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Team Incentives in Relational Contracts

by

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Abstract: Incentive schemes for teams are compared. I ask: under which conditions are relational incentive contracts based on joint performance evaluation, relative performance evaluation and independent performance evaluation self-enforceable. The framework of Che and Yoo (2001) on team incentives is combined with the framework of Baker, Gibbons and Murphy (2002) on relational contracts. In a repeated game between one principal and two agents, I find that incentives based on relative or independent performance are expected to dominate when the productivity of effort is high, while joint performance evaluation dominates when productivity is low. Incentives based on independent performance are more probable if the agents own critical assets.

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1. Introduction

In the last decade we have seen a growth of group based incentive schemes such as profit sharing, gain sharing and employee ownership schemes to improve motivation and labour relations. It is recognized in private industry that by introducing group incentive compensation systems, it may be possible to induce workers to work both harder and more cooperatively, in a way that enhance their productivity (see Banker, Field, Schroeder and Sinha, 1996; Gerhardt, Minkoff and Olsen, 1995). Yet there has been paid little attention to group incentives in the economics literature. The main reason for this lack of theoretical interest is that it has been difficult to prove the efficiency of such schemes. Economists studying teams, beginning with Alchian and Demsetz (1972), have argued that group incentives are ineffective due to free-riding problems. If you award an equal bonus to each member of a team on the basis of the team’s overall performance, it may give each member an incentive to shirk (free ride). It is not difficult to see that if the members of a team are interacting only once (for example in one project only) and each one knows that this is the one and only project of which they will work together, the free riding problem can easily occur. In such a static one period relationship an incentive scheme based on relative performance evaluation (RPE) is optimal (see Lazear and Rosen, 1981; Holmström, 1982; Mookherjee, 1984). An RPE scheme rewards team members that perform relatively better than their peers. That way an employer can make a worker’s compensation independent from good or bad outside factors (common noise components). This lowers the cost of providing a given level of incentives. The RPE scheme also has the advantage that one only needs to detect relative performance, which can be easier than measuring absolute performance.

In recent years, however, economists have been able to show that joint performance evaluation (JPE) may come out as an optimal solution. A JPE scheme compensates the group members if the group as a whole performs well. Hence, a worker is rewarded if his peers perform well. The value of encouraging employees’ cooperation is emphasized in works such as Holmström and Milgrom (1990), Itoh (1992, 1993), Macho-Stadler and Perez-Castrillo (1993) and particular Kandel and Lazear (1992) who emphasize the effect of peer pressure. In
addition, the folk theorem of repeated games has for some years now provided a possible answer to the free rider critique of group incentives (see Radner, 1986; Weitzman and Kruse, 1990; and FitzRoy and Kraft, 1995). In a repeated setting where agents interact in an unknown number of periods, shirking from some desired, co-operative solution can be deterred by social sanctions including withholding co-operation in the future. This idea is most elegantly formalized by Che and Yoo (2001). They show how an implicit contract between the agents of a team can generate implicit incentives and thus make joint performance evaluation optimal.

But even if group incentives have gained popularity, we still see a high frequency of individual compensation schemes based on relative or independent performance evaluation (IPE). This is especially the case for white collar workers (Prendergast, 1999) and workers in higher levels of organizations (see Appelbaum and Berg, 1999). In the present paper I combine the framework of Che and Yoo on team incentives with the framework of Baker, Gibbons and Murphy (BGM) on relational contracts¹ to show when relative performance evaluation and independent performance evaluation (IPE) is expected to dominate in repeated transactions. Like Che and Yoo I study a repeated game between one principal and two agents, but contrary to them, I model a self-enforcing relational contract between the principal and his agents. Contrary to Che and Yoo, I assume that the quality of the agents’ output is non-verifiable, so that legal enforcement is impossible. Like Che and Yoo, I compare three types of incentive schemes: Relative performance evaluation, joint performance evaluation and independent performance evaluation. But instead of focusing on optimality conditions, I focus on enforceability conditions: under which conditions are the various incentive schemes implementable? Since the parties only can choose incentive schemes among the enforceable ones, the results of my model often differs from Che and Yoo’s results.

In Section 2 and 3 I analyse an environment where collusion is impossible, and where the principal owns the assets so that the agents do not have the possibility to hold-up any values

¹ ‘Relational’ contracts and ‘implicit’ contracts are used synonymously in the literature. MacLeod and Malcomson (1989), Baker, Gibbons and Murphy (1994) and Schmidt and Schnitzer (1995) used ‘implicit’, while Bull (1987) used both ‘implicit’ and ‘relational’. Newer papers such as Baker, Gibbons and Murphy (2002) and Levin (2003) use ‘relational’, inspired by the legal literature, particularly MacNeil (1978). Since implicit contracts can be interpreted as vaguer than relational contracts (due to the antonym implicit versus explicit), I will in this paper use the term ‘implicit’ on the contract between the agents (like Che and Yoo), since it is most natural to think about this contract as a verbal informal agreement. But I will use ‘relational’ on the contract between the principal and his agents, since this most likely is a formally written wage contract.
ex post production. The main result from these sections is that when productivity of effort increases, the enforceability of IPE and RPE rises relatively to the enforceability of JPE. In general, an increase in productivity raises the cost of deviation, and the discount factor is allowed to decrease without running the risk of deviation. But JPE is vulnerable to low discount factors. Since the efficiency of JPE is dependent on the possibility for repeated interaction among the agents, the necessary JPE incentives increase with lower discount factors. Hence, higher productivity weakens the enforceability of JPE relatively to RPE and IPE.

In Section 4 I briefly discuss the implications of collusion. In particular, I show that independent performance evaluation may turn out as optimal if the agents are able to collude. In Section 5 I consider the situation when the agents own the critical assets. I show that the possibilities for agents to renegotiate the terms of trade ex post the realizations of values, makes incentives based on non-independent performance evaluation harder to enforce. This opens for independent performance evaluation to dominate.

The empirical relevance of the model is discussed in Section 6. Section 7 concludes.

2. The model

First I will replicate the repeated setting in Che and Yoo’s model. Consider an economic environment consisting of one principal and two identical agents who each period produce either high, $Q_H$, or low, $Q_L$, values for the principal. Their effort level can be either high or low, where high effort has a disutility cost of $c$ and low effort is costless. The principal can only observe the realization of the agents’ output, not the level of effort they choose. But the agents can observe each other’s effort decisions. Their output depends on effort decisions as well as a common environmental shock. A favourable shock occurs with probability $\sigma \in (0,1)$, in which case both agents produce high values for the principal. If the shock is unfavourable, the probability for agent $i$ of realizing $Q_H$ is $q_H$ if the agent’s effort is high and $q_L$ if the agent’s effort is low, where $q_H > q_L$. 
Agent \(i\) receives a fixed payment, \(\alpha\), prior to (ex ante) value realizations,\(^2\) where \(\alpha\) can be both positive and negative. A bonus wage vector \(\beta^i = (\beta^i_{HH}, \beta^i_{HL}, \beta^i_{LH}, \beta^i_{LL})\) where the subscripts denote respectively agent \(i\) and agent \(j\)’s realization of \(Q_j\), \((i = H, L)\), is to be paid ex post the realizations of values. It assumed that all parties are risk neutral, except that the agents are subject to limited liability: the principal cannot impose negative bonus wages. Limited liability may arise from liquidity constraints or from laws that prohibit firms from exacting payments from workers.\(^3\)

Let agent \(i\) and \(j\) choose efforts \(k \in \{H, L\}\) and \(l \in \{H, L\}\) respectively. Agent \(i\)’s expected wage is then

\[
\pi(k, l, \beta^i) \equiv \alpha + (\sigma + (1-\sigma) q_k q_l)\beta^i_{HH} + (1-\sigma)[q_k (1-q_l) \beta^i_{HL} + (1-q_k) q_l \beta^i_{LH} + (1-q_k)(1-q_l)\beta^i_{LL}]
\]

For each agent, a wage scheme exhibits joint performance evaluation if \((\beta^i_{HH}, \beta^i_{HL}) > (\beta^i_{HL}, \beta^i_{LL})\), \(^4\) (I suppress superscript since the agents are symmetric). In this case \(\pi(k, H; \beta) > \pi(k, L; \beta)\) so an agent’s work yields positive externalities to his partner. A wage scheme exhibits relative performance evaluation if \((\beta^i_{HH}, \beta^i_{LH}) < (\beta^i_{HL}, \beta^i_{LL})\). In this case \(\pi(k, H; \beta) < \pi(k, L; \beta)\) so an agent’s work generate a negative externality on his partner. A wage scheme exhibits independent performance evaluation if \((\beta^i_{HH}, \beta^i_{LH}) = (\beta^i_{HL}, \beta^i_{LL})\) which implies \(\pi(k, H; \beta) = \pi(k, L; \beta)\), so an agent’s work has no impact on his partner.

It is assumed that high effort is sufficiently valuable to the principal that he always prefers to induce the agents to exert high effort. The principal’s problem is then to minimize the wage payments subject to the constraints that the agents must be induced to yield high effort. In a repeated setting, the agents can exploit the fact that they are able to observe each other’s effort decisions. In particular, they can play a repeated game where they both play high effort if the

\[^2\] Che and Yoo do not include fixed payments in their model.

\[^3\] Limited liability in terms of liquidity constraints does not conflict with the possibility of a negative fixed payment since the fixed payment is paid ex ante. Also, a law against exacting payment ex post can still permit voluntary payments from workers ex ante.

\[^4\] The inequality means weak inequality of each component and strict inequality for at least one component.
other agent played high effort in the previous period. In order for such a strategy to constitute a subgame perfect equilibrium, we must have

\[(2) \quad \frac{1}{1-\delta} (\pi(H,H;B) - c) \geq \pi(L,H;B) + \frac{\delta}{1-\delta} \min \{\pi(L,L;B), \pi(L,H;B)\},\]

where \(\delta\) is the discount factor. The left hand side shows the expected present value of playing high effort, while the right hand side shows the expected wage from unilaterally playing low effort in one period and being subsequently punished by the worst possible equilibrium payoff. Hence, (2) says that, given the strategy to play high effort if the other agent played high effort in the previous period, an agent will play high effort as long the present value from playing high effort is greater than the present value from playing low effort. Note that (2) is a necessary but not sufficient condition. For (2) to be sufficient, the punishment path specified on the right hand side must also be self-enforcing.

Observe that in a JPE scheme, \(\pi(L,H;B) > \pi(L,L;B)\). Thus the right hand side of (2) becomes \(\pi(L,H;B) + \frac{\delta}{1-\delta} \pi(L,L;B)\). In an RPE scheme, however, \(\pi(L,L;B) > \pi(L,H;B)\), so the right hand side of (2) is reduced to \(\frac{1}{1-\delta} \pi(L,H;B)\) which makes (2) coinciding with the static incentive constraint (see Che and Yoo). Hence, we see that repeated interaction between the agents can increase the punishment of playing low effort in a JPE scheme, but not in an RPE scheme. The intuition is straightforward: in the JPE scheme, low effort from agent \(i\) does not only imply a reduced chance for him to realize high values, but it also implies that his peer plays low effort and thus lower the chance of realizing high values. This is costly since a JPE scheme promises highest wage if both realize high values. Hence, the repeated interaction yields both direct and implicit incentives to yield high effort.

Now, for the principal to choose the most efficient wage scheme, he must solve

\[(3) \quad \min_{B;\delta^0} \pi(H,H;B), \text{ subject to (2)}.\]

This is a relaxed program since there also exists low-effort strategies that constitute subgame perfect equilibria.
The following lemma characterizes the solution to (3):

**Lemma:** Define \( \delta(\sigma) \equiv \frac{\sigma}{(1-\sigma)q_H q_L} \). If \( \delta \in \left[ \hat{\delta}(\sigma), 1 \right] \), then a JPE scheme \( B^{JPE} = (\beta_{HH}^{JPE}, 0, 0) \) where \( \beta_{HH}^{JPE} \equiv \frac{c}{(1-\sigma)(q_H + \delta q_L)\Delta q} \) where \( \Delta q = q_H - q_L \) solves (3). If \( \delta \in \left[ 0, \hat{\delta}(\sigma) \right] \), then the RPE scheme \( B^{RPE} \equiv (0, \beta_{HL}^{RPE}, 0, 0) \) where \( \beta_{HL}^{RPE} \equiv \frac{c}{(1-\sigma)(1-q_H)\Delta q} \) solves (3).

**Proof:** See appendix.

The lemma suggests that an extreme form of JPE is optimal for sufficiently high discount factors, while (under the no collusion assumption) an extreme form of RPE is optimal on sufficiently low discount factors. Intuitively, there must be a sufficiently high discount factor if an agent should have interests in assuring future high-effort from his peer, hence JPE is only optimal on high discount factors.\(^5\)

It can be shown that \( B^{JPE} \) makes the worst sustainable punishment - low effort from both workers (L,L)- self-enforcing. This makes high effort from both agents (H,H) a subgame perfect equilibrium (Che and Yoo call this a ‘team equilibrium’). Hence, the incentive constraint given by (2) is sufficient when \( B^{JPE} \) solves (3). When \( B^{RPE} \) solves (3), (2) is sufficient if the discount factors are sufficiently high (see Che and Yoo for details).

The JPE scheme, \( B^{JPE} \), in contrast to \( B^{RPE} \), has the virtue of being collusion proof since each agent’s work confers positive externalities to their peer, but \( B^{RPE} \) is susceptible to collusion since both agents can jointly be better off by playing low effort. There may, however, exist institutional constraints to the possibilities of engaging in collusion. Of course, there are no technical constraints to collusion since the agents can observe each other’s actions. But it is

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\(^5\) Not unsurprisingly then, the RPE scheme is optimal in the static version. In this sense Che and Yoo complements Holmström (1982) and Mookheerjee (1984). The optimality depends on the assumed specification of the common shock.
not unrealistic to assume that in an industrious corporate culture it is easier for workers to sustain a high effort culture than to initiate a low effort culture. A worker that initiates low effort collusion may risk great personal costs in loss of prestige and respect from his peers. Hence, before we proceed to the collusion problem, I will simply assume that there is a social cost associated with initiating low effort collusion that exceeds the benefits from such collusion.

2.1 Relational contract between principal and agents

Unlike Che and Yoo I will now assume that the value realizations are not verifiable to a third party. Hence, the contract between the agents and the principal must therefore be self-enforcing, and thus ‘relational’ by definition. I consider a multilateral relational contract, which implies that any deviation by the principal triggers low effort from both agents. The principal honours the contract only if both agents honoured the contract in the previous period. The agents honour the contract only if the principal honoured the contract with both agents in the previous period. A natural explanation for this multilateral feature is that the agents interpret a unilateral contact breach (i.e. the principal deviates from the contract with only one the agents) as evidence that the principal is not trustworthy (see Bewley, 1999).

The contract is self-enforcing if the present value of honouring is greater than the present value of reneging. Ex post realizations of values, the principal can renege on the contract by refusing to pay the promised wage, while the agents can renege by refusing to accept the promised wage. In this section I consider a relational employment contract, to use the terms of BGM. This means that the principal owns the critical assets. Asset ownership conveys ownership to the values produced, hence, the principal can ex post take the values even if he is not paying bonus wages.

The parties play trigger strategies. Like BGM, I assume that if one of the parties renege on the relational employment contract, the other insist on spot employment forever after. Spot employment implies that the agents exert low effort \( q_L \), but receives zero wage (neither bonus wage nor fixed payment).
The condition for the optimal JPE contract, $B^{JPE}$, to be self-enforcing is (see appendix)

(4) \[ 2\beta_{III}^{JPE} \leq \frac{2A}{1-\sigma} \left[ (1-\sigma)\Delta q \Delta Q - c \right], \]

where $\Delta Q = Q_H - Q_L$. The condition for the optimal RPE contract, $B^{RPE}$, to be self-enforcing is (see appendix)

(5) \[ \beta_{III}^{RPE} \leq \frac{2A}{1-\sigma} \left[ (1-\sigma)\Delta q \Delta Q - c \right]. \]

The expression in the square brackets shows the productivity of an agent’s effort. Hence, the right hand side of (4) and (5) shows the present value of both agents yielding high effort. We clearly see the difference between the self-enforcing conditions of $B^{JPE}$ and $B^{RPE}$. While the principal ‘risks’ paying both agents in the JPE scheme, he only risks paying one if his agents in the RPE scheme. Levin (2002) argues that multilateral relational contracts makes relative performance evaluation favourable, since the principal only has to satisfy the sum of individual constraints (not every separate incentive constraint) and is thus committed to reward only one of his agents. But Levin does not allow for implicit contracting between the agents. In the present model, this possibility can make the necessary JPE wage, $\beta_{III}^{JPE}$, lower-powered and thus easier to implement. Hence, there is a trade-off between enforcing a double set of small JPE wages and a single, but larger RPE wage.

3. Comparative analysis

We can formally compare the self-enforcing conditions of $B^{JPE}$ and $B^{RPE}$. Solving (4) for $\delta$ and inserting for $\beta_{III}^{JPE}$ yields

(6) \[ \delta \geq \frac{1}{2Aq} \left[ -1 - Aq_L + \sqrt{(1 + 2Aq_H + A^2q_H^2 + 4Aq_L)} \right] = \delta^{JPE}, \]
where \( A = \left[ \frac{(1-\sigma)\Delta q \Delta \sigma}{ \alpha } - 1 \right] (1-\sigma) \Delta q. \)

Solving (5) for \( \delta \) and inserting for \( \beta_{HL}^{RPE} \) yields

\[
\delta \geq \frac{1}{1+2\Delta (1-\sigma/q)} \delta^{RPE}. 
\]

Expression (6) and (7) show the critical discount factors for the optimal JPE contract and the optimal RPE contract, respectively, to be self-enforcing. If \( \delta^{RPE} < \hat{\delta}(\sigma) < \delta < \delta^{JPE} \), then \( \beta^{JPE} \) is optimal, but not enforceable, while \( \beta^{RPE} \) is enforceable. The principal must then either choose a second best JPE contract, or \( \beta^{RPE} \) (a second best JPE contract implies a less extreme JPE contract where agent \( i \) can be paid even if \((H,H)\) is not realized, that is \( \beta^{JPE}_{LH} > \beta^{JPE}_{LL} \) and/or \( \beta^{JPE}_{HH} > \beta^{JPE}_{HL} > 0 \) (strict inequality if \( \beta^{JPE}_{LH} = \beta^{JPE}_{LL} \)), where

\[
\beta^{JPE}_{HH} < \frac{1}{(1-\sigma x_H, q_H) \Delta \sigma}. 
\]

Since a second best JPE contract is more costly than the optimal JPE contract, \( \beta^{JPE} \), this will increase the critical discount factor, \( \hat{\delta} \), for when an incentive scheme that exhibits JPE is chosen. Hence, if \( \delta^{RPE} < \hat{\delta}(\sigma) < \delta^{JPE} \), then there exist discount factors \( \hat{\delta}(\sigma) < \delta < \hat{\delta} \), where \( \beta^{RPE} \) is chosen.

If \( \delta^{JPE} < \delta < \hat{\delta}(\sigma) < \delta^{RPE} \), then \( \beta^{RPE} \) is optimal, but not enforceable, while \( \beta^{JPE} \) is enforceable. The principal must then either choose a second best RPE contract or \( \beta^{JPE} \) (a second best RPE contract implies a less extreme RPE contract where agent \( i \) can be paid even if \((H,L)\) is not realized, that is \( \beta^{RPE}_{LH} < \beta^{RPE}_{LL} \) and/or \( \beta^{RPE}_{HH} \geq \beta^{RPE}_{HL} > 0 \) (strict inequality if \( \beta^{RPE}_{LH} = \beta^{RPE}_{LL} \)), where

\[
\beta^{RPE}_{HL} < \frac{1}{(1-\sigma x_L) \Delta \sigma}. 
\]

Since a second best RPE contract is more costly than the optimal RPE contract, \( \beta^{RPE} \), this will decrease the critical discount factor \( \hat{\delta} \) for when an incentive scheme that exhibits JPE is chosen. Hence, if \( \delta^{JPE} < \hat{\delta}(\sigma) < \delta^{RPE} \), then there exist discount factors \( \hat{\delta} < \delta < \hat{\delta}(\sigma) \) where \( \beta^{JPE} \) is chosen.

I will now show that increasing levels of productivity increases the enforceability of
\( b^{RPE} \) relatively to \( b^{JPE} \). In other words, for sufficiently high levels of productivity, \( \delta^{RPE} < \delta^{JPE} \). We can rewrite (4) and (5) as

\[
(4') \quad \frac{1}{q_H + \delta q_L} \leq \frac{\delta}{1 - \delta} A
\]

\[
(5') \quad \frac{1}{2(1 - \eta)} \leq \frac{\delta}{1 - \delta} A.
\]

where \( A = \left[ \frac{(1 - \sigma)\Delta Q}{\epsilon} - 1 \right] (1 - \sigma) \Delta q \) is a proxy for productivity.

**Proposition 1:** There is a critical \( \tilde{A} = A \) such that \( \delta^{JPE} = \delta^{RPE} \), that is

\[
\frac{1}{2\eta} \left[ -1 - \tilde{A} q_L + \sqrt{(1 + 2\tilde{A} q_H + \tilde{A}^2 q_H^2 + 4\tilde{A} q_L)} \right] = \frac{1}{1 + 2(1 - \eta)}.
\]

For given levels of \( q_H \) and \( q_L \), if \( A > \tilde{A} \), then there exist discount factors \( \delta^{RPE} \leq \delta < \delta^{JPE} \) where \( b^{RPE} \) is a self-enforcing incentive scheme, and \( b^{JPE} \) is not. If \( A < \tilde{A} \), then there exist discount factors \( \delta^{JPE} \leq \delta < \delta^{RPE} \) where \( b^{JPE} \) is a self-enforcing incentive scheme, and \( b^{RPE} \) is not.

Hence, if the productivity of effort increases through an increase in \( \Delta Q \), and/or a decrease in \( \sigma \), and/or a decrease in \( c \), we can move from a situation where there exists discount factors where \( b^{JPE} \) is enforceable and \( b^{RPE} \) is not, to a situation where \( b^{RPE} \) is enforceable and \( b^{JPE} \) is not. Hence, we obtain an empirical testable hypothesis: when the productivity of effort is high, RPE is more common. When productivity of effort is low, JPE is more common. The reasoning goes as follows: the higher the productivity, the easier it is for the principal to offer credible incentive schemes, hence the critical discount factors decrease. When the critical discount factors decrease, the necessary RPE wage, \( b^{RPE}_{HL} \) is not affected, but the necessary JPE wage \( b^{JPE}_{HL} \) increases since it is harder for the agents to enforce an implicit contract between them on low discount factors. Hence, higher productivity makes the enforceability of RPE rises relatively to JPE. Note that higher productivity does not mean that JPE is harder to enforce. High productivity implies however that there is scope for relational contracts even on low discount factors. But \( b^{JPE} \) is difficult to implement on low discount factors due to the implicit contract between the agents.
The proposition can be demonstrated graphically:

![Graph](image)

**Figure 1**

The downward-sloping curve corresponds to the left hand side of (4’) (hereafter referred to as the ‘JPE curve’) and the horizontal curve the left hand side of (5’) (hereafter referred to as the ‘RPE curve’) when \( q_H = 0.6 \) and \( q_L = 0.3 \). The thick upward-sloping curve corresponds to the right hand side of (4’) and (5’) when \( A = 0.5 \), while the thin upward-sloping curve, corresponds to the right hand side of (4’) and (5’) when \( A = 1 \). We see that when \( A = 0.5 \), \( \delta_{JPE} < \delta_{RPE} \). Now, if productivity increases (\( A \) increases), given constant probabilities, the upward-sloping curve gets steeper and we move to a situation where \( \delta_{RPE} < \delta_{JPE} \), as shown from the thin upward-sloping curve.

Note also that a decrease in \( q_L \), given \( q_H \), increases \( A \) (the upward sloping curve gets steeper), while the left hand side of (4’) increases so the JPE curve shifts upwards. Hence, a decrease in \( q_L \), given \( q_H \), makes the enforceability of \( \Phi^{RPE} \) increase relatively to the enforceability of \( \Phi^{JPE} \). An increase in \( q_H \), given \( q_L \), also increases \( A \), but now the JPE curve shifts downwards while the RPE curve shifts upwards. Hence, \( \delta_{JPE} \) decreases, while \( \delta_{RPE} \).
decreases if $A(1-q_H)$ increases. Hence, an increase in $q_H$, given $q_L$, makes the enforceability of $B^{RPE}$ increase relatively to the enforceability $B^{JPE}$, given sufficiently high levels of $\Delta Q$, and/or low levels of $\sigma$, and/or low levels of $c$.

If neither $B^{JPE}$ nor $B^{RPE}$ is enforceable, the principal must either choose a second best JPE or RPE scheme, or he can chose an incentive scheme based on independent performance evaluation (IPE). The optimal IPE scheme solves (3) subject to $(\beta_{HH}, \beta_{HL}) = (\beta_{HL}, \beta_{LL})$. The agents’ incentive constraint is $(\sigma + (1-\sigma)q_H)\beta - c \ge (\sigma + (1-\sigma)q_L)\beta$. Solving for $\beta$ yields $\beta \ge \frac{(\sigma + (1-\sigma)q_H)}{(1-\sigma)Aq} = \beta_{HH}^{IPE} = \beta_{HL}^{IPE}$. On low realizations, the principal will pay zero. Hence the optimal IPE scheme is $B^{IPE} = (\beta_{HH}^{IPE}, \beta_{HL}^{IPE}, 0, 0)$. The condition for the optimal IPE contract, $B^{IPE}$, to be self-enforcing is (see appendix)

$$(9) \quad 2\beta_{HH}^{IPE} \le \frac{1}{1+\delta}[(1-\sigma)\Delta Q - c],$$

which is equivalent to the JPE condition. In IPE, as in JPE, the principal ‘risks’ paying both agents. Solving (9) for $\delta$ and inserting for $\beta_{HH}^{IPE} = \beta_{HL}^{IPE}$ yields

$$(10) \quad \delta \ge \frac{1}{1+\delta} = \delta^{IPE}.$$  

We then have $\delta^{IPE} < \delta^{RPE}$ when $q_H > \frac{1}{2}$ and $\delta^{IPE} < \delta^{PE}$ for sufficiently high productivity. We can write (9) as

$$(9') \quad 1 \le \frac{1}{1+\delta} A$$

**Proposition 2:** There is a critical $A = \bar{A}$ such that $\delta^{IPE} = \delta^{IPE}$, that is

$\frac{1}{2Aq_L} \left[ -1 - \bar{A}q_L + \sqrt{(1+2\bar{A}q_H + \bar{A}^2 q_H^2 + 4\bar{A}q_L^2)} \right] = \frac{1}{1+\delta}$. For given levels of $q_H$ and $q_L$, if $A > \bar{A}$, and $q_H > \frac{1}{2}$, then there exist discount factors $\delta^{IPE} \le \delta < \delta^{JPE}$ and $\delta^{IPE} \le \delta < \delta^{RPE}$ where $B^{IPE}$ is self-enforcing and $B^{JPE}$ and $B^{RPE}$ are not.
The proposition shows that the enforceability of $B_{IPE}$ relative to $B_{JPE}$ increases with effort productivity, for the same reasons as $B_{RPE}$ to $B_{JPE}$. Moreover the enforceability of $B_{IPE}$ relative to $B_{RPE}$ increases on high probabilities for positive outcome: while the necessary IPE wage $\beta_{IPE}^{Hj}$ is independent of $q_H$, the necessary RPE wage, $\beta_{RPE}^{HL}$, increases in $q_H$, given $q_L$.

We can demonstrate proposition 2 graphically:

![Figure 2](image-url)

The parameters are the same as in Figure 1, with $A=1$. The thick horizontal line represents the ‘IPE curve’ (corresponding to the left hand side (9’)), while the thin horizontal line is the ‘RPE curve’. We see that for $q_H > \frac{1}{2}$ and a sufficiently high A, then $\delta_{IPE} < \delta_{RPE} < \delta_{JPE}$.

So far we have seen that high effort productivity strengthens the self-enforcing conditions of RPE and IPE schemes relatively to the optimal JPE scheme. The analysis reveals that the outcomes in Che and Yoo’s model are sensitive to the assumption that the parties can write
legally enforceable contracts. Once contracts have to rely on self-enforcement, the optimal choice of team incentives becomes more complicated, since the parties can only choose between self-enforcing incentive contracts. Some numerical examples elucidate this:

Assume that the discount factor is constant over time and has the value $\delta = 0.75$

**Example 1**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c = 1$</td>
<td>$\delta_{IPE} = 0.74$</td>
</tr>
<tr>
<td>$\sigma = 0.05$</td>
<td>$\delta_{RPE} = 0.70$</td>
</tr>
<tr>
<td>$q_h = 0.4$</td>
<td>$\delta_{JPE} = 0.83$</td>
</tr>
<tr>
<td>$q_L = 0.2$</td>
<td>$\hat{\delta} = 0.66$</td>
</tr>
<tr>
<td>$\Delta Q = 15$</td>
<td></td>
</tr>
</tbody>
</table>

$JPE$ is optimal but not enforceable, while $RPE$ is enforceable.

**Example 2**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c = 1$</td>
<td>$\delta_{IPE} = 0.74$</td>
</tr>
<tr>
<td>$\sigma = 0.2$</td>
<td>$\delta_{RPE} = 0.83$</td>
</tr>
<tr>
<td>$q_h = 0.7$</td>
<td>$\delta_{JPE} = 0.74$</td>
</tr>
<tr>
<td>$q_L = 0.4$</td>
<td>$\hat{\delta} = 0.89$</td>
</tr>
<tr>
<td>$\Delta Q = 10$</td>
<td></td>
</tr>
</tbody>
</table>

$RPE$ is optimal, but not enforceable, while $JPE$ is enforceable.
Example 3

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c = 0.7$</td>
<td>$\delta^\text{IPE} = 0.72$</td>
</tr>
<tr>
<td>$\sigma = 0.4$</td>
<td>$\delta^\text{RPE} = 0.81$</td>
</tr>
<tr>
<td>$q_H = 0.7$</td>
<td>$\delta^\text{JPE} = 0.77$</td>
</tr>
<tr>
<td>$q_L = 0.1$</td>
<td>$\hat{d} = 9.52$</td>
</tr>
<tr>
<td>$\Delta Q = 4$</td>
<td></td>
</tr>
</tbody>
</table>

$\beta^\text{RPE}$ is optimal, but neither $\beta^\text{RPE}$ or $\beta^\text{JPE}$ is enforceable, while $\beta^\text{IPE}$ is enforceable.

4. Collusion

As noted, in contrast to IPE and JPE, RPE is exposed to collusion. If the principal offers $\beta^\text{RPE}$, then the agents, for $\delta$ close to 1, are better off if they both play low effort ($L,L$) than if they both play high effort ($H,H$). And for any value of $\delta$, the agents can play a correlated randomisation of ($L,H$) and ($H,L$) and generate a higher joint payoff than ($H,H$). Hence, if there are no institutional constraints to collusion between the agents, then the cost of using RPE to generate ($H,H$) raises. It is complicated to analyse the optimal scheme in the region where the principal ‘would have chosen’ RPE if it was not for the collusion problem. In order to prevent collusion, the principal has to offer incentives which make high effort from both agents ($H,H$) the only subgame perfect equilibrium. A correlated randomisation strategy between ($H,L$) and ($L,H$) provides the ‘hardest’ equilibrium to prevent. It can be shown (see appendix) that the condition that makes this randomisation strategy a non equilibrium is given by \(^6\)

\[
\beta_{HL} \geq \frac{(2 - \delta)c}{(1 - \sigma)(2 - 2q_H - \delta)\Delta q} = \beta_{RPE}^\text{RPEC}.
\]

\(^6\) I thank Yeon-Koo Che for help with deducing this expression.
It will of course be more difficult to enforce RPE schemes if the agents can collude. But the basic implications from proposition 1 and 2 are not altered. If we compare \( \beta_{RPE}^{HL} \) to \( \beta_{RPEC}^{HL} \) we see that the first order effects of \( \sigma, \Delta q, q_H \) and \( \epsilon \) is corresponding. Moreover, a decrease in \( \delta \) decreases \( \beta_{RPEC}^{HL} \), while it increases \( \beta_{IPE}^{HL} \). Hence, the economics of the observations in the previous section is robust to the threat of collusion.

Two things are happening to optimality conditions when we allow for collusion. First, since RPE is now more expensive (that is \( \beta_{RPE}^{HL} < \beta_{RPEC}^{HL} \) for \( \delta > 0 \)), the region where \( \beta_{IPE}^{HL} \) is optimal expands. Second, in contrast to the no collusion case, relative performance evaluation does not always dominate independent performance evaluation, since the expected cost per agent in RPE, \( (1 - \sigma)q_H (1 - q_H) \beta_{RPEC}^{HL} \), exceeds the expected cost per agent in IPE, \( \sigma + (1 - \sigma)q_H \beta_{IPE}^{HL} \), when \( \delta > \frac{2\sigma (1 - q_H)}{q_H (1 - \sigma) + \sigma} = \delta^* \). And since the expected cost per agent in IPE is lower than the expected cost per agent in JPE, \( \sigma + (1 - \sigma)q_H q_H \beta_{IPE}^{HL} \) when \( \delta < \frac{\sigma (1 - q_H)}{q_L (1 - \sigma) + \sigma} = \delta^* \), there are levels of discount factors where IPE is optimal.

**Proposition 3:** If the agents are able to collude, there exist discount factors \( \delta^* > \delta > \delta_0 \) where independent performance evaluation is optimal.

Observe that the larger \( q_H - q_L = \Delta q \), the larger is the region \( (\delta^*, \delta^*_0) \) where IPE is optimal. The reason is that JPE is costly when \( q_L \) is low and RPE is costly when \( q_H \) is large.

5. Asset ownership

So far I have considered the principal to be the owner of the assets involved. Since the principal was able to utilize the values that the agents created even after reneging on the bonus-payment, the assumption was made that the principal owned the critical assets involved in producing values. This section discusses the self-enforcing conditions of the incentive

---

7 Observe that when \( \delta = 0 \), \( \beta_{RPE}^{HL} = \beta_{RPEC}^{HL} \). As \( \delta \) approaches \( 2 - 2q_H \), \( \beta_{RPEC}^{HL} \) approaches infinity. For \( \delta \in (2 - 2q_H, 1) \), RPE can never succeed in preventing the randomization collusion.
schemes when the agents own critical assets so that they are able to renegotiate the terms of trade ex post the realization of values. I will argue that independent performance evaluation is more likely in this situation.

Assume that there are two assets involved in the production, and that each agent uses one asset to produce values for the principal. Asset ownership conveys ownership of the values produced. In the previous section it was implicitly assumed that the principal owned both assets (‘principal-ownership’). In this section I assume that the agents own one asset each (‘agent-ownership’). This does not necessarily mean that the agents are independent suppliers owning physical assets. It can also be interpreted as agents in an employment relationship, where the critical assets are human capital. The main difference from the previous sections is that the agents are able to renegotiate the terms of trade ex post the realization of values.

In agent-ownership, if the agents deviate from the relational contract by refusing to accept the promised bonus, they can “hold up” the values they have produced and renegotiate the price. Assume that there exist an alternative market for the output, and that the price in this market is either high, \( P_H \), or low \( P_L \), where \( Q_H > Q_L > P_H > P_L \). The favourable shock, which occurs with probability \( \sigma \in (0,1) \), makes both agents produce high values both for the principal and for the alternative market. I assume one-dimensional effort. This means that the agents cannot take actions that increase the probability of realizing high- alternative use values \( P_H \), without also increasing the probability of realizing high values for the principal \( Q_H \). Hence, if the shock is unfavourable, the probability of realizing \( P_H \) is \( q_H \) if the agent’s effort is high and \( q_L \) if the agent’s effort is low. The one dimensional effort constraint ensure that agent \( i \)’s wage vector \( \mathbf{B}^i \equiv (\beta^{iH}, \beta^{iHL}, \beta^{iLH}, \beta^{iLL}) \) still applies since there is then no point in letting wage depend on the realization of alternative use values. Hence, the optimal schemes in principal-ownership, \( \mathbf{B}^{JPE}, \mathbf{B}^{RPE}, \) and \( \mathbf{B}^{IPE} \), also applies in agent-ownership. Note that the one-dimensional effort constraint is most natural in the human capital interpretation of asset ownership, where the principal can decide the agent’s behavioral pattern even though the agents own the critical assets.

In general, the principal’s reneging temptations are weaker in agent-ownership than in
principal-ownership, since he in agent-ownership cannot just take the goods, but has to bargain with the agents if he reneges. I assume that 50:50 Nash bargaining determines the price, that is \( \frac{Q_1 + Q_2}{2} \). The agents’ reneging temptations are stronger, however, since they can receive the Nash price if they renegade. This ‘outside opportunity’ is especially binding with non-independent performance evaluation since the agents risk getting low wages from the relational contract, while high realization assure them a high Nash price if they renegotiate.

The condition for the optimal JPE contract, \( B^{JPE} \), to be self-enforcing is (see appendix):

When \( 2\beta_{HH}^{JPE} > \Delta Q \)

\[
(13a) \quad 2\beta_{HH}^{JPE} - \frac{1}{2} \Delta Q + \Delta P \leq \frac{2\Delta}{1-\sigma} \left[ (1-\sigma) \Delta Q Q - c \right]
\]

When \( 2\beta_{HH}^{JPE} < \Delta Q \)

\[
(13b) \quad \frac{1}{2} \Delta Q + \Delta P \leq \frac{\Delta Q}{1-\sigma} \left[ (1-\sigma) \Delta Q Q - c \right]
\]

where \( \Delta P = P_H - P_L \). Compared to (13a,b) to (4) shows that it is easier to enforce \( B^{JPE} \) in agent-ownership than in principal-ownership only if \( \Delta P < \frac{1}{2} \Delta Q \). A disadvantage with the optimal JPE scheme, \( B^{JPE} \), in agent-ownership is that if only one of the agents realize high values, he receives no payments and is therefore tempted to renge; that is to renegotiate a Nash price with the principal. This ‘non-independence’ problem of agent-ownership becomes even more severe with the optimal RPE scheme \( B^{RPE} \). The conditions for the RPE contract \( B^{RPE} \) to be self-enforcing is (see appendix):

When \( \beta_{HL}^{RPE} > \frac{1}{2} \Delta Q \)

\[
(14a) \quad \beta_{HL}^{RPE} + \frac{1}{2} \Delta Q + \Delta P \leq \frac{2\Delta}{1-\sigma} \left[ (1-\sigma) \Delta Q Q - c \right]
\]

When \( \beta_{HL}^{RPE} < \frac{1}{2} \Delta Q \)

\[
(14b) \quad \Delta Q + \Delta P \leq \frac{\Delta Q}{1-\sigma} \left[ (1-\sigma) \Delta Q Q - c \right]
\]
Comparing (14a,b) and to (5) shows that it is more difficult to enforce $B^{RPE}$ in the agent-ownership than in principal-ownership. Now, if both agents realize high values, they get zero, and both have high temptations to renege on the contract since the Nash price is high (due to the high output-realizations.)

This non-independent problem is eliminated with the $B^{IPE}$ scheme. Since each agent gets paid according to their own output, independent from their peer’s output, they do not risk receiving no payments within the contract while the Nash price is high. The condition for the IPE contract $B^{IPE}$ to be self-enforcing is (see appendix):

\[(15) \quad \left|2\beta_{HH}^{IPE} - \Delta Q\right| + \Delta P \leq \frac{2\delta}{1-\delta} \left[(1-\sigma)\Delta q\Delta Q - c\right]\]

Comparing (15) to (9) we see that it is easier to enforce $B^{IPE}$ in agent-ownership than in principal-ownership when $\Delta Q > \Delta P$, hence if effort is more productive internally than externally (effort specificity).

A comparison of the constraints above suggests that the relative enforceability of $B^{IPE}$ to $B^{JPE}$ and $B^{RPE}$ is strengthened if the agents own the assets. Since IPE, in contrast to JPE and RPE do not exhibit the ‘non-independent problem’ we would expect that independent performance evaluation is relatively more common if agents own the critical assets (or have essential human capital) so that they are able to renegotiate the terms of trade ex post the realization of values, than if the principal owns the critical assets.

6. Relevance

There are two important features with the model discussed in the previous section that decides its applicability. First, the agents can observe each other’s actions. Consequently, the model is best applied on smaller organizations, or on subdivisions of larger organizations. Second, the relational contract is multilateral, which implies that the agents cooperatively decide
whether or not to honour the contract. Hence, the model is best applied in environments or corporate cultures where worker-coordination is relatively easy.

Do we find empirical support for the model’s predictions? A common wage/incentive structure of hierarchical organizations is that low-wage ‘blue collar’ workers in the bottom of a hierarchy enjoy some sort of group incentives, while ‘white collar’ workers higher up in the organization typically enjoy incentives based on individual or relative performance (see Prendergast, 1999 and Appelbaum and Berg, 1999). Baker, Gibbs and Holmström (1993), and Treble, van Gameren, Bridges and Barmby (2001), find that the salary range is much greater at the higher levels of an organization than at lower levels, which indicates a higher frequency of individual and relative performance based compensation structure on higher levels.

The model help explain these observations: Proposition 1 and 2 suggest that RPE or IPE are more common when the productivity of effort is high, while JPE is more common when productivity of effort is low. Since wages are expected to reflect the workers’ productivity, we would therefore expect to see higher occurrence of IPE and RPE in higher levels of organizations where wage and productivity presumably are highest.

The higher frequency of IPE in highly paid jobs can also be explained by the non-independence problem of RPE and JPE discussed in Section 5. When the workers own the critical assets, or have essential human capital so that they are able to renegotiate the terms of trade ex post the realization of values, incentives based on IPE has the advantage that it balances the value of relational contracting with the value of independent ex post bargaining. With incentive schemes based on non-independent performance evaluation, there is a greater probability of getting low payments inside the relation and high payments outside the relation, which increases the possibility of contract breach. This theoretical result is not surprising. It helps explain the higher frequency of individual compensation packages and IPE incentives in human capital-intensive industries.
It is argued that RPE discourages cooperative work morale.\(^8\) Human recourses managers often claim that salaries must be compressed to maintain internal harmony in a firm. If the difference between the winner’s salary and the loser’s salary is too great, morale suffers. Since the high positions in an organization tend to be dominated by individuals who have managed to fight the “corporate war”, and thus presumably tend to be more aggressive and more willing to engage in sabotage, one should find it necessary to reduce the incentives for those workers to compete with each other (see for instance Lazear, 1989, 1998); in other words reduce: the use of relative performance evaluation. But the model in this paper shows that it is easier to implement incentives based on relative performance evaluation than joint performance evaluation in high ability/low trust environments (using the discount factor as a proxy for trust; see Hart 2001).

7. Conclusion

It can be efficient to reward a group on the basis of the group’s joint performance if the problem of free riding can be deterred by mutual peer monitoring and social sanctions. But incentives based on relative performance evaluation and independent performance evaluation are still quite common even in industries were peer monitoring is possible. Individual compensation based on IPE and/or RPE is especially common in the higher levels of organizations. In this paper I have shown how the absence of legally enforceable contracts can explain this. In a model with self-enforcing relational contracts between principal and agents it is shown that we can expect a relatively higher frequency of incentive schemes based on RPE and IPE when the productivity of effort is high. Moreover it is shown that we can expect to see a relatively higher frequency of IPE schemes if agents own critical assets so that they can renegotiate the terms of trade ex post realization of values.

This paper has not fully characterized optimal solutions, but showed the important implications of the relational contract’s enforceability constraints when dealing with team

\(^8\) It is also argued that RPE, in addition to discourage cooperative work morale, also encourages employees to adopt restricted work norms (see Baron and Kreps, 1999). RPE can also distort agents’ incentives if they carry out multiple activities (multitasking). Gibbons and Murphy (1990), and more formally Baker (1992) show that if workers can take actions that effect the output of their peers, and in addition are able to "game" the compensation scheme to their benefit, RPE can distort incentives and thus make it less efficient.
incentives. In formal studies of relational contracts it is important to bear in mind the subjective nature of the discount factor. It is not necessarily optimal for a principal to choose the optimal incentive scheme among the enforceable ones at a given date. If the discount factors vary over time, and there are costs associated with shifting from one scheme to another, it may be optimal to choose the incentive scheme with the lowest critical discount factor.

**APPENDIX**

1. Sketch to proof of Che and Yoo’s Lemma

First we can write out the objective function:

\[
\pi (H, H, B) = (\sigma + (1 - \sigma)q_H q_H) \beta_{HH} + (1 - \sigma)q_H (1 - q_H) (\beta_{HL} + \beta_{LH}) + (1 - \sigma) (1 - q_H)^2 \beta_{LL}
\]

(A.1)

As noted, in an RPE scheme, \(\pi (L, L; B) > \pi (L, H; B)\), so the incentive constraint (2) is reduced to \(\pi (H, H; B) - c \geq \pi (H, L; B)\). We can write this out:

\[
(\sigma + (1 - \sigma)q_H q_H) \beta_{HH} + (1 - \sigma)q_H (1 - q_H) \beta_{HL} + (1 - q_H)^2 \beta_{LL} \geq (\sigma + (1 - \sigma)q_H q_H) \beta_{HH} + (1 - \sigma)q_H \beta_{HL} + (1 - q_H)^2 \beta_{LL} - c
\]

and simplify it to

(A.2) \(q_H \beta_{HH} + (1 - q_H) \beta_{HL} - q_H \beta_{LH} - (1 - q_H) \beta_{LL} \geq \frac{c}{(1 - \sigma)q_H}
\)

The left hand side of the constraint is decreasing in \(\beta_{LL}\), while the objective function is increasing in \(\beta_{LL}\). Hence, it is optimal to set \(\beta_{LL} = 0\), which from \(\pi (L, L; B) > \pi (L, H; B)\) implies that \(\beta_{LH} = 0\). Since both the objective function and the constraint are linear in \(B\), only \(\beta_{HH}\) or \(\beta_{HL}\) are strictly positive, which from \(\pi (H, L; B) > \pi (H, H; B)\) implies \(\beta_{HH} = 0\). Hence, the optimal RPE scheme is an the extreme form \(B^{RPE} = (0, \beta_{HL}^{RPE}, 0, 0)\) where solving
(A.2) for $\beta_{HL}$ yields $\beta_{HL}^{RPE} \equiv \frac{c}{(1 - \sigma)(1 - q_H)\Delta q}$.

In a JPE scheme, $\pi(L, H; \mathcal{B}) > \pi(L, L; \mathcal{B})$. Thus the incentive constraint (2) becomes

$$\frac{1}{1 - \delta} \pi(H, H; \mathcal{B}) - c \geq \pi(L, H; \mathcal{B}) + \frac{\delta}{1 - \delta} \pi(L, L; \mathcal{B})$$

We can write this out:

$$\frac{1}{1 - \delta} \left\{ (\sigma + (1 - \sigma)q_H)\beta_{HH} + (1 - \sigma)(q_H (1 - q_H)\beta_{HL} + (1 - q_H)(1 - q_H)\beta_{LL}) - c \right\}$$

$$\geq (\sigma + (1 - \sigma)q_L)\beta_{HH} + (1 - \sigma)(q_L (1 - q_L)\beta_{HL} + (1 - q_L)(1 - q_L)\beta_{LL})$$

$$+ \frac{\delta}{1 - \delta} \left\{ (\sigma + (1 - \sigma)q_L)\beta_{HH} + (1 - \sigma)(q_L (1 - q_L)\beta_{HL} + (1 - q_L)(1 - q_L)\beta_{LL}) \right\}$$

and simplify it to

$$(q_H + \delta q_L)\beta_{HH} + (1 - q_H - \delta q_L)\beta_{HL} + (\delta - q_H - \delta q_L)\beta_{LL} - (1 - q_H + \delta (1 - q_L))\beta_{LL} \geq \frac{c}{(1 - \sigma)(q_H + \delta q_L)\Delta q}$$

(A.3)

The left hand side of the constraint is decreasing in $\beta_{LL}$, while the objective function is increasing in $\beta_{LL}$. Hence, it is optimal to set $\beta_{LL} = 0$. Observe that the coefficient of $\beta_{HL}$ is weakly greater than that of $\beta_{HH}$ in the left hand side of (A.3), but that their coefficients are the same in the objective function (A.1). Suppose $\beta_{HL} > 0$. Then lowering $\beta_{HH}$ and raising $\beta_{HL}$ simultaneously so that the left hand side of (A.3) remains the same will reduce the value of the objective function. Hence, it is optimal to set $\beta_{HH} = 0$. Since only $\beta_{HH}$ or $\beta_{HL}$ are strictly positive $\beta_{HH} = 0$ from $\pi(H, L; \mathcal{B}) < \pi(H, H; \mathcal{B})$. Hence, the optimal JPE scheme is an extreme form $\mathcal{B}^{JPE} = (\beta_{HH}, 0, 0)$ where solving (A.3) for $\beta_{HH}$ yields

$$\beta_{HH}^{JPE} \equiv \frac{c}{(1 - \sigma)(q_H + \delta q_L)\Delta q}.$$ 

Now, $\mathcal{B}^{JPE}$ solves (3) if $\pi(H, H, \mathcal{B}^{JPE}) < \pi(H, H, \mathcal{B}^{RPE})$. That is

$$(\sigma + (1 - \sigma)q_H q_H) \frac{c}{(1 - \sigma)(q_H + \delta q_L)\Delta q} < (1 - \sigma)q_H (1 - q_H) \frac{c}{(1 - \sigma)(1 - q_H)\Delta q}$$
Solving for $\delta$ yields

$$\delta > \frac{\sigma}{(1-\sigma)q_L q_H} \equiv \hat{\delta}(\sigma)$$

For further details see Che and Yoo (2001).

2. The conditions for self-enforcing relational contracts when the principal owns the assets

In JPE, the binding constraint for the principal is when both agents realize high values so that he has to pay wage $\beta^{JPE}_{HH}$ to both agents. The binding constraint for the principal to honour the contract $B^{JPE}$ is then given by

$$(A.4) \quad -2\beta^{JPE}_{HH} + \frac{2\Delta Q}{1-\alpha} \left[ Q_L + (\sigma + (1-\sigma)q_H)\Delta Q - (\sigma + (1-\sigma)q_H q_H)\beta^{JPE}_{HH} - \alpha \right] \geq \frac{2\Delta Q}{1-\alpha} \left[ Q_L + (\sigma + (1-\sigma)q_L)\Delta Q \right]$$

where the left hand side shows the expected present value from honouring and the right hand side shows the expected value from reneging. The square brackets show the principal’s expected revenue per agent per period.

For the agents, the constraints bind for realization of zero bonus wages. Each agent will thus simply honour the contract if expected future wage exceeds zero:

$$(A.5) \quad \frac{1}{1-\alpha} (\alpha + (\sigma + (1-\sigma)q_H q_H) \beta^{JPE}_{HH} - c) \geq 0$$

Combining (A.4) with (A.5) for both agents, that is multiplying (A.5) with 2 and add with (A.4) yields:

$$(4) \quad 2\beta^{JPE}_{HH} \leq \frac{2\Delta Q}{1-\alpha} \left[ (1-\sigma)\Delta Q - c \right]$$
For (4) to be a sufficient condition for the relational contract to hold, either (A.4) or (A.5) must hold with equality. The fixed payment can always be chosen in a way that either (A.4) or (A.5) holds with equality. From the incentive constraint (2), we see that \( \alpha \) must be negative for (A.5) to hold with equality. Note however, I have not included ex ante participation constraints in the model. But observe that if the agents have ex ante outside opportunities \( \widetilde{w} \), then the principal must satisfy (A.5’) \( \frac{\beta^{\text{RPE}}}{1 - \beta} (\alpha + (\sigma + (1 - \sigma)q_H)q_H) \beta^{\text{RPE}}_{HL} - c \geq \widetilde{w} \) for both agents to honour the contract, implying that the fixed salary need not be negative for a (A.5) to hold with equality. As long as low effort is costless, the agents would not have incentives to *not participate* if the participation constraint is satisfied. Moreover, the agents cannot use ‘no participation’ as a threat in the relational contract if the principal then can contract with other agents at the same cost.

In RPE, the principal only has to pay wage to one of the agents, and this occurs only when one realizes high values and one realizes low values. The binding constraint for the principal to honour the contract \( B^{\text{RPE}} \) is thus

\[
\begin{align*}
\frac{\beta^{\text{RPE}}}{1 - \beta} + \frac{2\beta^{\text{RPE}}}{1 - \beta} \left[ Q_L + (\sigma + (1 - \sigma)q_H)\Delta Q - (1 - \sigma)q_H (1 - q_H) \beta^{\text{RPE}}_{HL} - \alpha \right] \\
\geq \frac{2\beta^{\text{RPE}}}{1 - \beta} \left[ Q_L + (\sigma + (1 - \sigma)q_H)\Delta Q \right]
\end{align*}
\]

The binding constraint for each agent is

\[
\begin{align*}
\frac{\beta^{\text{RPE}}}{1 - \beta} (\alpha + (1 - \sigma)q_H (1 - q_H) \beta^{\text{RPE}}_{HL} - c) & \geq 0
\end{align*}
\]

Combining (A.6) and (A.7) for both agents yields

\[
\beta^{\text{RPE}}_{HL} \leq \frac{2\beta^{\text{RPE}}}{1 - \beta} \left[ (1 - \sigma)\Delta q \Delta Q - c \right]
\]

In IPE, the binding constraint for the principal to honour the contract \( B^{\text{IPE}} \) is given by
The binding constraint for each agent is

\[
(A.9) \quad \beta_{hh}^{\text{JPE}} + \frac{\delta}{1-\delta} (\alpha + (1-\sigma)q_h) \geq 0
\]

Combining (A.8) and (A.9) for both agents yields

\[
(9) \quad 2\beta_{hh}^{\text{JPE}} \leq \frac{2\delta}{1-\delta} \left[(1-\sigma)\Delta q\Delta Q - c\right]
\]

3. The conditions for self-enforcing relational contracts when the agents own the assets.

The principal will honour the contract \( \beta_{hh}^{\text{JPE}} \) if

\[
(A.10) \quad -2\beta_j^{\text{JPE}} + \frac{2\delta}{1-\delta} \left[Q_L + (\alpha + (1-\sigma)q_H)\Delta Q - (\alpha + (1-\sigma)q_H)\beta_{hh}^{\text{JPE}} - \alpha\right] \\
\geq \frac{(Q_L + P_L)}{2} + \frac{(Q_j + P_j)}{2} + \frac{2\delta}{1-\delta} \left[Q_L + (\alpha + (1-\sigma)q_L)\Delta Q - \gamma\right]
\]

where \( \gamma = \frac{Q_L + (\alpha + (1-\sigma)q_L)\Delta Q + P_L + (\alpha + (1-\sigma)q_L)\Delta P}{2} \) is the expected Nash bargaining price. To see which realizations bind, we must study (i) \( 2\beta_j^{\text{JPE}} \leq \frac{Q_L}{2} + \frac{Q_j}{2} \). Observe that (H,L) and (L,H) never bind since the right hand side of (i) is then higher than (L,L) while both (L,L), (H,L) and (L,H) yields zero wage. Hence, we must compare (ii) \( 2\beta_{hh}^{\text{JPE}} \leq \frac{Q_L}{2} + \frac{Q_h}{2} \) with (iii) \( 0 \leq \frac{Q_L}{2} + \frac{Q_j}{2} \). We see that if the difference between the left hand side of (ii) and (iii) are smaller than the difference between the right hand side of (ii) and (iii), that is \( 2\beta_{hh}^{\text{JPE}} - 0 < Q_H - Q_L \), then (L,L) binds since (iii) are then weaker than (ii). Hence, when \( 2\beta_{hh}^{\text{JPE}} > \Delta Q \) (A.10) binds on high realizations of \( Q \), and when \( 2\beta_{hh}^{\text{JPE}} < \Delta Q \) (A.10) binds on low realizations.

We see that the only difference from (A.4) where the principal has all the ex post bargaining
power, is that the deviation payoff on the right hand side is smaller since he cannot just take the good, but has to pay for it after deviation. Note also that for a relational contract to exist, the expected Nash bargaining price cannot satisfy the incentive constraint, because then would either the principal, if it was sufficiently low, or the agents if it was sufficiently high, insist on spot trading, instead of long term contracts. Hence, if there exists a relational contract, the agent’s will play low effort after deviation.

The binding constraint for agent $i$ is when he realizes high output as agent $j$ realizes low output. The optimal JPE scheme, $B^{JPE}$, then yields no wage to the agent, while Nash bargaining yields a higher deviation payoff. Hence, the pair of constraints that bind are

$$\begin{align*}
(A.11) & \quad \beta_{HH}^{JPE} \geq \frac{\delta}{1-\delta} (\alpha + (1- \sigma) q_H q_H) \beta_{HH}^{JPE} - c \geq \frac{Q_H + \eta_H}{2} + \frac{1}{1-\delta} \gamma \\
(A.12) & \quad \beta_{HH}^{JPE} \geq \frac{\delta}{1-\delta} (\alpha + (1- \sigma) q_H q_H) \beta_{HH}^{JPE} - c \geq \frac{Q_H + \eta_H}{2} + \frac{1}{1-\delta} \gamma
\end{align*}$$

When $2\beta_{HH}^{JPE} > \Delta Q$, combining (A.10) (A.11) and (A.12) yields

$$\begin{align*}
(13a) & \quad 2\beta_{HH}^{JPE} - \frac{1}{2} \Delta Q + \Delta P \leq \frac{2\delta}{1-\delta} [(1- \sigma) \Delta q \Delta Q - c]
\end{align*}$$

When $2\beta_{HH}^{JPE} < \Delta Q$, combining (A.10) (A.11) and (A.12) yields

$$\begin{align*}
(13b) & \quad \frac{1}{2} \Delta Q + \Delta P \leq \frac{2\delta}{1-\delta} [(1- \sigma) \Delta q \Delta Q - c]
\end{align*}$$

In RPE, the principal will honour the contract $B^{RPE}$ if

$$\begin{align*}
(A.13) & \quad -\beta_y^{RPE} + \frac{2\delta}{1-\delta} \left[ Q_L + (\sigma + (1- \sigma) q_H) \Delta Q - (1- \sigma) q_H (1- q_H) \beta_{HH}^{RPE} - \alpha \right] \\
& \quad \geq -\frac{(Q_L + \eta_L)}{2} - \frac{(Q_H + \eta_H)}{2} + \frac{2\delta}{1-\delta} \left[ Q_L + (\sigma + (1- \sigma) q_L) \Delta Q - \gamma \right]
\end{align*}$$

Observe that (H,H) never binds, since this yields zero wage outlays if he honours, and high
wage outlays (in terms of Nash prices) if he deviates. Comparing \( \beta_{HL}^{RPE} \leq \frac{Q_L}{2} + \frac{Q_H}{2} \) to \( \beta_{LL}^{RPE} = 0 \leq \frac{Q_L}{2} + \frac{Q_H}{2} \) shows that when \( \beta_{HL}^{RPE} > \frac{1}{2} \Delta Q \), then (H,L) binds and when \( \beta_{HL}^{RPE} < \frac{1}{2} \Delta Q \), then (L,L) binds.

The binding constraint for both agents is

\[
(A.14) \quad \beta_{HL}^{RPE} + \frac{1}{1-\delta} \left( \alpha + (\sigma + (1-\sigma)q_H(1-q_H)) \right) \beta_{HL}^{RPE} - c \geq \frac{Q_H + P_L}{2} + \frac{1}{1-\delta} \gamma
\]

Observe that the constraint binds when both agents realize high output. The optimal RPE scheme, \( \beta^{RPE} \), then yields no wage to the agents, while high realizations yield a higher deviation payoff.

When \( \beta_{HL}^{RPE} > \frac{1}{2} \Delta Q \), combining (A.13) and (A.14) then yields

\[
(14a) \quad \beta_{HL}^{RPE} + \frac{1}{2} \Delta Q + \Delta P \leq \frac{2\delta}{1-\delta} \left[ (1-\sigma)\Delta Q - c \right]
\]

When \( \beta_{HL}^{RPE} \leq \frac{1}{2} \Delta Q \), combining (A.13) and (A.14) then yields

\[
(14b) \quad \Delta Q + \Delta P \leq \frac{2\delta}{1-\delta} \left[ (1-\sigma)\Delta Q - c \right]
\]

In IPE, the principal will honour the contract \( \beta^{IPE} \) if

\[
(A.15) \quad -2\beta_{ij}^{IPE} + \frac{2\delta}{1-\delta} \left[ Q_L + (\sigma + (1-\sigma)q_H) \Delta Q - (\sigma + (1-\sigma)q_H) \beta_{HL}^{IPE} - \alpha \right] \\
\geq -\frac{(Q_L + P_L)}{2} - \frac{(Q_L + P_H)}{2} + \frac{8\delta}{1-\delta} \left[ Q_L + (\sigma + (1-\sigma)q_L) \Delta Q - \gamma \right]
\]

Each agent will honour the contract if

\[
(A.16) \quad \beta_{ij}^{IPE} + \frac{1}{1-\delta} (\alpha + (\sigma + (1-\sigma)q_H)) \beta_{HL}^{IPE} - c \geq \frac{Q_H + P_L}{2} + \frac{8}{1-\delta} \gamma
\]
We see that when \( 2\beta_{ihi}^{PE} > \Delta Q \), (A.15) binds on high realizations of \( Q \), and (A.16) binds on low realizations. Combining (A.15) with (A.16) for both agents yields \( 2\beta_{ihi}^{PE} - \Delta Q + \Delta P \leq \frac{2}{1-\delta} \left[ (1-\sigma)\Delta q\Delta Q - c \right] \). When \( 2\beta_{ihi}^{PE} < \Delta Q \), (A.15) binds on low realization of \( Q \), and (A.16) binds on high realizations. Combining (A.15) with (A.16) for both agents yields \( -2\beta_{ihi}^{PE} + \Delta Q + \Delta P \leq \frac{2}{1-\delta} \left[ (1-\sigma)\Delta q\Delta Q - c \right] \). Using absolutes, we can reduce it to one condition:

\[
(15) \quad \left| 2\beta_{ihi}^{PE} - \Delta Q \right| + \Delta P \leq \frac{2\Delta}{1-\delta} \left[ (1-\sigma)\Delta q\Delta Q - c \right]
\]

4. Deducing (12)

For \((H,H)\) to be an equilibrium, \( \beta_{hl} \) must be chosen so that the randomisation becomes a non-equilibrium. This means that the workers must find it profitable to deviate from the randomisation strategy. If the worker fulfils the strategy, his payoff is

\[
(A.17) \quad \alpha + \beta_{hl} (1-\sigma)q_L (1-q_H) + \frac{\delta}{1-\delta} \frac{1}{2} (\alpha + \beta_{hl} (1-\sigma) (q_H (1-q_L) + q_L (1-q_H)) - c)
\]

A worker may only deviate from the randomisation strategy when he gets a turn to play low effort. The worker can then deviate by playing high effort, and this will be punished in the future by \((H,H)^\infty\). Hence the deviation payoff will be

\[
(A.18) \quad \frac{1}{1-\delta} (\alpha + \beta_{hl} (1-\sigma)q_H (1-q_H) - c)
\]

To make the randomisation a non-equilibrium, \( \beta_{hl} \) must be chosen so that (A.18) exceeds (A.17). That is

\[
(12) \quad \beta_{hl} \geq \frac{(2-\delta)c}{(1-\sigma)(2q_H - \delta)\Delta q} = \beta_{hl}^{RPEC}
\]
REFERENCES


