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Quality and location choices under price regulation

by

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Quality and location choices under price regulation*

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Abstract

In a model of spatial competition, we analyse the equilibrium outcomes in markets where the product price is exogenous. Using an extended version of the Hotelling model, we assume that firms choose their locations and the quality of the product they supply. We derive the optimal price set by a welfarist regulator and find that this (second-best) price causes over-investment in quality and an insufficient degree of horizontal differentiation (compared with the first-best solution) if the cost of investing in product quality, or the transportation cost of consumers, is sufficiently high. Comparing with the case of price competition, we also identify a hitherto unnoticed benefit of regulation, namely improved locational efficiency.

Keywords: Spatial competition; Product quality; Location; Price regulation.

JEL classification: L13, L50, R30, R38

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1 Introduction

In this paper we study the strategic interaction between horizontal differentiation and the supply of quality in markets which are subject to price regulation. Imperfect competition does not generally guarantee an optimal supply of quality, nor locational efficiency. This could provide a rationale for regulation. In the present paper we characterise the optimal regulated price in markets where firms compete along both a vertical and a horizontal dimension.

It is well known that the market cannot always be relied upon to supply a socially efficient level of product quality. This is illustrated in a seminal paper by Spence (1975), within a monopoly framework.\(^1\) Introducing competition between firms, Ma and Burgess (1993) identify another potential inefficiency caused by the strategic interaction between quality and price competition, that will generally lead to sub-optimal product quality. When quality and price decisions are made sequentially, firms will under-invest in quality in order to dampen price competition. A regulator can then make the firms commit to a higher level of product quality by eradicating price competition.

In imperfectly competitive markets, though, an important part of the strategic interaction among firms also occur along a spatial dimension. It is well known that the location choices of firms, interpreted in either geographical space or product space, are highly dependent on whether or not prices are regulated. For instance, Anderson and Engers (1994) show that price-taking firms will agglomerate at the market centre in a spatial duopoly if demand is sufficiently inelastic, a result which corresponds with Hotelling’s (1929) prediction of minimum differentiation.\(^2\) On the other hand, if firms are allowed to compete in prices they can reduce competition by locating further apart. In another seminal contribution, D’Aspremont et al. (1979) show that, under certain conditions, price competition induces the firms to locate at either extreme of the Hotelling-line; a result often referred to as the ‘Principle of Maximum Differentiation’. From a welfare-point-of-view it is clear that neither location at the market centre nor location at the market borders are desirable.

\(^1\) The incentive to provide quality is related to the marginal willingness to pay for quality, for the marginal consumer in the case of a profit-maximising firm, and for the average consumer in the case of a social planner. Depending on the difference between the consumers’ marginal and average valuations, the supply of quality may be higher or lower than the social optimum.

\(^2\) In a related paper, Hinloopen (2002) analyses the location choices of firms in a price regulated spatial duopoly where consumers’ reservation prices may bind in equilibrium.
The case of location-quality competition has received relatively little attention in the literature, and is therefore less understood. The purpose of this paper is to examine the interaction between location and quality choices made by competing firms facing a fixed product price, and to explore welfare implications and optimal regulation of prices in such markets. To do so we employ the following three-stage spatial duopoly model: first, a welfarist regulator sets the price of the product (or the third party payment). Second, the firms choose location, or specialisation, of their product on the ‘unconstrained’ Hotelling-line. Third, the firms invest in quality before the consumers decide which product to purchase.

A prime example of where the situation analysed in this paper applies is the health care market. In response to the peculiarities of medical services or, more general, health goods, compensation of health care suppliers is, in most countries, set by some regulatory authority. In the absence of price as a strategic variable, profit maximising health care suppliers will resort to other variables to increase profits. As a patient’s decision on which supplier to attend crucially depends on the (perceived) quality levels provided and on the specialisations chosen, suppliers will set quality (vertical differentiation) and specialisation (horizontal differentiation) strategically. The horizontal dimension could also be interpreted in the geographical sense. Consider two physicians providing the same quality of care. A patient would then simply consult the medical practice he lives closer to.

Our main findings are the following: first, a higher price will increase the equilibrium level of quality, but it will also induce the firms to locate further apart. The higher the price-cost margin, the higher are the benefits, in terms of profits, of capturing a larger share of the market, inducing the firms to compete more intensively on quality. However, firms have then an incentive to locate further apart in order to dampen quality competition. Second, we find that if the cost of investing in product quality, or the transportation cost of consumers, is sufficiently high, the optimal (second-best) price causes over-investment in quality and an insufficient degree of differentiation, compared with the first-best.

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3 The market for prescription drugs may also serve as an example. In this market, drug prices are often regulated by the government, at least in most European countries, and pharmaceutical firms compete for consumers in terms of vertical and horizontal product differentiation.

4 Considering the market for primary care, the geographical interpretation of ‘distance’ is perhaps the most relevant. However, for secondary care we may think of distance as a measure of horizontal product differentiation. For instance, Calem and Rizzo (1995) interpret location choice as hospitals choosing a speciality mix, with the Hotelling-line reflecting patients’ preferences over different service mixes.
outcome.

Comparing with the case of price competition we are also able to identify a second source of inefficiency that provides an additional argument for the desirability of regulating prices. In our model, regulation will not only yield a higher supply of quality, but it will also generally lead to improved locational efficiency.

Finally, we also briefly consider the case of partial commitment, where the regulator is not able to commit to a price before locational decisions are made. Optimal regulation in this regime yields an efficient supply of quality, but too much differentiation.

This paper relates to the following literature: in the aforementioned paper by Ma and Burgess (1993) it is shown that price regulation reduces inefficiencies in the provision of quality in a spatial duopoly. Moreover, Wolinsky (1997) extends the former study both in terms of optimal market regime (managed competition versus regulated monopolies) and asymmetric information. However, in both studies locations are exogenous, and thus the interaction between quality and location is not investigated.

Economides (1989) considers both quality and location choices under price competition, whereas Bester (1998) analyses the effect of imperfect information about quality on firms’ location choices in a similar model. Price regulation, however, is not an issue in either paper.

Two other related papers, which are applied to health care markets, are Gravelle (2000) and Nuscheler (2002). In both cases, though, attention is directed towards entry of firms in a circular model, which means that the distance between firms are determined by the number of firms entering the market, so the focus of these papers are quite different from the present one in this respect. Finally, in a paper applied specifically to the hospital market, Calem and Rizzo (1995) consider the interaction between location and quality choices under the assumption that hospitals cover a fraction of their patients’ transportation costs. This paper differs from ours in two important ways, though. Firstly, they do not consider optimal regulation, which is a major issue in the present paper. Secondly, the very particular assumptions in their model reduce the applicability beyond hospital markets.

The remainder of the paper is organised as follows. In Section 2 we present the main ingredients of the model. In Section 3 we analyse the strategic relationship between quality and location choices when the firms face an exogenous product price. In Section 4 we derive the optimal regulated price and the corresponding equilibrium outcome, whereas a comparison between competition and regulation is discussed in Section 5. In section 6 we also briefly consider the case of partial commitment.
Finally, some concluding remarks are offered in Section 7.

2 The model

A unit mass of consumers are distributed uniformly on the line segment [0, 1]. Each of two identical single-product firms, indexed by \( i = 1, 2 \), choose a location \( x_i \in R \) and a quality level \( q_i \geq 0 \). Both firms charge the same exogenous (regulated) price \( p \) for the product.\(^5\) Without loss of generality, we assume that \( x_1 \leq x_2 \).

Each consumer demands one unit of the good. The utility derived by a consumer located at \( z \) from getting a unit of the product from firm \( i \) is given by

\[
U(z, x_i, q_i) = v + q_i - t\ (z - x_i)^2 - p. \tag{1}
\]

This utility specification implies that consumers always prefer higher quality. We assume that the gross utility, \( v + q_i \), is always large enough for the whole market to be covered, even at \( q_i = 0 \). Given that \( v \) is assumed equal for all consumers, and that both firms charge the same price, the location \( \bar{z} \) of the consumer who is indifferent between buying the product from either firm is the solution to

\[
q_1 - t\ (\bar{z} - x_1)^2 = q_2 - t\ (x_2 - \bar{z})^2 \tag{2}
\]

and given by

\[
\bar{z} = \frac{1}{2}\ (x_1 + x_2) + \frac{q_1 - q_2}{2t\ (x_2 - x_1)}. \tag{3}
\]

Let \( y_i \) be the total demand facing firm \( i \). With a uniform distribution of consumers, the distribution of market shares between the two firms is given by \( y_1 = \bar{z} \) and \( y_2 = 1 - \bar{z} \).

The marginal cost of production, denoted \( c \), is assumed to be constant and independent of locations. The cost of achieving a quality level \( q_i \) is determined by a quadratic cost function \( C(q_i) = k q_i^2 \), where \( k > 0 \).\(^6\) The profit of firm \( i \) is thus

\[
\pi_i = (p - c)\ y_i - k q_i^2, \quad i = 1, 2. \tag{4}
\]

\(^5\) Alternatively, we can think of this as the payment transferred from a third party (e.g. an insurer or a governmental agency) to the firms. The analytical exposition is simplified by considering a single price for both firms. Due to the symmetric nature of the model, the equilibrium outcome is obviously not affected by this simplification.

\(^6\) The assumption that the firms’ costs are separable in quality and quantity implies that quality has the characteristics of a public good for the consumers. This is a standard assumption in the literature (see e.g. Economides, 1989, 1993; Calem and Rizzo, 1995; Gravelle and Masiero, 2000). Allowing also for production-dependent quality costs mitigates the inefficiency in quality provision somewhat, but does not qualitatively change the analysis. Due to analytical tractability, we focus on the special case of completely production-independent quality costs.
We consider the following three-stage game:
Stage 1: The regulator sets a price $p$.
Stage 2: The firms simultaneously choose locations $x_1$ and $x_2$.
Stage 3: The firms simultaneously choose the quality levels $q_1$ and $q_2$.

This sequence of moves relies on the assumptions that (i) the regulator is able to pre-commit to a regulatory policy,\textsuperscript{7} and (ii) choice of location is more of a long-term decision than choice of product quality.\textsuperscript{8}

3 Equilibrium qualities and locations

We start out by deriving the Nash equilibrium outcome for a given price $p$, in order to analyse how the firms’ choices of location and quality are determined by the regulated price. As usual, the game is solved by backwards induction.

3.1 Quality competition

For a given pair of locations $(x_1, x_2)$ and a given price $p$, firm $i$’s choice of quality is found by maximising (4) with respect to $q_i$, yielding\textsuperscript{9}

$$q_i^*(x_1, x_2, p) = \frac{p - c}{4tk\Delta}, \quad i = 1, 2,$$

(5)

where

$$\Delta \equiv x_2 - x_1.$$

The first observation to be made is that the equilibrium levels of quality depend only on relative, and not absolute, locations. In other words, only the distance between the firms, $\Delta$, matters.\textsuperscript{10} Thus, the firms will always invest equally much in quality, even if they are asymmetrically located. This is due to the absence of price competition. When prices

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\textsuperscript{7}The assumption of commitment can be justified by a reputation argument. Commitment can also be obtained by creating institutional mechanisms that makes it costly, or otherwise difficult, to change the regulated price. In Section 5 we will briefly consider the case of partial commitment, where the regulator is not able to commit to a price prior to location decisions.

\textsuperscript{8}If location is interpreted in product space, the assumption that location decisions precede quality decisions seems to be more logically consistent than the alternatives.

\textsuperscript{9}The second-order conditions are satisfied since $\frac{\partial^2 q_i^*}{\partial q_i^2} = -2k < 0$ for $i = 1, 2$.

\textsuperscript{10}From (5) we also see that $q_i^* \to \infty$ when $\Delta \to 0$. This illustrates a special feature of quality competition in this setting, namely that firms could earn negative profits if they are located too close together. In the two-stage equilibrium, where firms choose locations, this will only be the case if the price-cost margin, $p - c$, is very large (cf. eq. (9)). A similar example of ruinous competition is found by Calem and Rizzo (1995). In the equilibrium with optimal price regulation, to be derived later, this is not a problem unless $t$ or $k$ is extremely small.
are exogenous, there is only a market share effect of quality investments. By increasing the level of quality, firm $i$ is able to capture a larger share of the market by ‘pushing’ the indifferent consumer in the direction of the rival firm. Since consumers are uniformly distributed, this effect does not depend on absolute locations.

The optimal level of quality is decreasing in the distance between the firms. This is due to the convexity of transportation costs. From the viewpoint of either firm, the further apart the firms are located, the smaller is the market share captured by a marginal increase in quality. Thus, differentiation softens quality competition. A similar kind of argument applies for the negative relationship between $q_i^*$ and $t$. The more costly it is for consumers to ‘travel’, the smaller are the benefits, in terms of increased market shares, for either firm of investing in quality improvements. This implies that the local monopoly power of firms increase as $t$ increases.\footnote{Note that an increase in $t$ is equivalent to an increase in market size. If we use the product space interpretation of horizontal differentiation, an increase in transportation costs can be interpreted as more heterogeneous consumer preferences.}

Obviously, the optimal level of quality depends on the direct costs of quality investments as well, and (5) confirms the expected negative relationship between $q_i^*$ and the cost parameter $k$. Finally, we also observe from (5) that the optimal level of quality is increasing in the price level, $p$. With the assumption of constant marginal costs, this result is quite intuitive. The higher the price-cost margin, the higher are the benefits, in terms of profits, of capturing a larger share of the market. Consequently, the stronger is the incentive to increase the level of quality. Indeed, a positive price-cost margin is a necessary condition for the firms to invest in quality. From (5) we see that $q_i^* = 0$ for $p = c$.

### 3.2 Location choice

At stage two of the game, the firms simultaneously choose their locations, anticipating the quality pair $(q_1^*(x_1, x_2, p), q_2^*(x_1, x_2, p))$ at the subsequent stage of the game. Inserting (5) into (4), the first-order condition for the optimal location of firm 1 is given by

$$\frac{\partial \pi_1}{\partial x_1} = \frac{p - c}{8} \left( 4 - \frac{p - c}{kt^2 \Delta^3} \right) = 0.$$ 

We are looking for a Nash equilibrium in symmetric locations. Setting $x_2 = 1 - x_1$ (which implies $\Delta = 1 - 2x_1$), the symmetric Nash equilibrium
is given by\(^\text{12}\)
\[
x_1^*(p) = \frac{1}{2} (1 - \Delta^*)
\]
(6)
and
\[
x_2^*(p) = \frac{1}{2} (1 + \Delta^*),
\]
(7)
where
\[
\Delta^* \equiv x_2^* - x_1^* = \left( \frac{p - c}{4t^2k} \right)^{\frac{1}{3}}.
\]
(8)

An important observation is that quality competition induces the firms to locate apart. In the absence of quality competition, we know that exogenous prices cause the firms to agglomerate at the market centre. In this model, the absence of quality competition can be thought of as prohibitively high investment costs. Indeed, from (8) it is confirmed that \(\lim_{k \to \infty} \Delta^* = 0\). However, the possibility of quality-enhancing investments introduces a degree of competition that the firms are able partly to avoid by locating away from each other. The less costly it is to increase the quality of the product, i.e. the lower is \(k\), the stronger are the incentives to avoid quality competition, and consequently, the larger is the distance between the firms in equilibrium. Furthermore, the higher the local monopoly power of firms, i.e. the higher \(t\), the smaller are differentiation incentives.

Inserting (8) into (5), the equilibrium levels of quality, for a given price level, are given by
\[
q_i^*(p) = \left( \frac{(p - c)^2}{16tk^2} \right)^{\frac{1}{3}}, \quad i = 1, 2.
\]
(9)

For exogenous prices, the comparative statics results for location and quality can be summarised as follows:

**Proposition 1** The equilibrium levels of quality, as well as the equilibrium distance between the firms, are decreasing in \(k\) and \(t\), and increasing in \(p\).

\(^{12}\)The second order conditions are satisfied, since \(\frac{\partial^2 q_i}{\partial x_i^2} = -\frac{3}{k} \frac{(p-c)^2}{8t^2k^2} < 0\). Note that although this is the unique symmetric equilibrium, there are also asymmetric equilibria. The first-order conditions reveal the existence of a continuum of equilibria with the same distance between the firms:
\[
x_1^* = a \in \left( \frac{1}{2} - \Delta^*, \frac{1}{2} \right),
\]
\[
x_2^* = a + \Delta^*.
\]

The choice of the symmetric equilibrium can be justified by a focal point argument.
Proof. Follows immediately from (8) and (9). ■

From the discussion of the last subsection, we know that an increase in the price level will, ceteris paribus, induce the firms to increase quality, implying that the competition between the firms intensifies. The firms have incentives to dampen this effect, though, by locating further apart. However, Proposition 1 confirms that the latter (indirect) effect is smaller than the former (direct) effect. Consequently, an increase in the product price leads to increased quality in equilibrium. There are similar mechanisms at work for the comparative statics results regarding the other two parameters. When locations are endogenous, the direct negative effect on quality from an increase in \( t \) or \( k \) is partly mitigated by a smaller distance between the firms in equilibrium, resulting in stronger incentives for quality investments. The overall effect, though, is a decrease in the equilibrium levels of quality.

4 Optimal price regulation

In this section we analyse how a regulator should optimally set the price in this particular market. The desirability of price regulation can arise for several reasons. Importantly, allowing for price competition generally leads to suboptimal equilibrium levels of quality, as well as socially inefficient locations, in this type of model, due to the strategic interaction between the firms. This could, in itself, create a potential role for regulation. However, we also want to treat this model as a depiction of markets in which price regulation is viewed as desirable due to e.g. distributional considerations or the presence of insurance, like in health care markets.

The vertical and horizontal dimensions of the product (quality and locations) are assumed to be non-contractible,\(^{13}\) leaving the product price as the only regulatory instrument. We assume that the regulator maximises the sum of consumers’ and producers’ surplus.\(^{14}\) Due to the symmetric features of the model, the first-best solution must also necessarily be symmetric. Setting \( q_1 = q_2 = q \) and \( x_2 = 1 - x_1 \), social welfare is given by

\[
W = q (1 - 2kq) + \frac{t (6\Delta x_1 - 1)}{12} - c. \tag{10}
\]

\(^{13}\)Due to measurement problems related to vertical and horizontal differentiation, these variables will typically be non-verifiable in a contractual sense.

\(^{14}\)If we interpret the model in the context of health care markets with third-party payers, this particular specification of the welfare function relies implicitly on the assumption that the third party (i.e. the regulator) is able to raise the necessary funds in a non-distortionary manner.
4.1 The first-best solution

For comparative purposes, we start out by considering the socially optimal first-best solution. With the assumption of unit demand, there is no efficiency loss associated with a price in excess of marginal costs, so the only relevant variables are locations and quality. The first-best solutions are easily calculated as\(^\text{15}\)

\[
x_1^{fb} = \frac{1}{4}, \quad x_2^{fb} = \frac{3}{4}
\]

and

\[
q_1^{fb} = q_2^{fb} = \frac{1}{4k}.
\]

The first-best solution is characterised by a pair of locations that minimises total transportation costs for consumers. When consumers are uniformly distributed on the line segment \([0, 1]\) this pair of locations is given by \((\frac{1}{4}, \frac{3}{4})\). The first-best solution also requires a quality level that equates marginal revenues and marginal costs.

4.2 The second-best solution

When the regulator is not able to control locations and quality directly, but only indirectly through the price level, the equilibrium outcome is generally expected to fall short of the first-best solution. Before scrutinising whether this is indeed the case, we will first consider the case of exogenous locations.

4.2.1 Exogenous locations

If locations are exogenous, the socially optimal levels of quality can be achieved at all possible locations, by imposing the appropriate price level. For simplicity, we will consider the case of symmetric locations. Substituting from (5) into (10), the first-order conditions for a welfare-maximising price \(p^*\) is given by

\[
\frac{\partial W}{\partial p} = \frac{\Delta t - (p - c)}{4t^2k\Delta^2} = 0,
\]

which yields\(^\text{16}\)

\[
p^* = c + \Delta t.
\]

An almost trivial, yet important, observation is that optimal price regulation implies a price in excess of marginal production costs. A

\(^{15}\)The second-order conditions are satisfied, since \(\frac{\partial^2 W}{\partial x_1^2} = -2t < 0\), \(\frac{\partial^2 W}{\partial q^2} = -4k < 0\) and \(\frac{\partial^2 W}{\partial x_1 \partial q} = 0\).

\(^{16}\)The second-order condition is satisfied since \(\frac{\partial^2 W}{\partial p^2} = -\frac{1}{4k^2\Delta^2} < 0\).
positive mark-up is necessary in order to induce the firms to undertake quality investments. More interesting, though, is the following result:

**Proposition 2** With exogenous symmetric locations, the optimal regulated price is an increasing function of the distance between the firms.

**Proof.** Follows immediately from (13).

The intuition is relatively straightforward. Although the distance between the firms influences the incentives for quality investments, the socially optimal level of product quality is independent of locations. For a given price, the further apart the firms are located, the less intense is quality competition, and consequently, the lower are the equilibrium levels of quality. The regulator can stimulate quality investments by increasing the price, which increases the marginal revenue of such investments. Thus, the further apart the firms are located, the higher is the price that is required to provide the firms with sufficient incentives to invest at the socially optimal quality level.

**4.2.2 Endogenous locations**

If the firms are able to choose their locations, the regulator must take into account how the regulated price affects not only quality, but also the choice of locations. From Proposition 1 we know that a higher price induces higher quality and more horizontal differentiation. Before solving explicitly for the optimal price we can use the previously established results to characterise the second-best solution. Assuming the Nash equilibrium to be symmetric in locations, we are able to state the following:

**Proposition 3** When locations are endogenous, the first-best outcome is achieved only if \( t = \frac{1}{k} \). For \( t \neq \frac{1}{k} \), the second-best outcome is characterised by (i) under-investment in quality and too much differentiation if \( t < \frac{1}{k} \), and (ii) over-investment in quality and insufficient differentiation if \( t > \frac{1}{k} \).

**Proof.** Assuming a symmetric equilibrium, the first-order condition for an optimal price \( p^* \) is given by

\[
\frac{\partial W(x_1(p), q(p))}{\partial p} = \frac{\partial W}{\partial x_1} \frac{\partial x_1}{\partial p} + \frac{\partial W}{\partial q} \frac{\partial q}{\partial p} = 0, \tag{14}
\]

where \( x_1(p) \) and \( q(p) \) are given by (6) and (9), respectively. Denote the price that yields first-best locations by \( \hat{p} \). We can calculate this price by solving (6) for \( p \) with \( x_1 = x_1^{fb} = 4 \). This yields \( \hat{p} = c + \frac{1}{2}kt^2 \). Inserting \( p = \hat{p} \) into (9), we find the equilibrium quality at this price to
be \( q(\bar{p}) = \frac{1}{4} t \). Comparing with the first-best level of quality, from (12), we find that \( q(\bar{p}) - q^b = \frac{t_k - 1}{4k} \). Thus,

\[
q(\bar{p}) < (>) q^b \text{ if } t < (>) \frac{1}{k}.
\]

(15)

Consider the case of \( t > \frac{1}{k} \). Since \( q(\bar{p}) > q^b \), this means that \( \frac{\partial W}{\partial q} < 0 \) at \( q = q(\bar{p}) \). From Proposition 1 we also know that \( \frac{\partial x_1}{\partial p} < 0 \) and \( \frac{\partial q}{\partial p} > 0 \). Furthermore, at \( x_1 = x_1^b \) it follows that \( \frac{\partial W}{\partial x_1} = 0 \). Thus, we have that

\[
\frac{\partial W}{\partial x_1} \frac{\partial x_1}{\partial p} + \frac{\partial W}{\partial q} \frac{\partial q}{\partial p} < 0 \quad \text{at} \quad p = \bar{p}.
\]

(16)

Consequently, no \( p \) can ensure that \( \frac{\partial W}{\partial x_1} = \frac{\partial W}{\partial q} = 0 \). For the first-order condition to hold, the first term in (16) must be positive. This can only be achieved by setting \( p < \bar{p} \), which yields \( x_1 > x_1^b \) and implies that \( \frac{\partial W}{\partial x_1} < 0 \). The second-best outcome is thus achieved by setting a price \( p^* \) where \( \frac{\partial W}{\partial x_1} < 0 \) and \( \frac{\partial W}{\partial q} < 0 \) at the equilibrium pair \((x_1(p^*), q(p^*))\), implying \( x_1(p^*) > x_1^b \) and \( q(p^*) > q^b \). By symmetry, the opposite result applies for \( t < \frac{1}{k} \), and the first-best outcome is only achieved at \( t = \frac{1}{k} \). ■

In general, first-best locations can only be achieved at the cost of a suboptimal level of quality, and vice versa, from a viewpoint of social welfare. Consequently, the regulator faces a trade-off between quality and horizontal differentiation in implementing the second-best solution. Proposition 3 states that if it is sufficiently costly to improve the quality of the product, or if it is sufficiently costly for consumers to ‘travel’, then the second-best solution is characterised by too much quality and insufficient differentiation. Conversely, if \( k \) or \( t \) are sufficiently low, the opposite result applies.

To understand the intuition behind this result, consider the price \( p = c \) as a candidate optimal price. In this case equilibrium quality will be zero and the firms will agglomerate at the market centre. By increasing the price above \( c \) the regulator can induce the firms to invest in a higher level of quality, and as a response to increased quality competition the firms will also choose to differentiate horizontally, in order to dampen competition. We can call this the ‘quality effect’ and ‘centrifugal effect’, respectively. Obviously, from an initial situation of \( q = 0 \) and \( x_1 = x_2 = \frac{1}{2} \), both effects will increase social welfare. The characteristics of the second-best equilibrium depend \textit{inter alia} on the relative strength of these two effects. If \( t \) is high, then the ‘centrifugal effect’ is relatively weak, because it only takes a small increase in the distance between
the firms in order to dampen competition considerably. Consequently, the price level necessary to induce first-best locations is so high that it provides incentives for over-investment in quality. This is also the case if \( k \) is relatively high, but for partly different reasons. If the cost of improving the quality of the product is high, then the first-best level of quality is relatively low. Thus, first-best quality incentives are achieved at a relatively low price, which is not high enough to induce a sufficient degree of differentiation.

Substituting from (6), (8) and (9) into (10), and maximising with respect to \( p \), we find the following expression for the optimal price:\^{17}

\[
p^* = c + \frac{1}{32} \left( 4t + 6 \left( \frac{t}{k} \right)^{\frac{1}{2}} (\Phi + \Phi^{-1}) \right),
\]

where\^{18}

\[
\Phi = \left( (tk)^{\frac{1}{2}} + (tk - 1)^{\frac{1}{2}} \right)^{\frac{1}{3}}.
\]

The relationship between the cost parameters, \( t \) and \( k \), and the optimal regulated price is given by the following comparative statics result:

**Proposition 4** The optimal price \( p^* \) is increasing in \( t \) and decreasing in \( k \).

A proof is given in the appendix.

The intuition behind these results is related to Proposition 1. Higher transportation costs mean that the intensity in quality competition is reduced, since it becomes more difficult to 'steal' market shares from the competitor, and the benefits of quality investments are thus decreased. This also implies that the firms’ incentives to differentiate horizontally is reduced. Consequently, it is necessary to increase the regulated price in order to counteract these effects.

The negative relationship between the optimal price and the investment cost parameter, \( k \), is not a straightforward result, since there are contradicting forces at play. We know from Proposition 1 that a higher

\[
\frac{\partial^2 W}{\partial p^2} = -\frac{2^{\frac{1}{3}} \left[ 2k^{\frac{1}{3}} (4 (p - c) + t) + 2^{\frac{1}{3}} 3 (t (p - c))^{\frac{1}{3}} \right]}{72 (tk)^{\frac{2}{3}} (p - c)^{\frac{1}{3}}} < 0
\]

\^{17}It can be shown that \( (\Phi + \Phi^{-1}) = 2 \cos \frac{\theta}{3} \), where \( \theta = \arccos (tk)^{-\frac{1}{2}} \). Thus, \((\Phi + \Phi^{-1}) \in R\) for all \( t, k \geq 0 \), even though \( \Phi \) is a complex number for \( t < \frac{1}{k} \). See also the appendix.
cost of quality reduces the firms’ incentives for quality investments and horizontal differentiation. *Ceteris paribus*, this should lead to a higher optimal price. However, if quality investments become more costly, then the first-best level of product quality is reduced. Proposition 4 confirms that the latter effect dominates, so that the optimal price is a decreasing function of \( k \).

Tables A and B illustrate numerically how the equilibrium outcome under the optimal regulatory regime depends on the parameters \( t \) and \( k \). In Table A we show how the optimal price, and the corresponding equilibrium values of quality, horizontal differentiation and profits, vary with \( t \), when \( k \) and \( c \) are set equal to 1. The equivalent results for a fixed value of \( t \) is presented in Table B.

Higher transportation costs imply that the regulator has to increase the price in order to increase quality investments. From Table A, we see that the firms are in some sense over-compensated in the optimal regulatory regime, so that the level of product quality is increasing in \( t \). We also observe that even though firms spend more resources on quality investments, the price increase is sufficiently large to secure higher profits for higher values of \( t \).

A similar pattern is found in Table B. We see that the price effect is the important one in determining equilibrium profits, so that a lower price means lower profits, even though the firms spend less resources on quality investments.

<table>
<thead>
<tr>
<th>( t )</th>
<th>Price</th>
<th>Quality</th>
<th>Distance</th>
<th>Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>1.181</td>
<td>0.218</td>
<td>1.043</td>
<td>0.043</td>
</tr>
<tr>
<td>0.4</td>
<td>1.377</td>
<td>0.229</td>
<td>0.756</td>
<td>0.086</td>
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<td>0.237</td>
<td>0.629</td>
<td>0.123</td>
</tr>
<tr>
<td>0.8</td>
<td>1.431</td>
<td>0.244</td>
<td>0.552</td>
<td>0.156</td>
</tr>
<tr>
<td>1</td>
<td>1.500</td>
<td>0.250</td>
<td>0.500</td>
<td>0.188</td>
</tr>
<tr>
<td>1.2</td>
<td>1.565</td>
<td>0.255</td>
<td>0.461</td>
<td>0.217</td>
</tr>
<tr>
<td>1.4</td>
<td>1.628</td>
<td>0.260</td>
<td>0.431</td>
<td>0.246</td>
</tr>
<tr>
<td>1.6</td>
<td>1.688</td>
<td>0.264</td>
<td>0.407</td>
<td>0.274</td>
</tr>
<tr>
<td>1.8</td>
<td>1.746</td>
<td>0.268</td>
<td>0.386</td>
<td>0.301</td>
</tr>
<tr>
<td>2</td>
<td>1.803</td>
<td>0.272</td>
<td>0.369</td>
<td>0.328</td>
</tr>
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</table>
Table B: Equilibrium outcomes for \( c = 1, t = 1 \).

<table>
<thead>
<tr>
<th>( k )</th>
<th>Price</th>
<th>Quality</th>
<th>Distance</th>
<th>Profits</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1.043</td>
<td>0.217</td>
</tr>
<tr>
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<td>0.756</td>
<td>0.215</td>
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<tr>
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<td>1.596</td>
<td>0.395</td>
<td>0.629</td>
<td>0.205</td>
</tr>
<tr>
<td>0.8</td>
<td>1.539</td>
<td>0.305</td>
<td>0.552</td>
<td>0.195</td>
</tr>
<tr>
<td>1</td>
<td>1.500</td>
<td>0.250</td>
<td>0.500</td>
<td>0.188</td>
</tr>
<tr>
<td>1.2</td>
<td>1.471</td>
<td>0.213</td>
<td>0.461</td>
<td>0.181</td>
</tr>
<tr>
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<td>0.431</td>
<td>0.176</td>
</tr>
<tr>
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<td>1.430</td>
<td>0.165</td>
<td>0.407</td>
<td>0.171</td>
</tr>
<tr>
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<td>1.415</td>
<td>0.149</td>
<td>0.386</td>
<td>0.167</td>
</tr>
<tr>
<td>2</td>
<td>1.402</td>
<td>0.136</td>
<td>0.369</td>
<td>0.164</td>
</tr>
</tbody>
</table>

5 Regulation versus competition

In this section we want to elaborate somewhat on the benefits of price regulation in this model, by contrasting the equilibrium derived in the previous sections with the case where the firms are allowed to compete in prices.

For the case of price competition, we assume that the firms simultaneously set prices at a new third stage of the game, after locations and quality investments have been decided. The choice of this particular timing of the game rests on the assumption that prices are more flexible than qualities.

We can simplify the exposition by assuming that the firms are confined to choosing locations within the market boundaries, i.e. that \( x_i \in [0, 1], i = 1, 2 \). Furthermore, in order to secure an equilibrium in the location-quality-price game we also make the assumption that \( k > \frac{2}{9} \).

The case of price competition in this model is highly similar to Economides (1989), so the derivation of the full equilibrium will be kept fairly short. If \( p_i \) is the price charged by firm \( i \), the indifferent consumer is located at

\[
\tilde{z} = \frac{1}{2} (x_1 + x_2) + \frac{q_1 - q_2 - (p_1 - p_2)}{2t (x_2 - x_1)},
\]

whereas profits are given by

\[
\pi_i = (p_i - c) \tilde{y}_i - kq_i^2, \quad i = 1, 2,
\]

where \( \tilde{y}_1 = \tilde{z} \) and \( \tilde{y}_2 = 1 - \tilde{z} \).

Solving the game backwards, we derive the following expressions for
prices and quality levels as functions of locations:

\[
p_1 = \frac{c - t \left[ x_2 - x_1 \right] \left[ 9ck + 3tk \left( 2 + x_2 + x_1 \right) (x_2 - x_1) - 1 \right]}{1 - 9tk \left( x_2 - x_1 \right)}, \tag{20}
\]

\[
p_2 = \frac{c + t \left[ x_2 - x_1 \right] \left[ 1 + 3tk \left( x_1 + x_2 - 4 \right) (x_2 - x_1) - 9ck \right]}{1 - 9tk \left( x_2 - x_1 \right)}, \tag{21}
\]

\[
q_1 = \frac{1 - 3tk \left( 2 + x_1 + x_2 \right) (x_2 - x_1)}{6k \left[ 1 - 9tk \left( x_2 - x_1 \right) \right]}, \tag{22}
\]

and

\[
q_2 = \frac{1 + 3tk \left( x_1 + x_2 - 4 \right) (x_2 - x_1)}{6k \left[ 1 - 9tk \left( x_2 - x_1 \right) \right]}. \tag{23}
\]

Due to the symmetric nature of the model, we can focus attention towards symmetric locations. The partial derivative of firm 1’s profit function with respect to its own location \((x_1)\), evaluated at \(x_2 = 1 - x_1\), is given by

\[
\frac{\partial \pi_1}{\partial x_1} = -\frac{(72t^2k + 18ktx_1 + 2x_1 - 9kt + 2)t}{6 \left[ 1 + 9kt \left( 2x_1 - 1 \right) \right]}. 
\]

It is easily confirmed that \(\frac{\partial \pi_1}{\partial x_1} < 0\) for all \(x_1 \in \left[ 0, \frac{1}{2} \right]\) when \(k > \frac{2}{9t}\). Thus, equilibrium locations are given by \(x_1^* = 0, x_2^* = 1\). In other words, price competition induces maximal horizontal differentiation, with the firms locating at the market borders. This result mirrors Economides (1989).

Inserting the equilibrium locations into (20)-(23), we obtain the following simple expressions for prices and qualities:

\[
p_i = c + t, \tag{24}
\]

\[
q_i = \frac{1}{6k}. \tag{25}
\]

Comparing (12) and (25), we observe that price competition does not yield sufficient incentives for quality investments, compared with the socially optimal level.\(^{19}\) This is due to the strategic effect of quality investments on price competition. If a firm improves the quality of its product, the competing firm has a stronger incentive to reduce prices in order to mitigate the loss of market share. Consequently, the firms can dampen price competition by offering lower-quality products.\(^{20}\)

\(^{19}\)This result is in fact more general, since it is easily confirmed that (25) holds for any exogenous symmetric location \(x_2 = 1 - x_1\).

\(^{20}\)This strategy is somewhat related to the classic ‘puppy-dog ploy’ introduced by Fudenberg and Tirole (1984).
In a similar model with fixed locations, Ma and Burgess (1993) show that price competition yields sub-optimal quality levels if there are some fixed (i.e. production-independent) costs associated with quality investments. This provides a rationale for price regulation. A quite simple, but still important, point that can be added within the context of the present model is that *endogenous* locations provide a second argument for price regulation. Since price competition yields too much horizontal differentiation, locational efficiency can be improved by introducing a fixed (regulated) price.

The isolated effect of price regulation can perhaps most clearly be seen by letting a regulator impose a price level which is equal to the equilibrium price that would emerge if the firms were allowed to compete on prices.

Inserting $p = c + t$ into (5) yields

$$q_i^* = \frac{1}{4k\Delta}.$$  \hspace{1cm} (26)

Comparing (25) and (26) we see that the effect of regulation *per se* is an increase in quality. Furthermore, by inserting $p = c + t$ into (8) we obtain

$$\Delta^* = \left(\frac{1}{4tk}\right)^{\frac{1}{3}}.$$  \hspace{1cm} (27)

We see that $\Delta^*$ is strictly less than 1 if $t > \frac{1}{4k}$. Thus, price regulation does not only provide higher-quality products, it also ensures improved locational efficiency for a substantial set of parameter values.

From this exercise we see that the benefits of price regulation do not only emerge from the effects of a lower price level. Even if regulation does not change the product price, there are isolated benefits of using regulation as a means of preventing price competition. By eradicating competition along one dimension (prices), the strategic responses from the firms ensure a higher degree of competition along other dimensions (quality and location).

### 6 Partial commitment

Before the paper is concluded, let us briefly discuss the issue of regulatory commitment. So far we have assumed that the regulator is able to commit to a particular regulatory regime at the start of the game, before the firms make their decisions. However, in some circumstances (e.g. in the absence of long-term contracts), this may not be the most plausible assumption. When full commitment is not possible for the regulator, we may suspect the equilibrium outcome to be quite different and perhaps less desirable from a welfare point-of-view.
In this section we focus on the case of partial commitment, where the regulator is not able to commit to a price before firms decide on locations. Thus, the following game is considered: at stage one firms choose locations; at stage two the regulator sets the price; finally, at stage three firms choose the level of quality.

From the discussion in Section 4 we know that the regulator in this case will set a price that induces a socially optimal supply of quality. This price is given by (13), with the corresponding equilibrium quality levels given by (12).

At the first stage of the game, each firm simultaneously chooses its location, anticipating the optimal regulated price. Solving the firms’ profit maximisation problems with \( p = c + \Delta t \) and \( q_i = \frac{1}{4k} \), we find a unique pair of equilibrium locations, given by

\[
x_1 = 0 \text{ and } x_2 = 1.
\]

Thus, each firm locates at either end of the Hotelling-line if they can precommit to a location before the regulator sets the price, or conversely, if the regulator is unable to commit to a price before firms decide on locations. Note that this equilibrium outcome is not dependent on whether or not we allow the firms to locate outside the market.\(^{21}\)

Since the regulator chooses a price schedule which implements first-best quality irrespective of the firms’ locational choices, the incentives to differentiate horizontally are not caused by a desire to avoid fierce head-on quality competition, as in the previous sections. Horizontal differentiation is rather a strategy for firms to achieve a higher regulated price, since the optimal price is increasing in distance.

However, the incentives for differentiation is still limited by the standard market share effect, which represents a centripetal force in the model. A unilateral relocation away from the market centre by one of the firms implies, \( \textit{ceteris paribus} \), a loss of market share to the other firm. The size of this loss is larger the further away from the market centre a firm is located. Consequently, when firms decide locations non-cooperatively, there are two counteracting forces which prevents locations outside the market borders.

The equilibrium locations yield an equilibrium regulated price

\[
p = c + t,
\]

\(^{21}\)In the product space interpretation of the model, locations outside the market borders mean that the firms offer product variants that do not correspond with the ideal variant of any consumer. This is also a way to portray, albeit rather crudely, a certain degree of consumer concentration in the market.
which is incidentally equal to the equilibrium price under price competition when firms are restricted to locate within the market borders (cfr. (24)). This is also true for the equilibrium locations. Consequently, regulation is still socially beneficial, due to a more efficient supply of quality, even if full regulatory commitment is not possible. However, social welfare under partial commitment will be (weakly) lower than under full commitment. The reason is straightforward: under full commitment, the regulator can always pick the same policy as under partial commitment. The regulator’s ability to trade off quality benefits against locational benefits enables her to improve social welfare.

7 Concluding remarks

In this paper we have analysed the strategic interaction between firms’ choices of product quality and location in a spatial duopoly that is subject to price regulation. When the firms are unable to compete in prices, the degree of horizontal differentiation is determined by the intensity of quality competition, which in turn is determined by the firms’ price-cost margins, as well as transportation and investment costs.

We have derived the socially optimal price under the assumption of pre-commitment by the regulator, and find that the first-best solution will generally not be achieved. If consumers’ transportation costs, or the cost of quality investments, are above a certain threshold level, optimal price regulation yields over-investment in quality, and an insufficient degree of horizontal differentiation, compared with the first-best solution.

In real life, price regulation is often motivated by a desire to avoid excessively high prices in markets with a low degree of competition. This is not an issue in our model. Instead we have focused on the potential benefits of price regulation purely as a means of avoiding price competition. We have identified two different efficiency gains of regulation. In addition to the positive effect on quality investments which has previously been pointed out by Ma and Burgess (1993), we have also provided a second argument for the desirability of price regulation, namely that locational efficiency will in most cases be improved.

There are obviously several well known arguments against price regulation. Most of these are related to potential problems caused by asymmetric information. It should therefore be stressed that this paper does not address such issues. We have instead focused exclusively on the strategic implications of price regulation in a world of perfect information. In an overall evaluation of the desirability of introducing, or upholding, price regulation in a particular industry, the potential efficiency gains identified in this paper should therefore be weighed against the potential problems that have been addressed elsewhere.
A Proof of Proposition 4

From (17) we can derive the following comparative statics expressions:

\[
\frac{\partial p^*}{\partial t} = \frac{1}{32} \left[ 4 + 3(tk)^{-\frac{1}{2}} (\Phi + \Phi^{-1}) + (tk - 1)^{-\frac{1}{2}} (\Phi - \Phi^{-1}) \right] \quad (A.1)
\]

\[
\frac{\partial p^*}{\partial k} = -\frac{1}{32} \left[ 3(tk)^{\frac{1}{2}} k^{-2} (\Phi + \Phi^{-1}) - \frac{t}{k} (tk - 1)^{-\frac{1}{2}} (\Phi - \Phi^{-1}) \right] \quad (A.2)
\]

We have to show that \( \frac{\partial p^*}{\partial t} > 0 \) and \( \frac{\partial p^*}{\partial k} < 0 \) for all \( t > 0, k > 0 \). Assume first that \( t > \frac{1}{k} \). In this case \( \Phi \) is a positive real number, and \( (\Phi - \Phi^{-1}) > 0 \) is a sufficient condition for \( \frac{\partial p^*}{\partial t} > 0 \). Inserting the expression for \( \Phi \) yields

\[
\Phi - \Phi^{-1} = \frac{(tk)^{\frac{1}{2}} + (tk - 1)^{\frac{1}{2}}}{(tk)^{\frac{1}{2}} + (tk - 1)^{\frac{1}{2}}} - 1,
\]

which is unambiguously positive for \( t > \frac{1}{k} \).

Inserting the expression for \( \Phi \) in (A.2) yields

\[
\frac{\partial p^*}{\partial k} = \frac{1}{32} \left[ \frac{A + tk + B\frac{3}{2} (A - tk)}{(tk - 1)^{\frac{1}{2}} B\frac{1}{2} k^2} \right], \quad (A.3)
\]

where

\[
A = 3(tk)^{\frac{1}{2}} (tk - 1)^{\frac{1}{2}}
\]

and

\[
B = (tk)^{\frac{1}{2}} + (tk - 1)^{\frac{1}{2}}
\]

The denominator in the square brackets of (A.3) is always positive for \( t > \frac{1}{k} \), and we see that the numerator is minimized for \( t \to \frac{1}{k} \). It is easily checked, however, that the numerator approaches zero as \( t \to \frac{1}{k} \). Thus, \( \frac{\partial p^*}{\partial k} < 0 \) for \( t > \frac{1}{k} \).

Now assume that \( t < \frac{1}{k} \). In this case \( \Phi \notin R \), and can be expressed as

\[
\Phi = \left( (tk)^{\frac{1}{2}} + (1 - tk)^{\frac{1}{2}} i \right)^{\frac{1}{2}}.
\]

Furthermore, we have that

\[
(tk)^{\frac{1}{2}} + (1 - tk)^{\frac{1}{2}} i = \cos \theta + i \sin \theta,
\]

where

\[
\theta = \arccos (tk)^{\frac{1}{2}}. \quad (A.4)
\]
Note that $\theta$ is a positive real number for $t < \frac{1}{k}$. By De Moivre’s Theorem we have that

$$\Phi = \cos \frac{\theta}{3} + i \sin \frac{\theta}{3}. \quad (A.5)$$

We can then use (A.5) to compute

$$\Phi + \Phi^{-1} = 2 \cos \frac{\theta}{3}. \quad (A.6)$$

Similarly, for $t < \frac{1}{k}$ we have that

$$(tk - 1)^{-\frac{1}{3}} (\Phi - \Phi^{-1}) = (1 - tk)^{-\frac{1}{3}} i (\Phi - \Phi^{-1}),$$

which, by the use of (A.5) and the fact that $(1 - tk)^{\frac{1}{3}} = \sin \theta$, reduces to

$$\frac{2 \sin \frac{\theta}{3}}{\sin \theta}.$$

We can thus re-write (A.1) and (A.2) as

$$\frac{\partial p^*}{\partial t} = \frac{1}{32} \left[ 4 + 6 (tk)^{-\frac{1}{3}} \cos \frac{\theta}{3} + \frac{2 \sin \frac{\theta}{3}}{\sin \theta} \right] \quad (A.7)$$

and

$$\frac{\partial p^*}{\partial k} = -\frac{1}{32} \left[ 6 (tk)^{\frac{1}{3}} k^{-2} \cos \frac{\theta}{3} - \frac{t}{k} \left( \frac{2 \sin \frac{\theta}{3}}{\sin \theta} \right) \right] \quad (A.8)$$

It follows immediately that $\frac{\partial p^*}{\partial t} > 0$ for $t < \frac{1}{k}$.

Using (A.4), and defining $a = tk$, we can re-write (A.8) as

$$\frac{\partial p^*}{\partial k} = -\frac{\Omega k^{-2} a^{\frac{1}{3}}}{32},$$

where

$$\Omega = 3a^{\frac{1}{3}} (1 - a)^{\frac{1}{3}} \cos \left( \frac{1}{3} \arccos a^{\frac{1}{3}} \right) - a \sin \left( \frac{1}{3} \arccos a^{\frac{1}{3}} \right).$$

It follows that $\frac{\partial p^*}{\partial k} < 0$ if $\Omega > 0$. Solving $\Omega = 0$ we find that this equation has two roots: $a = 0$ and $a = 1$. Due to continuity, we can determine the sign of $\Omega$ by inserting numerical values for $a$. By this method it is easily confirmed that $\Omega > 0$ for $a \in (0, 1)$. It follows that $\Omega > 0$, and thus $\frac{\partial p^*}{\partial k} < 0$, for $t < \frac{1}{k}$.

It only remains to check the sign of $\frac{\partial p^*}{\partial k}$ for $t = \frac{1}{k}$. Using either (A.1)-(A.2) or (A.7)-(A.8), it is easily confirmed that $\lim_{t \to \frac{1}{k}} \frac{\partial p^*}{\partial t} = \frac{1}{3} > 0$ and $\lim_{t \to \frac{1}{k}} \frac{\partial p^*}{\partial k} = -\frac{1}{6k^2} < 0$. ■

21
References


