \alpha s^* + q^*) \text{ for } s^* = s 
\end{cases}
\]

In the case of \( s^* \in (s, \bar{s}) \) the equilibrium price may be determined as

\[
s^* = \frac{p(\bar{s}) - b - q^*}{\alpha}, \text{ where } s^* \in (s, \bar{s})
\]

In the case of \( s^* = s \), the demand for Green Certificates is less than \( \bar{s} \), and there is an excess supply of Green Certificates i.e. some of the Green Certificates may not be sold unless some third party intervenes and purchases the otherwise unsold Green Certificates at the minimum price \( s \). An institution for the repurchase of Green Certificates is assumed established in the Danish reform programme.

**Market for CO2-emission permits**

As stated earlier the supply of CO2-emission permits is exogenously given and indirectly determined by the choking factor \( \beta \), while the demand for CO2-emission permits is indirectly
determined by the marginal abatement cost through emission-preserving substitution. However there is a maximum price of permits equal to $\bar{r}$. At this price the producer is free to purchase any number of additional permits. (To keep it simple, we will not, however, consider this special case in the following).

Expanding production by one unit, the producer using non-renewable sources has two options; either to purchase a permit for the additional emission following from the expansion or to choose a more costly production method and expand production without additional emission. In equilibrium the cost of these two options must be the same. Hence, in equilibrium the price of permits needed for a marginal expansion of production may be expressed as

$$r(y^*) = \min[c'(y_j^*; \beta_j) - c'(y_j^*; 0), \bar{r}]$$

12)

3. Comparative statics in the short run

Using the system of equations following from the equilibrium conditions, we derive various comparative static results conditional of $s$ being binding or not. As the results vary according to whether external trade is allowed or not we split the analysis in two cases; the case of autarky and the case of external trade.

3.1. Results under autarky

Under autarky the external wholesale price of electricity will no longer determine the domestic wholesale price. Rather the domestic wholesale price will be determined along with the other endogenous variables of the model.

The comparative static results in this case are derived using the equilibrium conditions stated above. The case of $s \in (\bar{s}, \bar{s})$ is illustrated in Fig. 1. In Fig. 1 and the figures to follow the supply function (i.e. aggregate marginal cost functions) for electricity stemming from non renewable sources is denoted by $f'(y, \beta)$

The case of $s \in (\bar{s}, \bar{s})$.

$$p^* = b + \alpha s^* + q^*$$

13)

$$q^* = c'(y_j^*; \beta_j) = f'(y^*, \beta)$$

14)
The case of $s = \bar{s}$

\[ p(x^*) = b + \alpha \bar{s} + q^* \]  

\[ q^* = c'(y^*_j, \beta_j) = f'(y^*, \beta) \]  

\[ x > \frac{\bar{s}}{\alpha} \]  

The case of $s = \underline{s}$

\[ p(x^*) = b + \alpha \underline{s} + q^* \]  

\[ q^* = c'(y^*_j, \beta_j) = f'(y^*, \beta) \]  

\[ x < \frac{\underline{s}}{\alpha} \]  

The results are reported in Table 1, 2 and 3. We will not go through all of these results but rather concentrate on two cases that may give insight into the functioning of these markets.

**Harsher constraints on CO2-emission**

Harsher constraints of CO2-emission correspond to an increased level of $\beta$ in the model considered here. We investigate the case where the price of certificates is not binding. Intuitively one may think that an increased emphasis on reducing CO2-emission would be to the benefit of the producers of “green electricity”. This is, however not the case. On the contrary the producers of “green electricity” stand to lose. In order to explain this relationship it is important to observe that an increased level of $\beta$ leaves total consumption and electricity production constant since the level of allowable consumption is not affected. For this reason the market price $p$ will also remain constant. However, the marginal cost of electricity using non renewable sources will increase since the tightening of emission constraints leads to substitution in the direction of more clean - but yet more costly - technologies. This corresponds to a larger wholesale price $q$ and a larger implicit price of CO2-emission permits. However, this translates into a lower price of certificates since this is determined as a weighted difference between the market price and the wholesale price as shown by equation 11).
In this situation the producers of electricity using renewable sources are faced with an increased wholesale price and a reduced price of certificates, which then give opposite effects on profits. The net effect on their profit is, however, negative. According to the model the per unit profit of a producer of “green electricity” is given by \( q^* + s^* \). Substituting for \( s \) from the above expression gives the following expression for the per unit profit

\[
q^* + s^* = \frac{p(\bar{z}) - b - (1 - \alpha)q^*}{\alpha}
\]

An increased value of \( q^* \) leads unilaterally to a reduced value of \( q^* + s^* \) as long as \( \alpha < 1 \). The intuition for this result is that a 1$ increase of the wholesale price translates into \( 1/\alpha \) $ reduction of the price of certificates.

In short the tightening of CO2-emission leads to a redistribution of producers’ surplus from the producers of “green electricity” to producers of electricity using non-renewable sources. It should be noted, however, that this conclusion is not necessarily valid for the cases were the certificate price is on its upper- or lower bounds.

**Increased production of “green electricity”**

In the short run the production of “green electricity” may vary due to the variability of the wind power. In this case we investigate how an increase of “green electricity” affect the various endogenous variables and the surplus. Again we assume that the certificate price bounds are not binding.

The increased production of “green electricity” induces an increased production of “non renewable electricity” since the level of allowable consumption increases. This leads to a reduction of the end-use price, an increase of the wholesale price and an increase of the implicit price of CO2-emission permits. The price of Green Certificates will, however, be reduced. While both the consumers’ surplus and the producers’ surplus for “non renewable electricity” increase, the net effect on the producers’ surplus for “green electricity” is indeterminate. In a sense this result says that the marginal revenue for the producers of “renewable electricity” may turn out to be negative. The precise value of the marginal revenue is, in part, determined by the price elasticity of demand.
Table 1. Comparative static results under autarky when $s \in (\underline{s}, \bar{s})$, short run

<table>
<thead>
<tr>
<th>$x^*$</th>
<th>$y^*$</th>
<th>$p^*$</th>
<th>$q^*$</th>
<th>$r^*$</th>
<th>$s^*$</th>
<th>$\Pi(z^*)$</th>
<th>$\Pi(y^*)$</th>
<th>Consumer surplus</th>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>-</td>
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<td>+</td>
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<td>?</td>
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<tr>
<td>$\beta$</td>
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Table 2. Comparative static results under autarky when $s = \bar{s}$, short run

<table>
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<tr>
<th>$x^*$</th>
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<th>$q^*$</th>
<th>$r^*$</th>
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<th>$\Pi(y^*)$</th>
<th>Consumer surplus</th>
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<tbody>
<tr>
<td>$\alpha$</td>
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<td>$z$</td>
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<td>-</td>
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<td>+</td>
<td>?</td>
<td>?</td>
<td>x</td>
</tr>
</tbody>
</table>

x) May fall below $\bar{s}$

Table 3. Comparative static results under autarky when $s = \underline{s}$, short run

<table>
<thead>
<tr>
<th>$x^*$</th>
<th>$y^*$</th>
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<th>$q^*$</th>
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<th>Consumer surplus</th>
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<tbody>
<tr>
<td>$\alpha$</td>
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<td>x</td>
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<tr>
<td>$\beta$</td>
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<tr>
<td>$z$</td>
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<td>+</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>x</td>
</tr>
</tbody>
</table>

x) May increase above $\underline{s}$

3.2. Results under external trade

In this section we open for exports or imports at a fixed wholesale price $q_m$. The level of this price determines whether there will be imports or exports. If it is smaller than the autarky equilibrium wholesale price, imports will take place; otherwise exports will take place. The domestic wholesale price becomes identical to the fixed external wholesale price. The markets for Green Certificates and emission permits do not comprise external producers (i.e. foreign producers do not receive green certificates and imported electricity is not considered “green”).

10
The case of \( s \in (\underline{s}, \bar{s}) \)

\[
p^* = b + \alpha s^* + q^* \tag{23}
\]

\[
q^* = \frac{\beta}{y_j} = f^*(y^*, \beta) \tag{24}
\]

\[
x = \frac{\bar{s}}{\alpha} \tag{25}
\]

The case of \( s = \bar{s} \)

\[
p(x^*) = b + \alpha \bar{s} + q^* \tag{26}
\]

\[
q^* = \frac{\beta}{y_j} = f^*(y^*, \beta) \tag{27}
\]

\[
x > \frac{\bar{s}}{\alpha} \tag{28}
\]

The case of \( s = \underline{s} \)

\[
p(x^*) = b + \alpha \underline{s} + q^* \tag{29}
\]

\[
q^* = \frac{\beta}{y_j} = f^*(y^*, \beta) \tag{30}
\]

\[
x < \frac{\bar{s}}{\alpha} \tag{31}
\]

The three cases are illustrated in Fig. 2, 3 and 4. In the following analysis, it should be noted that we consider marginal changes and that we for simplicity assume that \( s \) remains in its initial interval after the changes considered. Also we concentrate on the case of imports. The results are reported in Table 4., 5 and 6.

In this case we also choose to only comment on a few cases. Firstly, one may observe that the negative effect on the profits of the producers of “green electricity” following from harsher constraints on CO2-emission, disappears when we consider an open economy. In fact, reducing the number of CO2-emission permits has no influence on the profits of these producers at all as both the price of Green Certificates and the wholesale price remain unaffected. Secondly, one may observe that an increase of the production of “green electricity”, definitely leads to a reduction in the per unit profit of the producers of “green electricity” when external trade is allowed. This follows from the fact that the wholesale price of electricity remains unaffected while the price of Green Certificates falls. For this reason it
will not be in the collective interest of the producers of “green electricity” to expand production.

*Increased import price*

We investigate the case where the upper- and lower price-bounds on certificates are not binding and ask which effects an increased import price will have on the profits of the two producer categories. One may think that such a price change would imply a reduced competition for the sheltered producers of “green electricity” and therefore that their profits would be increased. However, the opposite is true.

The increase of the import price leads to an increase of the domestic wholesale price and a smaller import. The implicit price of CO2-emission permits will also be increased. However, nothing is happening to allowable consumption wherefore total consumption remains constant and so does the market price $p$. The production of electricity using non-renewable sources will, however, be increased by the same amount as the import is reduced. The increase of the wholesale price by 1$ translates into a reduction of $1/\alpha$ of the certificate price. In this case producers’ surplus from the producers of “green electricity” are redistributed in the direction of the producers of using non-renewable energy sources.

Table 4. Comparative static results under external trade (imports) when $s \in (\bar{s}, \bar{s})$, short run

<table>
<thead>
<tr>
<th></th>
<th>$x^*$</th>
<th>$y^*$</th>
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<th>$m^*$</th>
<th>$q^*$</th>
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Table 5. Comparative static results under external trade (imports) when $s = \bar{s}$, short run

<table>
<thead>
<tr>
<th></th>
<th>$x^*$</th>
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<th>$\Pi(z^*)$</th>
<th>$\Pi(y^*)$</th>
<th>Consumer surplus</th>
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<tr>
<td>$\alpha$</td>
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</tr>
<tr>
<td>$\beta$</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>+</td>
<td>0</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>$z$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$q_{im}$</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>?</td>
<td>+</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 6. Comparative static results under external trade (imports) when $s = \bar{s}$, short run

<table>
<thead>
<tr>
<th></th>
<th>$x^*$</th>
<th>$y^*$</th>
<th>$p^*$</th>
<th>$m^*$</th>
<th>$q^*$</th>
<th>$r^*$</th>
<th>$s^*$</th>
<th>$\Pi(z^*)$</th>
<th>$\Pi(y^*)$</th>
<th>Consumer surplus</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>-</td>
<td>0</td>
<td>+</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>+</td>
<td>0</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$q_m$</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>?</td>
<td>+</td>
<td>0</td>
</tr>
</tbody>
</table>

4. The long-run solution

Up until now only short run problems have been investigated and, in particular, it has been shown that, the number of CO2-emission permits ($\beta$), the production of “green electricity” ($\gamma$) and the import wholesale price ($q_m$) may influence the remuneration to producers of “green electricity”. The remuneration for this group of producers will be determined by the current and future level of the sum of the wholesale price and the price of certificates corresponding to each kWh generated. For a producer to expand the capacity of “green electricity” or for new producers to enter the market, the expected present value of future revenue must be at least as large as the investment cost for new capacity. As capacity is expanded the sum of the wholesale price and the certificate price per kWh generated will, assuming a time-invariant demand function, presumably fall (even though this result is not definite in the above analysis) and lead to less investment as time passes. However, to investigate the path of investment one really needs a dynamic model and the static model considered here is, therefore, not suited for this kind of analysis. Still, the static model may be used to characterise the long run equilibrium once attained even though we do not consider the adjustments leading to this equilibrium. The essential difference from the short run analysis is that the capacity of “green electricity” is determined endogenously in a long run model. Hence, the effect on the long run capacity of “green electricity” is at the focus of the following analysis.

The model may easily be expanded to include the long run equilibrium by introducing a cost function for new capacity of “green electricity” as well as long run cost functions for electricity from non renewable sources. We denote long run values and cost functions by capital letters corresponding to the short-run notation and introduce the long run cost function for new capacity of “green electricity” as follows
\[ H = H(Z) \text{ where } H'(Z) > 0, H''(Z) \geq 0 \]

The long run optimisation problem of a producer of “green electricity” may then be formulated as
\[
\text{Max } [(S + Q)Z_k - H(Z_k)]
\]
subject to \( Z_k \leq \bar{Z}_k \)

The first order condition for a maximum is simply
\[
(S + Q) = H'(Z_k), \forall k
\]

### 4.1. Long run equilibrium

Recognising that the optimisation principles from the short-run carry over to the long-run by simply introducing long run cost functions, the long run equilibrium may be characterised as follows below.

**End-user market**

Equilibrium quantity: \( X(P^*) = Y^* + \bar{Z}^* + M^* \)

Equilibrium price: \( P^* = b + \alpha S^* + Q^* \); where \( X^* = \sum_i X_i^*, Y^* = \sum_j Y_j^*, Z^* = \sum_k Z_k^* \)

**Wholesale market**

Equilibrium quantity: \( X(Q^*) = Y(Q^*) + \bar{Z}(Q^* + S^*) + M^* \)

Equilibrium price: \( Q^* = Q_M = C_\gamma (Y_j^*; \beta_j) = H'(Z_k^*) - S^* \), \( \forall j, k \)

**Market for Green Certificates**

The equilibrium conditions for the market for Green Certificates are as follow

\[
G^* = \begin{cases} 
\alpha X (b + \alpha S + Q^*) \text{ for } S^* = \bar{s} \\
\alpha X (b + \alpha S^* + Q^*) = \bar{Z}^* \text{ for } S^* \in (s, \bar{s}) \\
\alpha X (b + \alpha s + Q^*) \text{ for } S^* = \bar{s}
\end{cases}
\]

In the case of \( S^* \in (s, \bar{s}) \) the equilibrium price may be determined as
\[ S^* = \frac{p(\frac{Z^*}{\alpha}) - b - Q^*}{\alpha}, \text{ where } S^* \in (\underline{s}, \bar{s}) \]

**Market for CO2-emission permits**

As stated earlier the supply of CO2-emission permits is exogenously given and indirectly determined by the choking factor \( \beta \), while demand is indirectly determined by the marginal abatement cost through emission-preserving substitution. However, there is a maximum price of permits equal to \( \bar{r} \). At this price the producer is free to purchase any number of additional permits. Hence, in equilibrium the price of permits needed for a marginal expansion of production may be expressed as

\[ R(Y^*) = \min \left[ C^*_Y(Y^*_j; \beta_j) - C^*_Y(Y^*_j; 0), \bar{r} \right] \]

**4.2. Comparative statics in the long run**

We will also in this case distinguish between autarky and external trade and investigate the effects of partial changes in the basic parameters of the model. As observed the main distinction between the short and the long run model is that the capacity for generating “green electricity“ is now itself an endogenous variable. In the following we restrict ourselves to consider only the case of interior solutions for \( S^* \) (i.e. \( S^* \in (\bar{s}, s) \)) and concentrate the analysis on the effects of the long run capacity of “green electricity“.

**Autarky**

To derive the comparative static results under autarky, we apply the following conditions:

\[ P^* = b + \alpha S^* + Q^* \]
\[ Q^* = C^*(Y^*_j; \beta_j) \]
\[ X^* = \frac{Z^*}{\alpha} \]
\[ (S^* + Q^*) = H'(Z^*_k), \forall k \]
Increase of the "percentage requirement"

The question we pose is: “What will happen to the long run capacity of “green electricity” if there is an increase of the “percentage requirement” for “green electricity”? To answer this question we consider the per unit payment that the producer of “green electricity” will get in a long run setting and investigate what change, if any, this will induce for optimal capacity. Applying the above equations we get

\[ H'(Z^*) = Q^* + S^* = \frac{p(Z^*) - b - (1 - \alpha)Q^*}{\alpha} \]  

Taking the implicit derivative with respect to \( \alpha \) we arrive at

\[ \frac{dZ^*}{d\alpha} = -\frac{p}{\varepsilon} + \alpha S^* \left[ \alpha^2 H'' - \frac{dP}{dX} + (1 - \alpha)^2 C_{yy} \right] \quad \text{with} \quad \varepsilon = \frac{dX}{X} \frac{dP}{P} \]

Inspection of signs shows that the denominator is positive while the numerator is indeterminate. Hence, it turns out that the effect on the capacity of “green electricity” following from an increased “percentage requirement” is inconclusive. Thus, while increasing the “percentage requirement” of electricity stemming from renewable energy sources definitely leads to reduced electricity consumption, it may very well also lead to a reduction of the capacity for producing “green electricity”. One essential element determining this relationship is the price elasticity of demand (\( \varepsilon \)). If demand is close to being inelastic (i.e. 0), the capacity of “green electricity” will increase, while the opposite is true if the elasticity is perfectly elastic (i.e. \(-\infty\)).

Harsher constraints on CO2-emission

To answer the question as to what will happen to the long run capacity of “green electricity” if the number of emission permits is reduced we again consider the change of the per unit payment to the producer of “green electricity”. Taking the implicit derivative of this expression with respect to \( \beta \) we find

\[ \frac{dZ^*}{d\beta} = -\frac{\alpha(1 - \alpha)C_{\beta\beta}}{\alpha^2 H'' - \frac{dP}{dX} + (1 - \alpha)^2 C_{yy}} < 0 \]  

16
Inspection of signs shows that this expression is negative. Hence, harsher constraints on CO2-emission leads to a reduction of the production capacity of “green electricity” and also a reduction of the production capacity for “non renewable electricity”. While the sum of $(S^*+Q^*)$ will be lower or remain constant in the new equilibrium, it is not clear whether $S^*$ and $Q^*$ separately will fall.

**Table 7. Comparative static results under autarky when $s \in (\underline{s}, \overline{s})$, long run**

<table>
<thead>
<tr>
<th></th>
<th>$X^*$</th>
<th>$Y^*$</th>
<th>$Z^*$</th>
<th>$P^*$</th>
<th>$Q^*$</th>
<th>$R^*$</th>
<th>$S^*$</th>
<th>$\Pi(Z^*)$</th>
<th>$\Pi(Y^*)$</th>
<th>Consumer surplus</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>$\leq 0 \times$</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

x) The equality applies if “green electricity” is a constant cost industry and the inequality applies if “green electricity” is an increasing cost industry.

**External trade**

To derive the comparative static results for the case of external trade and imports, we apply the following conditions

The case of $S^* \in (\underline{s}, \overline{s})$

\[ P^* = b + \alpha S^* + Q^* \] \hspace{1cm} (48)

\[ Q^* = Q_M = C(Y_j^*, \beta_j) \] \hspace{1cm} (49)

\[ X^* = \frac{Z^*}{\alpha} \] \hspace{1cm} (50)

\[ (S^*+Q^*) = H'(Z_k^*), \forall k \] \hspace{1cm} (51)

**Increase of the ”percentage requirement”**

We arrive at the same result as under the long run solution under autarky, i.e. that the effect of an increased “percentage requirement” on the capacity for “green electricity” is inconclusive and, in general, depends on the price elasticity of demand.
Harsher constraints on CO2-emission

The expression for the per unit payment that the producer of “green electricity” will get in a long run setting may be written

\[
Q_M + S^* = \frac{p(\overline{Z}^*) - b - (1-\alpha)Q_M}{\alpha}
\]

It is clear from this expression that it will not be affected by a change in \( \beta \) since \( Q_M \) is exogenously given. Hence, the initial value of \( \overline{Z}^* \) still satisfies the long run optimality conditions and \( S^* \) will remain the same as prior to the change of \( \beta \). There will, however, be a reduction in the generation of electricity from non-renewable sources that will be completely compensated by imports. Also, the price of CO2-emission permits will increase. These results are quite similar to the results in the corresponding short run analysis.

Increased import price

In order to investigate the effects of an increased price of imported electricity, we again consider the expression for the per unit payment of the producer of “green electricity” stated above while recognising that the supply of capacity for “green electricity” is a non-decreasing function of \( Q_M + S^* \). It is clear from the above expression that an increase of \( Q_M \) will, ceteris paribus, lead to a reduction of the unit payment of the producers of “green electricity”. Hence, the new long run equilibrium implies a reduction of \( \overline{Z}^* \) with a lower price of certificates. In this case there will also be a reduction of total electricity consumption. In the corresponding short run analysis, total electricity consumption remained unchanged.

| Table 8. Comparative static results under external trade (imports) when \( s \in (\overline{s}, \overline{s}) \), long run |
|------------------|---------|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
|                  | \( X^* \) | \( Y^* \) | \( Z^* \) | \( P^* \) | \( M^* \) | \( Q^* \) | \( R^* \) | \( S^* \) | \( \Pi(Z^*) \) | \( \Pi(Y^*) \) | Consumer surplus |
| \( \beta \)      | 0       | -     | 0     | 0      | +     | 0     | +     | 0     | 0     | -     | 0 |
| \( Q_m \)        | -       | +     | -     | +      | -     | +     | -     | +     | \( \leq 0(x) \) | +     | 0 |

x) The equality applies if “green electricity” is a constant cost industry and the inequality applies if “green electricity” is an increasing cost industry.
5. Conclusions

In this paper we have focused on some basic features of the proposed Danish Green Certificate system. We have applied a simple static model for the electricity market and derived results both for a short run and a long run setting, under autarky and under external trade. In particular it has been shown that:

- The effects of an increase of the “percentage requirement” (i.e. the obligatory number of “Green Certificates” per kWh consumed) are most inconclusive, both in the short and in the long run, under autarky and external trade. Hence, it is not generally true that an increase of the “percentage requirement” leads to a larger capacity of “green electricity” in the long run. However, the share of “green electricity” as compared to total consumption will increase.

- Harsher CO2-emission constraints will give a downward pressure on the certificate price and on the profits of the producers of “green electricity” both in the short and the long run under autarky. Thus, such a policy will lead to a reduced capacity of “green electricity” in the long run under autarky. Under external trade with imports of electricity, harsher CO2-emission constraints have no effects on the certificate price and on the profits of the producers of “green electricity”. This is true both in the short and in the long run. Thus, the long run capacity of “green electricity” will remain unaffected by such a policy.

- An increase of the import wholesale price for electricity, will lead to a downward pressure on the certificate price and on the profits of the producers of “green electricity”. This is true both in the short and in the long run. Hence, it is not in general true that a high import price will protect the producers of “green electricity” and lead to an expansion of the capacity for producing “green electricity”. On the contrary; an increase of the import wholesale price for electricity will - ceteris paribus - lead to reduced capacity for “green electricity”.

Quite a large number of problems remain to be investigated for the Green Certificate system, e.g. the effects of uncertainty, the separation between physical and financial trade of Green Certificates including forward and future markets, the welfare effects of the system, and the
effects of a simultaneously functioning international market for Green Certificates. Also the question of market power may be important as the producers of “green electricity” may collide and exercise market power by restricting their capacity expansion. These producers have really a strong protection as they may restrict the power generation from other producers by reducing their own capacity.
References


Fig. 1. The short run equilibrium under autarky. The case of $s \in (g, \bar{s})$. 

\[ p(x) - b \]

\[ q^* = p^* - \alpha s^* \]

\[ q^* - r^* \]
Fig 2. The short run equilibrium with external trade. The case of $s \in (\underline{s}, \bar{s})$
Fig. 3. The short run equilibrium with external trade. The case of $s = \bar{s}$. 
Fig. 4. The short run equilibrium with external trade. The case of \( s = s^* \).