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Asset Specificity and Vertical Integration

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Abstract: Asset specificity is usually considered to be an argument for vertical integration. The main idea is that specificity induces opportunistic behaviour, and that vertical integration reduces this problem of opportunism. In this article I show that asset specificity actually can be an argument for non-integration. In a repeated game model of relational contracts, based on Baker, Gibbons and Murphy, 2002, I show that asset specificity affects the temptation to renege on relational contracts between non-integrated parties, but not between integrated parties. If the parties are non-integrated, higher levels of specificity can provide relational contracts with higher-powered incentives.

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1. Introduction

The transaction cost theory entered the stage in the mid 1970s, partly as an attempt to explain the fundamental Coaseian question: Why do we have firms? The question acted as a headline for the general problem of economic organization: How do we explain the various observed ways of organizing economic activity? The factors leading to vertical integration have been a central issue in this literature. And a factor that has received a lot of attention is the degree of asset specificity. The traditional hypothesis is that asset specificity leads to vertical integration. This hypothesis is formulated through different lines of thought. Klein, Crawford and Alchian (1978) emphasize the problem of “hold-up”. A party that has invested in specific assets may be forced to accept a worsening of the terms of the relationship after the investment is sunk. Hence, asset specificity creates appropriable specialized quasi rents. Klein et al. claim that “integration by common or joint ownership is more likely the higher the appropriable specialized quasi rents of the assets involved.” Williamson (1985, 1991) emphasizes the problem of maladaptation. As investments in specific assets increase, disturbances requiring coordinated responses become more numerous and consequential. The high-powered incentives of markets may impede efficient coordination, since both parties want to appropriate as much as possible of the coordination gains. Vertical integration is a way of reducing this kind of maladaptation. The “property rights approach”, developed by Grossman, Hart and Moore (GHM) (1986, 1990), does not formulate an explicit hypothesis concerning asset specificity, but states that if assets are strictly complementary, then some form of integration is optimal. GHM show that if complementary or co-specialized assets operate under separate ownership, the parties owning the assets will underinvest in the relationship.

The three approaches introduced here share the common belief that there is a correlation between the degree of asset, or investment, specificity and the appearance of vertical integration. In the April 2000 edition of Journal of Law and Economics, Klein states that “...the rigidity costs associated with long term contracts increase as relationship-specific investments increase (...). Therefore, the greater the relationship-specific investments present in an exchange, the more likely vertical integration (that avoids the rigidity costs associated with long term contracts) will be chosen as the self-enforcing arrangement. All that is
required for this positive relationship between specific investments and the likelihood of vertical integration is that the relative inefficiency costs from weakening of incentives is not systematically positively related to the level of specific investments, and there is no reason to believe they are.”

In the present article I will show, however, that there may be a reason to believe that the “relative inefficiency costs from weakening of incentives” are systematically positively related to the level of asset and investment specificity. The analysis draws on a repeated game model of relational contracts developed by Baker, Gibbons and Murphy (BGM), 2002. A relational contract contains rules or standards that cannot be legally enforced. Hence, the contract must be self-enforceable in the sense that the present value of honouring the contract must be greater than the present value of reneging. BGM show how asset allocation matters in the presence of long-term relational contracts. An important result is that incentives in relational contracts between firms can be higher-powered than incentives in relational contracts within firms. In a modified version of BGM’s model, I show that this difference in incentive intensity is positively related to the degree of asset and investment specificity.

The repeated game model is one in which an upstream party in each period uses an asset to produce a good that could either be used in a specific downstream party’s production process, or put to an alternative use. Asset ownership conveys ownership of the good produced, so if the upstream party owns the asset (non-integration), the downstream party cannot use the good without buying it from the upstream party, whereas if the downstream party owns the asset (integration), then he already owns the good. Since the good’s value to the downstream party exceeds its value in the alternative market, the parties agree on a relational contract where the downstream party pays bonuses to make the upstream party improve the specific quality of the good. In order to analyse asset specificity within this framework, it is necessary to make modifications to BGM’s model. In their set up, the parties play grim trigger strategies in which deviation from the relational contract results in spot governance forever after. In spot governance, the parties cannot contract ex ante on ex post realizations, but they can negotiate ex post over the price of the good. BGM assume Nash bargaining, so the price depends on

1 Relational contracts are also called ‘implicit’ contracts (e.g. MacLeod and Malcomson, 1989).
2 Following Grossman and Hart’s (1986) terminology, seller ownership is called “non-integration”; buyer ownership is called “integration”.

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bargaining positions, but not necessarily on the level of asset specificity (since high and low levels of specificity can yield the same spot price). Asset specificity clearly matters, however, if the parties face the possibility of actual trade in the alternative market. I analyse so-called carrot and stick strategies where contract deviation results in a one-period trade in the alternative market before return to the relational contract. These kinds of strategies, also called mutual punishment (Myerson, 1997), are more complex to analyse, but are still more realistic than the standard grim strategies.

When the alternative market is a real alternative and the parties can choose between a relational employment contract (integration) and a relational outsourcing contract (non-integration), high levels of asset specificity induce relational outsourcing. The reason is that increased specificity reduces the temptation to renge on a relational outsourcing contract, since the benefit of external trade is reduced. In a relational employment contract, however, the downstream owner has the residual control right to the good produced, so the upstream party cannot hinder the downstream party to force internal trade. Hence, asset specificity does not affect the self-enforcing conditions of the employment contract. This difference between employment contracts and outsourcing contracts makes the relative efficiency of non-integration increase with the level of asset specificity. The reduced temptation to renge on the relational outsourcing contract, due to increased specificity, makes it possible to design higher-powered incentive schemes without running the risk of opportunistic behaviour.

This link between asset specificity, contract efficiency and asset allocation seems not to be addressed in the theoretical part of the literature. Repeated game models of economic organization acknowledge that relational contracts may be a substitute for vertical integration in dealing with the problem of opportunism. They also recognize the role of reneging temptation in the design of efficient incentive contracts. But the absence of a formal comparison of relational contracts between firms and relational contracts within firms, prior to Baker, Gibbons and Murphy’s important contribution, has made the separating effect of specificity hard to identify. Klein and Leffler analyse reputation effects in assuring product quality in their seminal 1981 paper. The buyer pays a price premium to the supplier to ensure that the supplier exerts effort to produce good quality. If the supplier reneges on the contract,

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3 The terms ‘relational employment’ and ‘relational outsourcing’ stem from BGM
all his potential customers get to know, and the supplier therefore loses all future sales. Hence, the alternative market disciplines against opportunistic behaviour. But Klein and Leffler do not compare relational contracts between independent parties with contracts between vertically integrated parties. Halonen (2002) recognizes the importance of reducing outside options in order to reduce the gain from contract deviation, in her dynamic version of the Hart/Moore (1990) game. But she does not relate the outside options to the difference between specific and alternative use. Since the supplier in her model only makes specific investments in human capital, she relates the outside option to the investing party’s dependency on the asset he invests in, i.e., to what extent it is important that the investing party manages the asset. Hence, Halonen does not make any statements concerning asset specificity and vertical integration, but she recognizes that separation of strictly complementary assets can be beneficial in providing maximum punishment for deviation.

The idea that putting parties in more adverse situations may promote efficiency is also discussed in Klein (1980) and Williamson (1983). Klein refers to the case where franchisers require franchisees to rent from them, rather than own the land on which their outlet is located. This prevents opportunism since the franchiser can require the franchisee to move if the franchisee cheats. Williamson uses the concept of hostages to emphasize the importance of credible commitment. By posting hostages, that is posting a value before the transaction in order to commit to the other party, one can reduce the possibility of opportunistic behaviour and negotiate a contract with better terms. Chiu (1998) relates the importance of credible commitment directly to the concept of investment specificity. He claims that “the theoretical prediction that integration is more likely in the presence of relationship-specific investments is not as robust as previously thought”. He shows that specific investments cause a threat to the relationship when outside options are attractive, not when outside options are unattractive, as the traditional hypothesis implies. But Chiu does not compare the effect specificity may have on contracts between integrated parties with the effect on contracts between non-integrated parties.

The rest of the paper is organized as follows: Section 2 briefly discusses the empirical research on the determinants of vertical integration. Section 3 presents the model. A comparative analysis is made in Section 4, while section 5 concludes.
2. The empiricism of vertical integration

There is an impressive body of empirical research that supports predictions of transaction cost economics (see Joskow 1988, and Shelanski and Klein 1995 for an overview). I believe, however, that the empirical work does not verify the hypothesis that asset specificity leads to vertical integration. It is a fact that a number of quantitative case studies and cross-sectional econometric analyses show a positive correlation between asset specificity and vertical integration. But these studies do not prove that asset specificity leads to vertical integration. The econometric models assume that organizational form is a function of asset specificity, uncertainty, complexity and frequency. Organizational form is the dependent variable while asset specificity is one of the independent variables. The causality between the variables is in general not discussed.

Even though many transaction cost economists claim that the vertical integration hypothesis has a substantial empirical foundation, a number of prominent economists question the empirical validity of the hypothesis. Ronald Coase has all since his famous contribution “The Nature of the Firm” (1937) doubted the importance of asset specificity in bringing about vertical integration. He is in fact sceptical to the concept of opportunism in analyses of economic organization. He argues (1988) that the importance of reputation makes it unlikely that a party would act opportunistically even if assets are specific. His experience is that businessmen find contractual arrangements to be a satisfactory answer to the possible problems of asset specificity. Holmström and Roberts (1998) point out “many of the hybrid organizations that are emerging are characterized by high degrees of uncertainty, frequency, and asset specificity, yet they do not lead to integration. In fact, high degrees of frequency and mutual dependency seem to support, rather than hinder, ongoing cooperation across firm boundaries.”

The economic organization of the international oil industry may serve as good example of separated specific assets. The oil companies and their main suppliers, who design and build installations that the oil companies use to extract oil, always operate with separate ownership. But the suppliers manage capital stock and produce inputs that are highly specific to the buying oil companies. The inputs may be valuable to a competing oil company, but the technology is often tailor-made for a specific field or a specific company. The parties usually
agree on a so-called EPCI-contract, in which the main suppliers are responsible for engineering, procurement, construction and installation. The parties normally agree on an even split of cost overruns and savings relative to a target sum. Hence, the contracts contain high-powered incentive schemes (for more details see Osmundsen, 1999). It is reasonable to assume that these incentive schemes would not have been feasible in an integrated solution. The specificity of the assets and the dependency between the parties makes it possible for the oil companies to design strong incentives without the risk of hold-up behaviour.

The classical empirical case of vertical integration has been the General Motors’ (GM) acquisition of Fisher Body in 1926. The standard view has been that GM merged vertically with Fisher Body because of concerns over specific investments and hold-up behaviour. Several economists now question this explanation. Coase (2000) points out that GM already owned 60 percent of the shares of Fisher Body before they acquired the remaining 40 percent. He claims that there is no evidence that hold-up occurred before the merger took place. Freeland (2000) states that “far from reducing opportunistic behaviour, the vertical integration in fact increased GMs vulnerability to rent seeking behaviour based in human asset specificity”. Casadesus and Spulber (2000) argue that the merger reflected economic considerations specific to that time, not some immutable market failure. The contractual arrangements and working relationship prior to the merger, they claim, exhibited trust rather than opportunism.

3. The model

Baker, Gibbons and Murphy analyse an economic environment consisting of an upstream party (U), a downstream party (D) and an asset, where both parties and the asset live forever or cease to exist simultaneously at a random date. The parties are risk neutral and share the discount factor, \( \delta \), per period. The upstream party uses the asset to produce a good that could either be used in the downstream party’s production process, or put to an alternative use. In each period the upstream party chooses a vector of \( n \) actions (or investments) \( \mathbf{a} = (a_1, a_2, \ldots, a_n) \) at a cost \( c(\mathbf{a}) \) which affects the value of the product both for the downstream party \( Q \) and for the alternative market \( P \). The downstream value is either high or low, where \( q(\mathbf{a}) \) is the probability that a high value \( Q_H \) will be realized and \( 1 - q(\mathbf{a}) \) is the
probability that a low value $Q_L$ will be realized. The alternative-use value can also be either high or low, where $p(a)$ is the probability that a high value, $P_H$, will be realized and $1 - p(a)$ is the probability that a low value $P_L$ will be realized. Given the upstream party’s actions, the downstream and the alternative-use values are conditionally independent. It is assumed that $c(0) = q(0) = p(0) = 0$, so when the upstream party decides not to take actions, he bears no costs but also has no chance of realizing the high values. It is further assumed that $P_L < P_H < Q_L < Q_H$ so that the value to the downstream party always exceeds its value in the alternative use. In other words, the asset is relationship specific. The first-best actions, $a^*$, maximizes the expected value of the good in its efficient use minus the cost of action, hence the total surplus from the transaction is given by

$$S^* = \max_a \left[ Q_L + q(a^*)\Delta Q - c(a^*) \right],$$

where $\Delta Q = Q_H - Q_L$.

The actions are unobservable to anyone but the upstream party, so contracts contingent on actions cannot be enforced. It is assumed that $Q$ and $P$ are observable, but not verifiable, so it is possible to design self-enforceable contracts, but not to contract on $Q$ or $P$ in a way that a third party can enforce.

The parties can organize their transactions through different choices of contract governance and ownership structure. With respect to ownership structure, it is assumed that asset ownership conveys ownership of the good produced, so if the upstream party owns the asset (non-integration), the downstream party cannot use the good without buying it from the upstream party, whereas if the downstream party owns the asset (integration), then he already owns the good. With respect to contract governance, the parties can agree on either a spot contract or a relational contract. In a spot contract, a spot price is negotiated for each period and is determined by ownership structure and bargaining positions. If the upstream party owns the asset, 50:50 Nash bargaining over the surplus from trade decides the spot price. If the downstream party owns the asset, he can just take the realized output without paying, so the upstream party will refuse to take costly actions. In a relational contract, the parties agree on a compensation contract $(s, b_L, b_H, \beta_L, \beta_H)$ where salary $s$ is paid by downstream to upstream at the beginning of each period, and $b_i$ is supposed to be paid when $Q_i$ is realized,
(i = H, L) and β_{j} when P_{j} is realized, (j = H, L). For example: If the upstream party produces a good which yields a high value in the specific relation, Q_{H}, and a low value in the alternative market, P_{L}, the downstream party should, according to the contract, pay the bonuses b_{H} + \beta_{L} to the upstream party. Such a contract induces the upstream party to yield effort even if he doesn’t own the asset. Since the contract cannot be enforced by a third party, the parties will honour the contract only if the present value of honouring is greater than the present value of reneging.

BGM’s taxonomy of organizational design is summarized as follows (see BGM, *QJE* pp.46):

<table>
<thead>
<tr>
<th></th>
<th>Non-integration</th>
<th>Integration</th>
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</thead>
<tbody>
<tr>
<td>Spot contract</td>
<td>Spot outsourcing (SO)</td>
<td>Spot employment (SE)</td>
</tr>
<tr>
<td>Relational contract</td>
<td>Relational outsourcing (RO)</td>
<td>Relational employment (RE)</td>
</tr>
</tbody>
</table>

So far, I have been following BGM’s set-up. In this paper I will compare relational outsourcing with relational employment using other player strategies than the grim trigger strategies analysed by BGM. In BGM, if a party reneges on a contract, the other party refuses to enter into a new relational contract with that party. Instead, they agree to trade in spot governance forever after. In this paper, however, if one of the parties reneges, they first agree on a spot price (as in BGM). In the next period, the party who did not renege punishes the other party by refusing to enter into any agreement (including a spot agreement) and instead chooses to trade in the alternative market. After this “punishment phase” the parties return to a relational contract (see strategy specifications below). These kinds of trigger strategies are in the literature referred to as mutual punishment strategies, carrot and stick strategies, or two-phase punishment strategies (see Gibbons, 1992).

BGM’s strategy specifications have the advantage of both being simple to analyse and making it possible to compare all four organizational forms within the same framework. In the modification studied here, it is simply assumed that specificity deters spot contracting from being a long-term option. Still, there are several reasons for making this modification.

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4 BGM start up with a more general contract \((s_{i}, b_{iH}, b_{iL}, b_{rH}, b_{rL})_{i,j=H,L}\), but restrict it to \((b_{iH} = b_{H} + \beta_{H}, b_{iL} = b_{H} + \beta_{L}, b_{rH} = b_{L} + \beta_{H}, b_{rL} = b_{L} + \beta_{L})\) in order to simplify the comparative analysis.
First, it can be argued that the carrot and stick strategy is more realistic than the grim strategy, especially in buyer/supplier relationships with high levels of asset or investment specificity. It is difficult to understand why the parties would stick to spot governance forever after a contract breach when specificity makes relational contracting significantly more efficient than spot contracting. Carrot and stick strategies are more in line with actual economic behaviour off the cooperation path. In the offshore industry, for example, contract breach often results in operators i) renegotiating the terms of the current project, (ii) searching for new long term trading partners, while trading directly in inferior spot markets iii) entering into a new long term contracts with either the old trading partner or a new one (see e.g the Norsok reports, 1995). Second, analysing carrot and stick strategy equilibria is more appropriate if asset specificity is regarded as a significant explanatory variable. In order to analyse the effect of asset specificity in long term contracts, the alternative market must be modelled as a real threat point, not merely a reference point for spot negotiations. In BGM the level of asset specificity does not affect the robustness of relational contracts. In the present paper, however, asset specificity does affect the parties’ temptation to renege on relational contracts. Third, both grim strategies and carrot and stick strategies yield the same surplus, for given actions, in equilibrium. But for sufficiently high levels of specificity, efficient relational contracts can be implemented for lower discount factors when the parties play the carrot and stick strategy than when they play the grim strategy. This provides an argument for studying carrot and stick strategies in the presence of specificity.

In this paper, the strategy for U (D) is specified as follows:

1. In period $t$, honour the terms of the relational contract $(s, b_L, b_H, \beta_L, \beta_H)$ if D (U) honoured in period $t-1$.
2. In period $t$, honour the terms of the relational contract $(s, b_L, b_H, \beta_L, \beta_H)$ if there was no trade with D(U) in period $t-1$.
3. In period $t$, refuse to trade with D (U) if the trade between the parties in period $t-1$ was accomplished by spot contracting.

To “honour the terms of the relational contract” means for the upstream party to accept the bonuses offered and for the downstream party to pay the promised bonuses. We enter this game ex post quality realizations in period $t$. When the parties are to decide whether to honour
or renege on the contract, they know the quality realizations of period $t$, but can only have expectations regarding the remaining periods. The parties honour the contract if the present value of honouring exceeds the present value of reneging. A relational contract is self-enforcing if both parties choose to honour the contract $(s, b_L, b_H, \beta_L, \beta_H)$ for all possible realizations of $Q_t$ and $P_t$. The critical part of the analysis is to deduce the conditions for when the relational employment contract and the relational outsourcing contract are self-enforcing. Technically, these are conditions for when the strategies specified above constitute subgame perfect Nash equilibria of relational contracts. See appendix on subgame perfection.

Before we proceed, consider four additional assumptions: First, it is assumed that both parties incur a switching cost $v$ by trading in the alternative market when the product has already been produced for the purpose of trading in the specific relation. They avoid this cost if they know ex ante that no trade will occur between the parties. Second, in contrast to BGM, it is assumed that ownership is fixed on the “punishment path”. This seems realistic as long as the strategies, in case of deviation, specify only one period of spot governance. Only small negotiation costs would make a one-period ownership transfer inefficient (Halonen (2002) fixes ownership forever after deviation even in grim trigger strategies). Third, BGM assume that $C(0)$ yields $Q_L$ always. But it is more realistic and thus assumed in this paper, that if the upstream party takes no costly actions, he can choose between realizing $Q_L$ and realizing zero values. This gives the upstream party a punishment possibility even if the downstream party owns the asset. Fourth, it is assumed that the downstream party’s valuation of the alternative market goods is equal to the price he has to pay. Hence, if the downstream party buys the good in the alternative market, he earns no surplus from this trade. None of these assumptions changes the quality of the results in this paper, but they are made both for analytical convenience and in order to make the upstream-downstream relationship as realistic as possible.

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5 We can view these costs as time costs or extra transport costs associated with the unexpected move from relational trade to alternative market trade.

6 This assumption will not change the downstream payoff function in relational contracts since $Q$ will always be realized in relational contracts equilibria. Also note that this does not mean that upstream can hold-up the good in relational employment ex post realization. The choice of realizing $Q_L$ or zero is taken ex ante.

7 BGM say nothing about this since the parties never trade in the alternative market in their model.
3.1 Relational employment

If the upstream party is confident that the downstream party will honour the contract \((s, b_L, b_H, \beta_L, \beta_H)\) then the upstream party will choose actions \(a^{RE}\) that solve:

\[
\max_a \left( s + b_L + \Delta b q(a) + \beta_L + \Delta \beta p(a) - c(a) \right) \equiv U^{RE}
\]

where \(\Delta b = b_H - b_L\), \(\Delta \beta = \beta_H - \beta_L\), and superscript \(\text{(RE)}\) denote relational employment. The expected downstream payoff is then

\[
E(Q_i - b_j - \beta_j - s | a = a^{RE}) = Q_L + \Delta Q q(a^{RE}) - s - b_L - \Delta b q(a^{RE}) - \beta_L - \Delta \beta p(a^{RE}) \equiv D^{RE}
\]

so total surplus under relational employment is \(S^{RE} \equiv U^{RE} + D^{RE} = Q_L + q(a^{RE}) \Delta Q - c(a^{RE})\).

Given that the downstream party always honours the contract, the upstream party will earn \(s + b_i + \beta_j - c(a^{RE})\) in period \(t\), and expect a total of \(\frac{s + \beta}{1-s}(s + b_L + \Delta b q(a^{RE}) + \beta_L + \Delta \beta p(a^{RE}) - c(a^{RE})) = \frac{s}{1-s} U^{RE}\) from future trade if he honours the contract. To make the different payoffs easy to compare, I distinguish between period \(t\), period \(t+1\), and all the remaining periods. The present value of honouring the relational employment contract is thus written

\[
s + b_i + \beta_j - c(a^{RE}) + \delta U^{RE} + \frac{\delta^2}{1-s} U^{RE}.
\]

If the upstream party reneges on the contract in period \(t\) by refusing to accept the promised payment \(b_i + \beta_j\) (or refusing to make a promised payment if \(b_i + \beta_j < 0\)), the trade is accomplished by spot contracting, where the downstream party, as the asset owner, just takes the good and leave the upstream party with nothing. According to the specified strategies, there is no trade between the parties in period \(t+1\), so the upstream party earns nothing and bears no investment costs. In period \(t+2\) the relational contract is re-established. The payoff after reneging is then

\[
s - c(a^{RE}) + \frac{\delta^2}{1-s} U^{RE}.
\]

The upstream party will thus honour rather than renege on the relational employment contract when, for all values of \(i\) and \(j\),
Given that the upstream party always honours the contract, the downstream party’s payoff from honouring the relational employment contract is

\[ Q_i = -b_i - \beta_j - s + \delta D^{RE} + \frac{\delta^2}{1-\delta} D^{RE} . \]

If the downstream party reneges on the contract in period \( t \), he will just take the realized value, \( Q_i \), and pay nothing. In period \( t+1 \) the upstream party will refuse to produce the good, so the downstream party has to buy the good in the alternative market. He will not gain a surplus on this trade, since his valuation of this non-specific good is equal to the price he has to pay. In period \( t+2 \) the relational employment contract is re-established. The present value of reneging on the contract is thus simply

\[ Q_i - s + \frac{\delta^2}{1-\delta} D^{RE} . \]

The downstream party will honour rather than renege on the relational employment contract when, for all values of \( i \) and \( j \),

\[ b_i + \beta_j \leq \delta D^{RE} . \]

(1) and (2) represent 8 constraints that have to hold in order for the relational employment contract to be self-enforcing. Combining these restrictions yields (see appendix):

\[ |\Delta b| + |\Delta \beta| \leq \delta S^{RE} . \]

This is both a necessary and a sufficient constraint for the relational contract \((s,b_L,b_H,\beta_L,\beta_H)\) to hold, since the parties can always choose a fixed salary \( s \) that satisfies both (1) and (2). The efficient relational employment contract maximizes total surplus, \( S^{RE} \), subject to (3).
3.2 Relational outsourcing

In relational outsourcing, if the upstream party is confident that the downstream party will honour the contract \((s, b_L, b_H, \beta_L, \beta_H)\) the upstream party chooses actions \(a^{RO}\) that solve \(\max_a \left( s + b L + \Delta bq(a) + \beta L + \Delta \beta p(a) - c(a) \right) \equiv U^{RO} \) where superscript \(RO\) denotes relational outsourcing. The downstream party’s payoff is then \(E(Q_i - s - b_j - \beta_j | a = a^{RO}) = Q_L + \Delta Q q(a^{RO}) - s - b_L - \Delta bq(a^{RO}) - \beta L - \Delta \beta p(a^{RO}) \equiv D^{RO}, \) so total surplus under relational outsourcing is \(S^{RO} \equiv U^{RO} + D^{RO} = Q_L + q(a^{RO}) \Delta Q - c(a^{RO}).\)

If the upstream party honours the relational outsourcing contract he will receive

\[
s + b_i + \beta_j - c \ (a^{RO}) + \delta U^{RO} + \frac{\delta^2}{1 - \delta} U^{RO} .
\]

If the upstream party reneges on the contract in period \(t,\) trade is accomplished by spot contracting. Since the upstream party now owns the asset, the downstream party cannot just take the good. I assume, like BGM, that the parties set prices by means of 50:50 Nash negotiations, which yields \(\frac{1}{2}(Q_i + P_j - v).\) In period \(t+1,\) downstream refuses to trade with upstream. Anticipating this, upstream chooses actions \(a^{AO},\) which solve \(\max_a \left( P L + \Delta P p(a) - c(a) \right) \equiv U^{AO}.\) In period \(t+2\) the parties re-establish their relational contract.

The upstream party’s payoff after reneging is then

\[
s + \frac{1}{2}(Q_i + P_j - v) - c \ (a^{RO}) + \delta U^{AO} + \frac{\delta^2}{1 - \delta} U^{RO} .
\]

8 The downstream party will pay the upstream party the alternative value \(P_j - v\) plus half the surplus from trade with the downstream party: \(\frac{1}{2}(Q_i - (P_j - v)),\) i.e. \(\frac{1}{2}(Q_i + P_j - v).\)

9 The strategy in which the no-trade-punishment is deferred until period \(t+1\) coincides with subgame perfect equilibrium for \(v\) exceeding a critical level (see Appendix). According to the specified strategies, the parties know that trade in the alternative market follows after spot governance. Hence, they avoid the switching cost \(v\) if they defer the trade in the alternative market from \(t\) until \(t+1.\)
The upstream party will thus honour the contract when, for all values of \( i \) and \( j \),

\[
(4) \quad b_i + \beta_j + \delta U^{RO} \geq \frac{1}{2}(Q_i + P_j - v) + \delta U^{AQ}.
\]

If the downstream party honours the contract he will earn

\[
Q_i - b_i - \beta_j - s + \delta D^{RO} + \frac{\delta^2}{1-\delta} D^{RO}.
\]

If the downstream party reneges in period \( t \), the parties agree on the 50:50 Nash price so that the downstream party earns \( Q_i - s - \frac{1}{2}(Q_i + P_j - v) \). In period \( t+1 \) upstream refuses to trade with downstream, who has to buy the good in the alternative market and thus gains no surplus. The downstream party’s payoff after reneging is then

\[
Q_i - s - \frac{1}{2}(Q_i + P_j - v) + \frac{\delta^2}{1-\delta} D^{RO}.
\]

The downstream party will thus honour the contract, for all values of \( i \) and \( j \), when

\[
(5) \quad \delta D^{RO} \geq b_i + \beta_j - \frac{1}{2}(Q_i + P_j - v)
\]

Combining (4) and (5) yields the following condition for the relational outsourcing contract to be self-enforcing (see appendix):

\[
(6) \quad |\Delta b - \frac{1}{2} \Delta Q| + |\Delta \beta - \frac{1}{2} \Delta P| \leq \delta (S^{RO} - U^{AQ})
\]

Like (3), (6) is both necessary and sufficient. The efficient relational outsourcing contract maximizes total surplus \( S^{RO} \) subject to (6).
4. Comparative analysis

We can now compare relational outsourcing with relational employment. First, observe that (3) and (6) underscore BGM’s main proposition: The parties’ temptation to renege on a given relational contract depends on asset ownership.\(^\text{10}\) I will now show how asset and investment specificity affect the parties reneging temptations under different types of ownership. Define \(Q_L - P_L\) as the level of asset specificity, and \(\Delta Q - \Delta P\) as the level of investment specificity. Now, observe that in relational outsourcing the value of the upstream party’s outside option, 
\[
U^{AO} \equiv P_L + \Delta P p(a^{AO}) - c(a^{AO}),
\]

is part of the relational contract constraint. In relational employment, however, the outside option is equal to zero for any level of \(P_H\) and \(P_L\). Hence, the levels of both asset specificity and investment specificity affect the relational outsourcing constraint, but not the relational employment constraint. In relational outsourcing the downstream party’s temptation to renege is lower than in relational employment, since he cannot just take the good, but has to bargain a spot price with the upstream owner. On the other hand, the upstream party’s temptation to renege is higher under relational outsourcing than under relational employment, because of his outside options. Under relational outsourcing, increased specificity will thus reduce the relative value of the upstream party’s outside option, and thereby give scope for better relational contracts.

From (3) and (6) we observe that increasing incentive intensity, given by \(\Delta b, \Delta \beta\), increases the total temptation to renege on a contract. Low bonuses may induce the upstream party to renege, while high bonuses may induce the downstream party to renege. Moreover, we observe that if \(U^{AO}\) is sufficiently low, then there is scope for higher-powered incentives in relational outsourcing than in relational employment. Hence, if the level of asset specificity is sufficiently high, which implies that \(S^{RO} - U^{RO}\) is high, and high-powered incentives are desirable, then relational employment is inefficient compared to relational outsourcing. We gain intuition by thinking through an incentive for downstream to increase the specificity of an asset. If the upstream party possesses an asset that is highly valuable to a broad market, downstream may wish to acquire the upstream party’s asset in order to avoid strategic behaviour. The problem then is that the downstream party’s incentive to cheat on upstream

\(^{10}\) Olsen (1996) has a related result, showing in a two-period model that the choice of renegotiating a contract depends on organizational form
increases, so upstream may call for lower-powered incentive schemes and higher fixed salaries. But if higher-powered incentives are desirable, he can make tailor-made investments in the asset in a manner that increases its internal, but not its external value. Then he can safely outsource the asset to upstream, achieving higher-powered incentives without running the risk of upstream opportunism.

I will now derive a formal result showing that relational outsourcing can be an efficient response to high levels of specificity. Assume that the two gradients of partial derivatives \( \frac{\partial a}{\partial c} (a^{FB}), \frac{\partial p}{\partial c} (a^{FB}), i = 1,2,\ldots, n \) are linearly independent (superscript FB denotes first-best). Then a first-best solution can only be achieved if \( \Delta b = \Delta Q \) and \( \Delta \beta = 0 \).

Given (3), first-best can be achieved under a relational employment contract if

\[
\delta \geq \frac{\Delta Q}{Q_L + \Delta Q (a^{FB}) - c(a^{FB})} = \delta^{RE}.
\]

Given (6), first-best can be achieved under a relational outsourcing contract if

\[
\delta \geq \frac{\frac{1}{2} (\Delta Q + \Delta P)}{Q_L + \Delta Q (a^{FB}) - c(a^{FB}) - P_L - \Delta P p(a^{AO}) + c(a^{AO})} = \delta^{RO}.
\]

Hence, to be able to implement first-best at equal or lower discount factors in the outsourcing contract than in the employment contract, we must have \( \delta^{RO} \leq \delta^{RE} \), that is

\[
(Q_L + \Delta Q (a^{FB}) - c(a^{FB})) \left( \frac{\Delta Q - \Delta P}{2\Delta Q} \right) - \left( P_L + \Delta P p(a^{AO}) - c(a^{AO}) \right) \geq 0.
\]

We can then state:
**Proposition 1:** i) Assume that there is investment specificity, defined as \( \Delta Q > \Delta P \). If asset specificity is sufficiently large, in the sense that \( Q_L \) is sufficiently high and/or \( P_L \) is sufficiently low, then there exist critical discount factors \( \delta^{RE} > \delta^{RO} > 0 \) such that for \( \delta > \delta^{RO} \) relational outsourcing is first-best and thus at least as efficient as relational employment, and for \( \delta^{RE} > \delta > \delta^{RO} \) relational outsourcing is strictly more efficient than relational employment. ii) Assume that there is no investment specificity (\( \Delta Q < \Delta P \)). Then for any level of asset specificity, i.e. for any level of \( Q_L \) and \( P_L \), there exist critical discount factors \( \delta^{RE} > 0 \) such that for \( \delta > \delta^{RE} \) relational employment is first-best and thus at least as efficient as relational outsourcing, and for \( \delta^{RO} > \delta > \delta^{RE} \) relational employment is strictly more efficient than relational outsourcing.

**Proof:** Given \( \Delta Q > \Delta P \), the left hand side of (7) is strictly increasing in \( Q_L \) and strictly decreasing in \( P_L \). Given \( \Delta Q < \Delta P \), (7) never holds.

I will show, for specific functions, that (7) is also a valid condition in second-best solutions. That is, given (7), relational outsourcing is always an equally efficient or more efficient solution than relational employment. I assume, like BGM, that the upstream party can take two actions: \( a = (a_1, a_2) \), and that the production functions are linear and the cost function quadratic:

\[
q(a_1, a_2) = q_1 a_1 + q_2 a_2 \\
p(a_1, a_2) = p_1 a_1 + p_2 a_2 \\
c(a_1, a_2) = \frac{1}{2} a_1^2 + \frac{1}{2} a_2^2
\]

where \( q_1, q_2, p_1, p_2 \geq 0 \) and \( q_1 p_2 \neq q_2 p_1 \).

The first-best actions are then \( a_1^{FB} = q_1 \Delta Q \) and \( a_2^{FB} = q_2 \Delta Q \). In both the outsourcing contract and the employment contract, the upstream party chooses to maximize \( s + b_L + (q_1 a_1 + q_2 a_2) \Delta b + \beta_L + (p_1 a_1 + p_2 a_2) \Delta \beta - \frac{1}{2} a_1^2 - \frac{1}{2} a_2^2 \), so that \( a_1 = q_1 \Delta b + p_1 \Delta \beta \).
and $a_2 = q_2 \Delta b + p_2 \Delta \beta$. A first-best solution can then only be achieved if $\Delta b = \Delta Q$ and $\Delta \beta = 0$

In order to keep it simple, I assume that $q_2 = p_1 = 0$. The agent can take one action that affects $Q$ and another action that affects $P$.

Given (8), (7) can be written

$$(7') \quad Q_L \left( \frac{\Delta Q - \Delta P}{2 \Delta Q} \right) - P_L - \left( \frac{1}{2} p_2^2 \Delta P^2 - \frac{1}{4} q_1^2 \Delta Q (\Delta Q - \Delta P) \right) \geq 0.$$

Given (7’), first-best cannot be achieved if

$$(9) \quad \delta < \frac{\frac{1}{2}(\Delta Q + \Delta P)}{Q_L + \frac{1}{2} q_1^2 \Delta Q^2 - P_L - \frac{1}{2} p_2^2 \Delta P^2} = \delta^{RO}.$$

**Proposition 2:** Given (7’) and (8), relational outsourcing is at least as efficient as relational employment if $\delta \geq \delta^{RO}$ and strictly more efficient than relational employment if $\delta < \delta^{RO}$.

**Proof:** see appendix.

The propositions suggest that outsourcing may be an efficient response to high levels of specificity. Note the relationship between asset specificity, investment specificity and governance in proposition 1. If there is no investment specificity, relational outsourcing is always an inefficient governance mechanism compared to relational employment. Moreover, if there is investment specificity, relational outsourcing is an efficient response to increased asset specificity.

The propositions help elucidate anecdotal empirical evidence and case studies showing that non-integration is highly compatible with asset/investment specificity. And further, that specificity can actually be beneficial for non-integrated solutions. The proposition may also cast some light on empirical studies questioning other aspects of “Williamsonian”
explanations of integration and outsourcing. Anderson, Glenn and Sedatole (2000) make an interesting empirical study of the relationship between asset complexity and outsourcing decisions. Using data on 156 sourcing decisions for process tooling (dies) of a new car program, they found that attributes that according to transaction cost economics favoured “insourcing”, favours outsourcing if the parties engage in relational contracting. In particular, they found that firms outsourced parts with high levels of complexity, and insourced simple parts with low levels of complexity. Also, parts with high levels of “design constraints” were more likely to be outsourced than parts with low design constraint levels. Problems of strategic behaviour from these relational-dependent external suppliers were relatively small, and field investigations suggested that the external suppliers were more responsive to incentives than internal suppliers.

5. Concluding remarks

The model in this paper identifies local non-monotonic relationships between asset specificity and vertical integration. In vertically integrated firms, there will always be some kind of complementarity between the assets, and the assets of an upstream party vertically integrated with its downstream buyer will always to a certain extent be specific to the downstream party’s needs. If there is no asset specificity or investment specificity, we have a competitive market with no need for contractual incentive schemes. Then the “old rule” would apply, saying that the best manager of an asset is its owner. So if the upstream party is the one taking actions, he should also own the asset. But if we are in an economic environment with significant levels of specificity, as assumed in the model of this paper, then the relationship between asset specificity and vertical integration becomes more complex. To a certain extent, specificity may induce integration, as the downstream party wishes to avoid unfavourable strategic behaviour from upstream. But if the level of specificity is sufficiently high, the relative value of external trade is reduced, and the incentive for upstream to behave opportunistically is reduced as well. Integration may then be an inefficient governance solution compared with non-integration: If the parties can engage in relational contracts, and the surplus from external trade is relatively small compared with the surplus from trade in the specific relationship, the parties will be able to design higher-powered incentive schemes if upstream owns the asset than if downstream owns the asset.
1. The conditions for honouring the relational employment contract

The upstream party’s condition is given by

\( b_i + \beta_j + \delta U^{RE} \geq 0 \)

The downstream party’s condition is given by

\( b_i + \beta_j \leq \delta D^{RE} \)

Since \( i=H,L \) and \( j=H,L \), each of these two conditions contains four constraints. We see that the high quality realisation always imposes the binding constraint on the downstream party, while low quality realisation imposes the relevant constraint on the upstream party. The relevant constraints are then:

\( (b_L + \beta_L) + \delta U^{RE} \geq 0 \)

\( (b_L + \Delta b + \beta_L + \Delta \beta) \leq \delta D^{RE} \)

Multiplying the upstream constraint by (-1) and adding the downstream constraint yields the following necessary and sufficient condition for honouring the relational employment contract:

\( |\Delta b| + |\Delta \beta| \leq \delta S^{RE} \)

2. The conditions for honouring the relational outsourcing contract

The upstream party’s condition is given by:

\( b_i + \beta_j + \delta U^{RO} \geq \frac{1}{2} (Q_i + P_j - v) + \delta U^{AO} \)
The downstream party’s condition is given by:

\[(5) \quad \delta D^{RO} \geq b_i + \beta_j - \frac{1}{2}(Q_i + P_j - \nu)\]

It is now less obvious which constraints are binding. But there will always be two constraints at most that are binding. We see that it depends on the differences: \(\frac{1}{2}\Delta Q - \Delta b\) and \(\frac{1}{2}\Delta P - \Delta \beta\).

When \(\frac{1}{2}\Delta Q > \Delta b\) and \(\frac{1}{2}\Delta P > \Delta \beta\), the relevant constraints are:

\[
\frac{1}{2}(Q_L + \Delta Q + P_L + \Delta P - \nu) + \delta U^{AO} \leq b_L + \Delta b + \beta_L + \Delta \beta + \delta U^{RO}
\]

\[
\frac{1}{2}(Q_L + P_L - \nu) + \delta \left( Q_L + \Delta Q q(a^{RO}) \right) \geq b_L + \beta_L + \delta \left( b_L + \Delta b q(a^{RO}) + \beta_L + \Delta \beta p(a^{RO}) \right)
\]

When \(\frac{1}{2}\Delta Q > \Delta b\) and \(\frac{1}{2}\Delta P < \Delta \beta\), the relevant constraints are:

\[
b_L + \Delta b + \beta_L + U^{RO} \geq \frac{1}{2}(Q_L + \Delta Q + P_L - \nu) + \delta U^{AO}
\]

\[
\delta D^{RO} \geq b_L + \beta_L + \Delta \beta - \frac{1}{2}(Q_L + P_L - \nu)
\]

When \(\frac{1}{2}\Delta Q < \Delta b\) and \(\frac{1}{2}\Delta P > \Delta \beta\), the relevant constraints are:

\[
b_L + \beta_L + \Delta \beta + U^{RO} \geq \frac{1}{2}(Q_L + \Delta Q + P_L - \nu) + \delta U^{AO}
\]

\[
\delta D^{RO} \geq b_L + \Delta b + \beta_L - \frac{1}{2}(Q_L + P_L - \nu)
\]

When \(\frac{1}{2}\Delta Q < \Delta b\) and \(\frac{1}{2}\Delta P < \Delta \beta\), the relevant constraints are:

\[
b_L + \beta_L + U^{RO} \geq \frac{1}{2}(Q_L + \Delta Q + P_L - \nu) + \delta U^{AO}
\]

\[
\delta D^{RO} \geq b_L + \Delta b + \beta_L + \Delta \beta - \frac{1}{2}(Q_L + P_L - \nu)
\]

Multiplying the downstream party’s constraints by \((-1)\) and adding the upstream party’s constraints yields a necessary and sufficient condition for each pair of constraints:

\[(6) \quad |\Delta b - \frac{1}{2}\Delta Q| + |\Delta \beta - \frac{1}{2}\Delta P| \leq \delta \left( S^{RO} - U^{AO} \right)\]
3. The conditions for subgame perfect equilibria:

From Selten (1965), a Nash equilibrium is subgame perfect if the players’ strategies constitute a Nash equilibrium in every subgame. In this game we have an infinite number of subgame divided into three categories: The games that start after trade governed by a relational contract, the games that start after trade governed by a spot contract, and the games that start after no trade between the parties / trade in the alternative market. The carrot and stick strategies constitute subgame perfect equilibrium if U (D), in case of D (U) deviation in period \( t \), finds it optimal to trade under spot governance, S, in period \( t \); refuses to trade with D (U), i.e trades in the alternative market, A, in period \( t+1 \); and returns to relational contracting, R, in period \( t+2 \). We can write this “punishment path” \( (S_t, A_{t+1}, R_{t+2}, R_{t+3} \ldots) \). U (D)’s feasible set of trade actions depends on D (U)’s offer. At the end of each period, the players have taken the same action, but in terms of feasibility A dominates S which dominates R.

There are an infinite number of strategies specifying punishment paths that could constitute subgame perfect equilibria. With the strategies specified here, we can, however reduce the relevant paths to \( (S_t, A_{t+1}, R_{t+2}, R_{t+3} \ldots) \) and \( (A_t, R_{t+1}, R_{t+2} \ldots) \). Recall that when identifying the conditions for subgame perfection, it is commonly assumed that U (D) assumes that D (U) follows his initial strategy after deviations. Hence, it is not possible for U (D) after D (U) deviation in period \( t \) to postpone the trade in the alternative market, for instance to play \( (S_t, S_{t+1}, A_{t+2}, R_{t+3}, R_{t+4} \ldots) \) since according to his initial strategy, D (U) will play A in period \( t+1 \). Hence, in addition to the strategy specified path \( (S_t, A_{t+1}, R_{t+2}, R_{t+3} \ldots) \), we are left with the “competing” path \( (A_t, R_{t+1}, R_{t+2} \ldots) \). No path will include more than one period of trade in the alternative market since A yields the lowest surplus. Also note that in the model it is assumed that a player who reneges on the relational contract offers spot contracting instead of direct trade in the alternative market. Then if \( (S_t, A_{t+1}, R_{t+2}, R_{t+3} \ldots) \) dominates \( (A_t, R_{t+1}, R_{t+2} \ldots) \), deviation starting with spot contracting dominates direct trade in the alternative market.

Relational outsourcing: If the downstream party reneges, the upstream party’s punishment path \( (S_t, A_{t+1}, R_{t+2}, R_{t+3} \ldots) \) dominates \( (A_t, R_{t+1}, R_{t+2} \ldots) \) if
(A.1) \[ \frac{1}{2}(Q_i + P_j - v) + \delta U^{AO} \geq P_j - v + \delta U^{RO} \]
i.e. \[ v \geq 2\delta(U^{RO} - U^{AO}) - (Q_i - P_j) \]

If the upstream party reneges, the downstream party’s punishment path \((S_i, A_{i+1}, R_{r+2}, R_{r+3}, \ldots)\) dominates \((A_r, R_{r+1}, R_{r+2}, \ldots)\) if

(A.2) \[ Q_i - \frac{1}{2}(Q_i + P_j - v) \geq -v + \delta D^{RO} \]
i.e. \[ v \geq \frac{3}{2} \delta D^{RO} - \frac{1}{2}(Q_i - P_j) \]

For sufficiently high switching costs, the upstream party (downstream party) will play \((S_i, A_{i+1}, R_{r+2}, R_{r+3}, \ldots)\) in case of downstream party (upstream party) deviation. For the strategies to constitute subgame perfect equilibrium, (A.1) and (A.2) must hold for the critical discount factor that is necessary for (6) to hold with equality. Note that for sufficiently high levels of specificity (A.1) and (A.2) hold for \(v=0\).

**Relational employment:** In relational employment, the upstream party cannot trade in the alternative market, but he can refuse to trade by not producing the good. But if the downstream party reneges on the contract in period \(t\), the upstream party cannot refuse to trade with the downstream party in this period, since he has already realized \(Q_i\). Hence the upstream party cannot play and is thus “forced” to follow the strategy-specified punishment path \((S_i, A_{i+1}, R_{r+2}, R_{r+3}, \ldots)\).

If the upstream party reneges in period \(t\), the downstream party have no incentive to play \((A_r, R_{r+1}, R_{r+2}, \ldots)\) since in period \(t\) he can just take the realized \(Q_i\). Hence, he follows the strategy-specified punishment path \((S_i, A_{i+1}, R_{r+2}, R_{r+3}, \ldots)\).

**Conclusion:** The strategies constitute a subgame perfect equilibrium if (3), (6), (A.1) and (A.2) hold.
4. Proof of proposition 2

For notational simplicity:
\[ \Delta b = x \]
\[ \Delta \beta = y \]
\[ Q_L = Q \]
\[ P_L = P \]
\[ \Delta Q = z \]
\[ \Delta P = w \]
\[ q_1 = q \]
\[ p_1 = p \]

Given the functional forms specified in (8) and the assumption that \( q_2 = p_1 = 0 \), the surplus from a relational contract is given by

\[ S(x, y) = Q + q^2xz - \frac{1}{2}q^2x^2 - \frac{1}{2}p^2y^2 \]

The outsourcing constraint is given by

\[ |x - \frac{1}{2}z| + |y - \frac{1}{2}w| \leq \delta(Q + q^2xz - \frac{1}{2}q^2x^2 - \frac{1}{2}p^2y^2 - P - \frac{1}{2}p^2w^2) \]

Geometry suggests that the solution is found in the area \( \frac{1}{2}z < x \) and \( \frac{1}{2}w > y \). The maximization problem can then be written

\[ \operatorname{Max}_{x,y} S(x, y) \]

subject to
\[ x - \frac{1}{2}z + \frac{1}{2}w - y \leq \delta(Q + q^2xz - \frac{1}{2}q^2x^2 - \frac{1}{2}p^2y^2 - P - \frac{1}{2}p^2w^2) \]

Solving for \( x \) and \( y \), and then substituting them into the surplus function yields
Let us now look at the relational employment contract. The constraint is given by

\[ |x| + |y| \leq \delta (Q + q^2 xz - \frac{1}{2} q^2 x^2 - \frac{1}{2} p^2 y^2) \]

Geometry suggests that \( y = 0 \). Assuming that \( x \geq 0 \), the maximization problem can be written

\[
\max_x S(x, 0)
\]

subject to \( x \leq \delta (Q + q^2 xz - \frac{1}{2} q^2 x^2) \)

Solving for \( x \) and then substituting into the surplus function yields:

\[
S_{RE} = \frac{\delta q^2 z}{q^2 \delta^2} - 1 + \sqrt{\delta^2 q^4 z^2 - 2 \delta q^2 z + 1 + 2 \delta^2 q^2 Q}
\]

Will now show that when (7') and (9) hold, we have

\[
S_{RO} \geq S_{RE}
\]

i.e.

\[
f(\delta, P, q, p, z, w, Q) \geq 0
\]

where

\[
f(\delta, P, q, p, z, w, Q) = 2 p^2 q^2 \delta^2 (S_{RO}(P) - S_{RE}) =
\]
\[-q^2\delta p^2 z + q^2\delta p^2 w + 2q^2\delta^2 p^2 P + q^2\delta^2 p^4 w^2 - 2q^2 \
+2\sqrt{(p^2 + q^2)}\sqrt{q^2\left[\delta^2 p^2 (p^2 - w^2 p^2) + 1 + 2\delta^2 (Q - P)p^2 - \delta (w + z) p^2\right]} + p^2 \
-2p^2\sqrt{\delta^2 q^2 z^2 - 2\delta q^2 z + 1 + 2\delta^2 q^2 Q}\]

From (7') we have

\[P \leq Q\left(\frac{w}{z}\right) - \left(\frac{1}{2} p^2 w^2 + \frac{1}{4} z w q^2 - \frac{1}{4} q^2 z^2\right) = P_0\]

Will now show that

\[f(\delta, P, q, p, z, w, Q) \geq f(\delta, p, q, p, z, w, Q) \geq 0\]

for every \(\delta\) for the special case \(p = q = 1\).

For \(p = q = 1\) we have

\[P_0(w, z, Q) = Q\left(\frac{w}{z}\right) - \left(\frac{1}{2} w^2 + \frac{1}{4} z w - \frac{1}{4} z^2\right)\]

and

\[f(\delta, 1, 1, z, w, Q) = \]

\[-\delta z + \delta w + 2\delta^2 P + \delta^2 w^2 - 2 + 2\sqrt{\delta^2 z^2 - \delta^2 w^2 + 2 + 2\delta^2 Q - 2\delta^2 P - \delta w - \delta z} \
-2\sqrt{\delta^2 z^2 - 2\delta z + 1 + 2\delta^2 Q}\]

Note that \(P_0 \geq 0\) requires \(w \leq z\). For \(P = P_0(w, z, Q)\) we get

\[f(\delta, P_0(w, z, Q), 1, 1, z, w, Q) = \]

\[\left[\delta - \delta^2 \left(Q + \frac{1}{2} z\right)\right]w - \delta z + \delta^3 \left(Q + \frac{1}{2} z\right) - 2 \
+2\sqrt{2 + \delta^2 \left(Q + \frac{1}{2} z\right) - \delta z + \left[\left(Q + \frac{1}{2} z\right)\delta^2 - \delta\right]w - 2\sqrt{\delta^2 (z^2 + 2Q) - 2\delta z + 1}\]

\[= A(\delta, z, Q)w + B(\delta, z, Q) - 2 + 2\sqrt{2 + B(\delta, z, Q) - A(\delta, z, Q)}w - 2\sqrt{C(\delta, z, Q) + 1}\]

where
\[ A(\delta, z, Q) = \left[ 1 - \delta \left( Q \frac{1}{z} + \frac{1}{2} z \right) \right] \delta \]
\[ B(\delta, z, Q) = - z A(\delta, z, Q) \]
\[ C(\delta, z, Q) = -2 z A(\delta, z, Q) \]

Hence, it is shown that

\[ f(\delta, P_0(w, z, Q), 1, 1, z, w, Q) = A(\delta, z, Q)(w - z) - 2 + 2 \sqrt{2} \sqrt{2 - A(\delta, z, Q)w + z} - 2 \sqrt{1 - 2 z A(\delta, z, Q)} \]

For second best solutions we have from (9) that

\[ \delta < \frac{\frac{1}{2} z + \frac{1}{4} w}{Q + \frac{1}{4} q} z^2 - P_0(w, z, Q) - \frac{1}{2} p^2 w^2 = 2 \frac{z}{2Q + z} = \delta_0(z, Q) \]

Hence

\[ A(\delta, z, Q) = \left[ 1 - \delta \left( Q \frac{1}{z} + \frac{1}{2} z \right) \right] \delta = \delta_0(z, Q) \]

where \( A_m = \frac{1}{4 \delta_0} = \frac{1}{\frac{1}{2} \left( 2Q + z^2 \right)} \)

Must also have expressions inside roots nonnegative

\[ 2 - A(\delta, z, Q)(w + z) \geq 0 \quad \text{i.e.} \quad w + z \leq \frac{2}{A(\delta, z, Q)} \]

and

\[ 1 - 2 z A(\delta, z, Q) \geq 0 \quad \text{i.e.} \quad A(\delta, z, Q) \leq \frac{1}{2z} \]

Hence, we must have

\[ A_m \leq \frac{1}{2z} \quad \text{i.e.} \quad \frac{1}{2z} \left( 2Q + z^2 \right) \leq 1 \]

Note that

\[ \frac{\partial}{\partial w} \left[ A(w - z) - 2 + 2 \sqrt{2 - A(w + z)} \right] = - A \frac{\sqrt{2 - A(w - Az)}}{2} < 0 \]

Hence, the expression is minimal when \( w \) is maximal, i.e. for
\[ w = \min \left\{ z, \frac{2}{\alpha^2(z, Q)} - z \right\} = z \]

where last equality follows because \( \frac{1}{\alpha} \geq 2z \). This yields

\[ f(\delta, P_0(w, z, Q), 1, 1, z, w, Q) \geq 2(1 + \sqrt{2 \sqrt{1 + a} - \sqrt{a}}) \geq 0 \]

where \( a = 1 - 2zA(\delta, z, Q) \in (0, 1) \), and the last inequality follows because expression is decreasing in \( a \) on \((0, 1)\).

It remains to consider

\[ \frac{\partial}{\partial w} f(\delta, P, 1, 1, z, w, Q) = -2\delta^2 \frac{-\sqrt{\delta^2 z^2 - \delta^2 w^2 + 2 + 2\delta^2 Q - 2\delta^2 P - \delta w - \delta z} + \sqrt{2}}{\sqrt{\delta^2 z^2 - \delta^2 w^2 + 2 + 2\delta^2 Q - 2\delta^2 P - \delta w - \delta z}} \]

Expression inside root is

\[ (\delta^2 z^2 - \delta^2 w^2 + 2 + 2\delta^2 Q - 2\delta^2 P - \delta w - \delta z) = 2 + \delta \left[ \delta (z^2 - w^2) + 2\delta (Q - P) - (w + z) \right] \]

We must have

\[ \delta < \frac{\frac{1}{2} z + \frac{1}{2} w}{Q + \frac{1}{2} q^2 z^2 - P - \frac{1}{2} p^2 w^2} = \frac{\frac{1}{2} z + \frac{1}{2} w}{Q + \frac{1}{2} z^2 - P - \frac{1}{2} w^2} \]

i.e.

\[ \delta (Q + \frac{1}{2} z^2 - P - \frac{1}{2} w^2) < \frac{1}{2} z + \frac{1}{2} w \]

It follows that expression inside root above is \(< 2\), and hence that \( \frac{\partial}{\partial w} f < 0 \).

Thus we have shown

\[ f(\delta, P, 1, 1, z, w, Q) \geq f(\delta, P_0, 1, 1, z, w, Q) \geq 0 \]

for \( P \leq P_0 \).
REFERENCES


