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CLIMATE CHANGE AND PRODUCTIVITY IN THE AQUACULTURE INDUSTRY

by

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1. INTRODUCTION ................................................................. 1
2. NATURAL CONDITIONS FOR PRODUCTION OF SALMON AND TROUT .... 2
   2.1 Ecological conditions for fish farming ........................................... 2
   2.2 Climate change .......................................................................... 7
   2.3 Growth functions ........................................................................ 8
       2.3.1 Von Bertalanffy’s growth function ........................................... 10
       2.3.2 Exponential growth function .................................................. 11
       2.3.3 The logistic growth function .................................................... 12
3. CHANGE IN ECOLOGY AND ECONOMIC EFFECTS .............................. 14
   3.1 Profit maximization in fish farming ............................................... 14
   3.2 Feeding, slaughtering and insurance costs ...................................... 16
   3.3 Optimal slaughtering time given different weight functions ............ 18
       3.3.1 Optimal slaughtering and rotation ........................................... 21
4. SEA TEMPERATURE, GLOBAL WARMING AND ECONOMIC EFFECTS .... 23
   4.1 Temperature dependent growth ................................................... 23
   4.2 Estimation of coefficients in the growth function ......................... 25
       4.2.1 Estimation of Bertalanffy’s growth function ............................ 26
       4.2.2 Estimation of the logistic growth function .............................. 29
       4.2.3 Estimation of the exponential growth function ....................... 32
   4.3 Optimal slaughtering time and temperature ................................... 35
5. ECONOMIC EFFECTS OF A TEMPERATURE CHANGE ............................. 37
   5.1 Comparative study between Lista and Skrova ................................. 37
       5.1.1 Temperature and optimal slaughtering weight .......................... 39
       5.1.2 Changes in quantity and frequency in slaughtering ................ 41
       5.1.3 Change in gross present value due to changes in average temperature .... 43
   5.2 Concluding remarks ................................................................. 46
6. GROWTH PATTERN AND SEASONALITY IN THE TEMPERATURE LEVEL .... 46
   6.1 Descriptive statistics ................................................................. 46
   6.2 Growth and seasonality ............................................................... 50
   6.3 Seasonal temperature oscillations and economic effect of global warming .... 53
   6.4 A comparative analysis between Lista and Skrova .......................... 54
7. SEASONAL TEMPERATURE OSCILLATIONS AND ROTATION ..................... 59
   7.1 Optimal fish farming and infinite time horizon ............................... 59
   7.2 Optimal rotation and the value of the firm .................................... 62
8. TEMPERATURE CHANGES IN THE SEA WATER OFF LISTA AND SKROVA –
   SINGLE COHORT AND ROTATION CASE ........................................ 64
   8.1 Introduction to the scenario analysis ............................................ 64
       8.1.1 Scenario I: Increase and reduction in amplitude ..................... 65
       8.1.2 Scenario II: Change in average temperature ......................... 68
       8.1.3 Scenario III: Simultaneous change in amplitude and average temperature .... 72
   8.2 The economic effect in the rotation case with infinite time horizon .... 76
   8.3 Conclusion ................................................................................. 81
1. INTRODUCTION

It is anticipated that global warming will increase the temperature in the Northeast Atlantic and that the future temperature in the waters off the coast of Norway will be affected (IPCC 2001, Stenevik and Sundby 2004, ACIA 2004 and NERSC 2005). This is likely to affect the salmon aquaculture industry in Norway. Cold-blooded animals are particularly sensitive to temperature in the environment. Wild species avoid areas where the temperature is outside their natural temperature range, but farmed fish cannot do so, as they are confined to their cages. Environmental conditions in each location determine whether the sea water is suitable for aquaculture production or not. In this report we will discuss and analyse climate induced changes in sea temperature and their potential effects on the Norwegian salmon and trout farming industry.1

The report is structured as follows. In the first section we present the problem to be analyzed. In the next section we describe the natural conditions for production of salmon and trout. Different growth functions are presented and it is shown how environmental conditions can be integrated into these functions. In Section Three we analyse the profit maximization behaviour of the fish farmer and derive the optimal slaughtering time of the fish as a function of environmental and economic parameters. Section Four analyses the empirical relationship between sea temperature and growth of salmon, presenting results which confirm the alleged dependency between temperature and growth of farmed salmon. In Section Five we estimate the ecologically dependent parameters for three different growth functions and show explicitly how optimal slaughtering time depends on temperature. This section analyses the productivity effect of change in temperature for farmers located at Skrova, Nordland county.

1 See also SNF-discussion paper no. 59/05 Climate Change and Future Expansion Paths for the Norwegian Salmon and Trout Industry where we analyse more broadly climate change and future expansion paths for the salmon and trout industry in Norway.
In section Six we analyse how the seasonal variations in temperature affect the growth of fish and the value of production. Section Seven analyses the effect increased future temperature could have on farmers located in the southernmost counties of Norway. Section Eight analyses the economic effect with repetitive releases and slaughtering of salmon. Finally, Section Seven concludes.

2. NATURAL CONDITIONS FOR PRODUCTION OF SALMON AND TROUT

2.1 Ecological conditions for fish farming

The typical fish farming company is assumed to maximize the net present value of its profits. To this end, the managers control a number of variables, i.e., feed ratio, type of feed, pattern of feeding, input of labour, number of smolts purchased and the stocking density of the fish, harvest time, etc. On the other hand the firm is exposed to forces and factors that are not under direct control or less easily controlled, i.e., exogenous factors which are both economic and environmental such as fish prices, governmental regulations, feed and other input prices, and site environment (temperature, sea current, waves, salinity, local temperature variation, depth, mortality from disease and algae blooms, number of hours with daylight, etc). The farmers normally have little control over environmental factors once a farm site has been chosen (Bjørndal and Uhler, 1993). This section discusses the natural conditions for the production of salmon and trout and clarifies the interdependence between production and natural conditions.

The quality of the water in a given environment will largely determine which species of fish can survive or can be farmed there. The principal areas of the world in which salmon farming has developed successfully, Norway, Scotland, Ireland, Canada and Chile, all have the kind of environment necessary for this
type of aquaculture. Changes in climate could change the production conditions for salmon in the areas where they are now farmed successfully and open up new areas for salmon aquaculture which currently are not suitable.

The quality of the water at any given site determines its production performance and indeed whether or not production is possible at all (Wallace 1993). What, then, is meant by water quality? Salmonids favour fairly low temperatures; the normal temperature for salmon farms usually lies within the range 5-20 degrees.\(^2\) Physiological investigations have shown that fast, efficient growth in salmon is best achieved in water temperature of 13-17 degrees (Wallace 1993). Outside this range, production becomes less efficient, either due to slower growth or to temperature stress problems. This means that the maximum oxygen content of the water in freshwater culture will be between 12.8 mg/l and 9.2/l, assuming that the water is 100% saturated (1ATP equal one atmospheric pressure), while the corresponding values for seawater are about 30% lower (Wallace 1993). Water used for salmon production should have a pH value between 6 and 8.

Sea temperature affects all metabolic processes in fish. Necessary information required for being able to estimate the production and carrying capacity of a site is the minimum water flow (cubic meters per minute) and maximum water temperature. The density of oxygen decreases with temperature, and the worst combination of these factors is high sea water temperature and low water flow. As to low temperatures, salmon will die when ice crystals begin to form in the body fluids, which occurs at about -0.5 degrees. Climate change is expected mainly to affect the sea temperature. Table 1 summarizes some of the vital ecological conditions for farming of some species which are expected to expand in the coming years.

---

\(^2\) All temperatures in this paper are expressed in centigrade (Celsius).
Table 1: Ecological condition for different species

<table>
<thead>
<tr>
<th>SPECIES</th>
<th>OPTIMAL DEPTH</th>
<th>OPTIMAL CURRENT</th>
<th>SALINITY PER THOUSAND</th>
<th>OPTIMAL TEMPERATURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atlantic salmon and trout</td>
<td>&gt;50m</td>
<td>10-20 cm/sec (5-20)</td>
<td>&gt;30 (&gt;20)</td>
<td>Atlantic salmon: 12-14 °C (&gt;2 °C) Trout: 15-17 °C (&gt;2 °C)</td>
</tr>
<tr>
<td>Cod</td>
<td>&gt;30m</td>
<td>10-20 cm/sec (5-50 cm/sec)</td>
<td>&gt;30 (&gt;5)</td>
<td>12-14 °C (&gt;2 °C)</td>
</tr>
<tr>
<td>Halibut</td>
<td>&gt;15m</td>
<td>25-30</td>
<td>6-14 °C (0-18 °C)</td>
<td></td>
</tr>
<tr>
<td>Mussel</td>
<td>10-30m</td>
<td>25-75 cm/sec</td>
<td>17-32 (&gt;5)</td>
<td>10-20 °C (&gt;0 °C)</td>
</tr>
<tr>
<td>Oyster</td>
<td>1-6m</td>
<td>25-75 cm/sec (≥75 cm/sec)</td>
<td>&gt;24-33 (&gt;16)</td>
<td>16-20 °C (≥3 °C)</td>
</tr>
<tr>
<td>Scallop Drooping-culture</td>
<td>10-20m</td>
<td>&lt;15 cm/sec</td>
<td>≥31</td>
<td>15-18 °C &gt;4 °C</td>
</tr>
<tr>
<td>Bottom-culture</td>
<td>5-40m</td>
<td>10-20 cm/sec</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Turbot</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


The aquaculture production of salmon and trout is industrialized. Even though most of the production is controlled by advanced technologies, production nevertheless depends critically on natural conditions. The sea temperature is one of the essential parameters for the growth of the fish, but for obvious reasons the fish farmer must take the temperature as given. Table 1 shows the optimal temperature range for salmon and trout. Sea temperature influences the metabolism of the fish, but fish can only survive within a certain temperature range which varies from species to species. High temperature reduces the saturation of oxygen in the water, and the fish cannot utilize the food. Changes in temperature therefore affect the growth and mortality rate of the fish. The low density of oxygen is also due to higher concentration of algae. Anadromous fish such as salmon, which are exposed to high temperature for a long period, also show a tendency to organic or phenotypical deformation. For a given time period \( \Delta t > 0 \) and weight \( w(t_0) \) at initial time \( t = t_0 \), it follows from the previous arguments that the growth in body weight as a function of temperature is parabolic. Figure 1 shows the relationship between growth and sea temperature during a given time interval.
Figure 1: Growth as a function of temperature

The figure shows that the relationship between growth and temperature in the interval \( t^C \in (t_{LB}, t_{UB}) \) is non-linear, and that the temperature which maximises growth is \( t = t_{OPT} \). In practice the sea temperature fluctuates with the seasons. As to the Norwegian coast, the average sea temperature decreases as one moves north; it is highest in the southernmost coastal areas and lowest off the coast of Troms and Finnmark counties. Given that the average temperature is within the open interval \( t^C \in (t_{LB}, t_{UB}) \) it will affect growth and therefore also the value of the firm. Differences in temperature and growth are a source of differential rent. Notice that plants located in regions with relatively high sea temperature, to the right of \( t_{OPT} \) in Figure 1, do not necessarily have a higher value than plants located in areas with lower temperature (to the left of \( t_{OPT} \)). Because of the parabolic relationship between growth and temperature, there exist two temperature levels which give identical growth rates.

Assume that the plants located along the coast are exposed to average temperature \( \bar{t} \in (t_{LB}, t_{OPT}) \), and that the plants located in the south are exposed to temperature closer to \( t_{OPT} \) than plants located in the northernmost counties. The implication is that fish in the south grow faster than fish in colder areas in the
north and that the value of the firm is highest in the south. Because of the higher rate of metabolism the feed is used more efficiently in the south, the fish absorb the food more efficiently, and the firm can utilize labour and capital more efficiently. The relationship between growth and fish weight, including the effect of water temperature, has been examined by Iwane and Tautz (1981), Brett, Shelbourn and Shoop (1969) and Elliott (1982). Growth and feed ratios has been studied by Austreng, Storebakken and Asgard (1987), and Storebakken and Austreng (1987).

The natural, environmental conditions on each site determine the carrying capacity, i.e., how much biomass that can be stored inside the cages. The mortality rate and frequency of diseases are likely to increase when the total biomass and density of fish increase. Higher density of fish also affects negatively the density of oxygen in the water in the cages and is likely, therefore, to have a negative effect of the growth rate (Fagerlund et al. 1981). Environmental conditions thus determine the productivity and the capacity of the chosen site.

The growth rate of the fish determines the optimal slaughtering time of the fish. As long as the net value of the relative growth rate of the biomass is higher than the rate of return on financial capital plus the opportunity cost of the site, the farming company will keep the fish in the cage. The relative growth rate of the fish depends on the weight of the fish, the feed factor, the density of fish in the cages, temperature, density of oxygen, salinity in the water, and more. What may make this condition difficult to apply is that temperature varies seasonally, and so the growth rate will also vary with the seasons.
2.2 Climate change

Climate change implies that the average temperature changes and possibly that the variance changes as well. It could be a permanent or a temporal change, i.e., a change limited to a given time period and then returning to the initial level. The climate change could be characterized as cyclical if it is recurring.

The effects of a climate change on fish growth can be illustrated by using Figure 1. A climate change will change the actual distribution of the sea temperature. Suppose that the distribution of the temperature during the year shifts to the right. The average temperature will increase. Figure 1 shows how the lower and upper bounds of the actual temperature $t_{\text{L}}$ and $t_{\text{U}}$ shift to the right. The figure also shows how the change in climate affects the growth rate. The change of the lower bound of the actual distribution of the temperature increases the growth rate from $G_1$ to $G_2$ while the change in the upper bound reduces the growth rate. The said change will raise the average temperature, but the amplitude could also increase (not shown in the figure). The latter could have a devastating impact on the possibility to farm fish.

The relationship between temperature and fish growth is assumed to be given by nature. In this part of the analysis we do not discuss the possibility to develop by genetic selection a salmon mutant which could survive at a higher temperature and less oxygen than today’s salmon. A permanent increase in the sea temperature could provide incentives to spend economic resources on a genetic program. The effect on the aquaculture industry depends on how strong the climate change is supposed to be. A priori one would expect that the stronger the change in climate and temperature, the more severe will be the impact on existing farms.

As the sea temperature is highest in the southernmost counties, a climate change that raises the temperature could mean that this area, wholly or partly, will no
longer be suited for production of salmon and trout. During the last five years the farmers of salmon located in Rogaland and Hordaland have occasionally experienced too high sea temperature in the summer months, i.e., to the right of $t_{\text{OPT}}$ in Figure 1. In the southernmost part the sea temperature fluctuates to the left and right of $t_{\text{OPT}}$ during the year. Fisheries biologists and commercial firms in the aquaculture business are testing a new technology for compensating artificially for lack of oxygen in the sea inside cages with salmon and trout. It was mentioned above that the implication of high temperature is lack of oxygen, partly as a consequence of increased density of oxygen consuming algae. The average temperature from Stadt in Møre og Romsdal county and to Lofoten in the northern part of Nordland county is today stable and close to optimum, but still on the left side of $t_{\text{OPT}}$. Sea areas north of Lofoten have sea temperature to the left of $t_{\text{OPT}}$, but on average closer to $t_{\text{LB}}$ than other areas. As a result of climate change, the colder areas will become more suitable for production of salmon and trout, as the growth rate will increase with higher temperature.

As already mentioned, an increase in the maximum temperature could be devastating for the fish. If the temperature occasionally reaches $t_{\text{UB}}$, the fish will die after a short time. If the temperature exceeds 20 degrees this is critical for salmon and trout. If the temperature occasionally reaches this level for a few hours, it could mean that the firm would lose its entire biomass.

2.3 Growth functions

The growth rate $\frac{dw}{dt}$ of the average fish can be expressed as a function of temperature $t_c^C$ at time $t$, volume of feed $F(t)$ at time $t$, density of fish in the cages $N(t)$ at time $t$, and a function of a set of ecological variables $E(t)$ including temperature at time $t$, i.e.
\[ \frac{dw}{dt} = g(F(t), N(t), E(t)) \]

Increase in feed per unit of time (and given that the other variables are constant) will increase growth, but only up to a limit, i.e., \( \frac{\partial g}{\partial F} \geq 0 \) and \( \frac{\partial^2 g}{\partial F^2} < 0 \). If the density of fish increases, we expect that it will affect the growth rate negatively, i.e., \( \frac{\partial g}{\partial N} \leq 0 \). As argued in the preceding section, temperature has a bell shaped effect on the growth rate. We also expect that other ecological parameters (salinity, density of oxygen, currents, pH-value) have a bell shaped effect on the growth rate of the fish.

As long as the functions \( F(t) \), \( N(t) \) and \( E(t) \) do not change, the growth function depends only on time \( t \):

\[ \frac{dw}{dt} = g(t) \]

The weight of the fish after a time period \( \Delta = t_2 - t_1 \) is \( w(t_2) = w(t_1) + \int_{t_1}^{t_2} g(t) \, dt \).

There exist different specifications of growth functions. In the following we will introduce three growth functions which we will estimate and apply in the analysis of the economic impacts of a temperature increase. The benefit of using these functions is that they have convergence property, i.e., the fish grows toward a genetically given maximum weight.
2.3.1 Von Bertalanffy’s growth function

Von Bertalanffy’s growth function can be expressed in the following way:

\[ w(t) = w_\infty (1 - \beta e^{-\alpha t})^3 \]

where \( w_\infty \) is the maximum weight the average fish reaches asymptotically. The constant \( \beta = 1 - \sqrt[3]{\frac{w(0)}{w_\infty}} \) and \( 0 < \beta < 1 \). The constant \( \beta \) does not reflect any ecological qualities, for example temperature, current, salinity etc. On the other hand the constant \( \alpha > 0 \) could indirectly reflect ecological or natural growth conditions at the site. The higher the value of \( \alpha \), the greater is the weight increase for any given time period. By differentiating the von Bertalanffy growth function with respect to time and dividing by the growth function, we get the following expression for the relative growth rate:

\[ \frac{\dot{w}}{w} = \frac{3\beta \omega e^{-\alpha t}}{(1 - \beta e^{-\alpha t})} \]

By differentiating this with respect to \( \alpha \), we can analyse how the relative growth rate changes by a marginal increase in \( \alpha \) for given value on \( t \).

\[ \frac{\partial}{\partial \alpha} \left[ \frac{\dot{w}}{w} \right] = \frac{3\beta \omega e^{-\alpha t}}{(1 - \beta e^{-\alpha t})} \left[ 1 - \frac{\alpha}{(1 - \beta e^{-\alpha t})} \right] \]
This expression shows that the sign of the change in the relative growth rate depends on time and the value of the parameters, i.e., the bracket to the right of the equality sign. The bracket is positive for $0 \leq t < t^c$ and negative for $t > t^c$. The growth rate will increase for an increase in $\alpha$ if $0 \leq t < t^c$, and an increase in $\alpha$ will have a negative effect on the growth rate if $t > t^c$. That is, young fish will grow more quickly.

An alternative (Bjørndal et al., 1987) is the exponential growth function, to which we now turn.

### 2.3.2 Exponential growth function

$$w(t) = e^{\alpha - \beta/t}$$

Here the weight of the average fish grows asymptotically towards $\lim_{t\to\infty} w(t) = e^\alpha$, $\lim_{t\to0} w(t) = 0$. Note that $t$ must be different from zero. The parameter $\alpha$ (or the asymptotic size of the fish) can be regarded as genetically given, and it is not affected by ecological variables. In practice we do not observe that farmers keep the fish as long as needed to reach its asymptotic size, because it is not economically optimal. A logarithmic transformation of the growth function and a proper transformation of $\beta/t$ gives a function which can be estimated by linear regression. The ecological properties are reflected in the $\beta$-coefficient, with a decreasing numerical value of $\beta$ reflecting better ecological conditions. The relative growth rate is:

$$\frac{\dot{w}}{w} = \frac{\beta}{t^2}$$
For a given value of $\beta$, the relative growth rate is a decreasing function of time. The function shifts up as $\beta$ increases, but the shift is greatest for time close to the starting point, i.e.

$$\frac{\partial}{\partial \beta} \left[ \frac{\dot{w}}{w} \right] = \frac{1}{t^2} > 0$$

2.3.3 The logistic growth function

The logistic growth function can be expressed in the following way:

$$w(t) = \frac{1}{\alpha + \beta \gamma'}$$

where $1 > \gamma > 0$. The function is nonlinear in the parameters $\alpha, \beta$ and $\gamma$ and can, as with von Bertalanffy’s function, be estimated with a nonlinear estimator. The logistic function is an S-shaped curve. By differentiating the function, the relative growth rate can be expressed in the following way

$$\frac{\dot{w}}{w} = -\frac{\beta \gamma' \ln \gamma}{\alpha + \beta \gamma'}$$

The relative growth rate depends on all the parameters and on time $t$ and can be transformed to a homogeneous, nonlinear differential equation with varying coefficients. The fact that the parameters $\alpha$ and $\beta$ determine the starting value, given $t = 0$, it is above all $\gamma$ which reflects the environmental conditions (see the
following sections which confirm this conclusion). The environmental effect on the relative growth rate can be shown by differentiating the function with respect to gamma, i.e.

\[
\frac{\partial}{\partial \gamma} \left[ \frac{\dot{w}}{w} \right] = -\frac{\beta (t \gamma^{-1} \ln \gamma + \gamma^{-1})}{\alpha + \beta \gamma'} + \frac{\beta^2 \gamma^{2r-1} \ln \gamma}{\left(\alpha + \beta \gamma'\right)^2}
\]

The partial derivative of the growth rate with respect to \( \gamma \) is a parabolic function. It implies that the function has maximum for a particular \( \gamma' \), given time \( t \) and the other parameters, i.e. \( \frac{\partial}{\partial \gamma} \left[ \frac{\dot{w}}{w} \right] > 0 \) for \( \gamma < \gamma' \) and \( \frac{\partial}{\partial \gamma} \left[ \frac{\dot{w}}{w} \right] < 0 \) for \( \gamma > \gamma' \).

Yet another alternative is to approximate the weight by a polynomial function of time, for example:

\[
w(t) = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \alpha_3 t^3
\]

This function is an approximation and it is only valid for \( t \leq t^* \). For \( t = t^* \) the fish has reached its maximum weight. The parameters \( \alpha_i \) for \( i = 1, 2 \) and 3 reflect indirectly ecological qualities for a particular area, with different ecological qualities giving different parameters in the function and different shape of the growth curve.
3. CHANGE IN ECOLOGY AND ECONOMIC EFFECTS

3.1 Profit maximization in fish farming

Assume that the price $p$ of fish is constant over time and independent of the weight of the fish, and that ecological parameters $E(t)$, temperature $(t_t)$, number of fish in the cages $(N(t))$, and the volume of feed $(F(t))$ are exogenously given. The objective of the firm is to select the slaughtering time $t = t'$ which maximizes the present value of the aquaculture firm. The number of fish in the cages at the starting point $t = t_0$ is given by $N(0)$. The natural mortality rate $M$ is constant over time. In the following we do not take into consideration that the mortality rate could be affected by different temperature levels or by density of the fish $N(t)$. The average weight of the fish at time $t$ is given by the growth function $w(t)$, with property $\dot{w} \geq 0$. The value of the biomass at time $t$ is (Bjørndal et al 1989);

$$V(t) = pN(t)w(t)$$

and the number of fish in the cages at time $t$ is

$$N(t) = N(0)e^{-Mt}$$

The continuous discounting factor is $e^{-rt}$, where $r$ is the real interest rate.

When the investment cost and other costs of producing the fish are sunk, the objective for the firm is to choose an optimal slaughtering time $t = t'$ of a year
class of fish, given that \( t^* \in [0 \leq t \leq T] \), which maximizes the present value or profit of the firm, i.e.

\[
\text{Max. } \pi(t) = V(t)e^{-rt} \quad \text{wrt. } t^* \in [0 \leq t \leq T]
\]

The first order condition for a maximum is:

\[
\frac{d\pi(t^*)}{dt} = \frac{\partial V(t^*)}{\partial t} e^{-rt} - rV(t^*)e^{-rt} = 0
\]

\[
= e^{-rt} \left[ \dot{w}N(0)e^{-\mu t}p - MN(0)e^{-\mu t}w(t^*)p \right] - rpN(0)e^{-\mu t}w(t^*)e^{-rt} = 0
\]

The first order condition implies that the value of the firm is maximized at a point in time where \( t^* \in [0 \leq t \leq T] \) following condition is realized:

\[
\frac{\dot{w}(t^*)}{w(t^*)} = r + M
\]

The rule of optimal slaughtering time \( t = t^* \) says that the present value of the firm is maximized if the value of the fish is realized (the fish is slaughtered and sold) at time \( t = t^* \) when the relative increase in the value of the biomass (in the sea) is equal to the opportunity cost. The opportunity cost has two elements, the value of time by keeping the bio-capital in the cages, expressed through the interest rate, and the mortality of the fish in the cages. If the relative growth rate \( \frac{\dot{w}(t)}{w(t)} \) is a
decreasing function of time, the second order condition will be satisfied and there exists a solution to the problem.

It bears mentioning that running an aquaculture plant optimally is also dependent on costs of equipment, labour costs, cost of energy, feeding, insurance, and slaughtering costs. In the following we will further discuss the last mentioned costs items.

3.2 Feeding, slaughtering and insurance costs

Feeding, slaughtering and insurance costs can be regarded as variable costs which are related to the volume and value of the biomass. Ecological or environmental conditions can be expected to affect feeding, slaughtering and insurance costs.

There exists a relationship between the feeding pattern and the growth of the fish. The feeding factor (Bjørndal et al., 1987) per fish can be defined as the quotient \( f^* \) between the volume of feed \( F(t) \) and growth of the fish \( \dot{w} \) per unit of time, i.e. \( f^* = \frac{F(t)}{\dot{w}} \). According to Bjørndal (op. cit.) the feed factor is constant and thus independent of environmental factors. On the other hand total feed consumption will be affected by ecological or environmental factors, i.e., total feed expenditure. \( SF_r \) (volume) is the sum of the product between, respectively, feed consumption per fish at time \( t \) and number of fish in the cages at time \( t \): \( SF_r = \int_0^\infty f^* \dot{w} N(0) e^{-Mt} dt \), and the discounted value of the feed expenditure is \( SF e^{-r} \).

The feeding cost increases the opportunity cost of keeping the fish in the cages. Taking account of feeding costs leads to earlier optimal slaughtering compared to no feeding costs. If the environment affects the mortality rate \( M \) and the growth rate, the feed expenditure can be affected by changes in ecology.
It can be shown that including slaughtering costs in the maximization of profit results in postponing of the optimal time of slaughtering compared to no slaughtering costs. Formally this relationship can be expressed in the following way: Suppose that the slaughtering costs per fish is $c_s$. If all the fish is slaughtered at point $t$, the costs is $c_s N(t)$. The costs of slaughtering are related to number of fish, and the number of fish in the cages at time $t$ depends on the mortality rate. If the ecological factors affect the mortality rate, then the slaughtering costs will be affected.

Environmental or ecological factors can be expected to affect the insurance costs if the probability of a breakdown is related to environmental and ecological factors. Suppose that the sea temperature increases over time, and that the temperature occasionally reaches critical values and the mortality rate increases dramatically. Under this scenario the insurance premium can be expected to increase. Suppose that the insurance costs at time $t$ are a constant fraction ($k$) of the value of the biomass at time $t$. The insurance premium at time period $t$ is $kV(t)$. The fish is insured through the life cycle, which implies that the total insurance costs are $P = \int_0^t kV(t)dt$. The insurance premium will most likely increase with the probability of a breakdown, i.e., the lower the quality of the environment, the higher the insurance premium. Higher insurance costs have the same effect on the optimal slaughtering time as a higher discount rate or a higher mortality rate, i.e., it leads to earlier slaughtering.

The optimal slaughtering time of the fish can be affected by whether the price per kilogram of the fish depends on the size of the fish. If the price is an increasing function of size, slaughtering is delayed, and the opposite if the price is a decreasing function of size. In practice there probably is a bell shaped relationship between price and size.
Above it was shown that environmental factors (salinity, temperature, current, pH-value etc.) influence growth and mortality and therefore indirectly the density of fish in the cages. Through these factors the environment also affects the costs of feeding, and feeding pattern, slaughtering, and insurance costs.

3.3 Optimal slaughtering time given different weight functions

In the preceding section we presented three specific growth functions. If we apply the von Bertalanffy growth function, we have the following optimum condition (from

\[
\frac{d\pi(t^*)}{dt} = 0
\]

\[
\frac{3\beta\alpha e^{-\alpha^*}}{(1 - \beta e^{-\alpha^*})} = r + M
\]

Solving for \( t^* \), we get:

\[
t^* = -\frac{1}{\alpha} \ln \left[ \frac{r + M}{\beta(3\alpha + r + M)} \right]
\]

We argued that \( \alpha \) reflects how growth depends on the ecological properties at each site, for example temperature. The expression for optimal slaughtering time can be plotted as a function of \( \alpha \). We can analyse how the optimal slaughtering time varies with \( \alpha \), but notice that \( \alpha \) must be different from zero.
Figure 2: Optimal slaughtering time as a function of $\alpha$

Figure 2 shows that the firms slaughter the fish earlier the higher the growth rate of the fish is. In other words, fish farms located at sites with good natural production conditions have an incentive to slaughter the fish at an earlier point in time compared to fish farmers located in geographical areas with poorer conditions. We can draw the same conclusion if we apply the growth function $w(t) = e^{(a-b)/t}$. Using the first order condition above we get the following optimal slaughtering time:

$$t^* = \sqrt{\frac{\beta}{r + M}}$$

As mentioned above the relative rate of growth increases as $\beta$ decreases. Low values of $\beta$ indicate more productive conditions for fish farming compared to slow growing areas associated with high values of $\beta$. The optimal slaughtering time $t^*$ is plotted in Figure 3 as a function of $\beta$ for a given value of $r + M$. 
Figure 3: Optimal slaughtering time as a function of $\beta$

The conclusion is the same as for von Bertalanffy’s weight function; better environmental conditions (lower $\beta$) lead to an earlier slaughtering of the fish.

If we differentiate the logistic growth function with respect to time and apply the optimality criterion, we get the following expression for the optimal slaughtering time

$$
\tau^* = \left[ \frac{1}{\ln \gamma} \right] \ln \left[ \frac{-\alpha(r + M)}{\beta(r + M + \ln \gamma)} \right]
$$

The optimal slaughtering time $\tau^*$ is plotted in Figure 4 as a function of $\gamma$ for a given value of $r + M$. 
3.3.1 Optimal slaughtering and rotation

The preceding analysis assumes production of only one cohort of fish. If we take into consideration that the farmer will as soon as he has slaughtered the fish put out a new cohort in the cages, we get an optimal rotation problem similar to what obtains in forestry economics. The maximization problem for an infinite number of rotations, i.e. $n \to \infty$ is

$$Max. \pi(t) = \sum_{n=1}^{\infty} V(t)e^{-rt}$$

If we apply the rotation principle derived by Faustmann to the optimal slaughtering problem, we get the following modified slaughtering rule (see for example Bjørndal et al., 1987, for a derivation):

$$\frac{\dot{w}(t^*)}{w(t^*)} = r + M + \frac{r}{\left(e^{\alpha^*} - 1\right)}$$
where \( t^* \) is the optimal rotation time. The last term is new compared with the original expression. The term is independent of the coefficient in the weight or growth function. That it is positive means that the opportunity cost of keeping the fish in the sea is increased. This additional opportunity cost arises because the fish in the cages can by substituted by a younger, faster growing cohort. The practical implication is a lowering of the optimal time of slaughtering. This point is shown in Figure 5, where the logistic growth function \( w(t) = \frac{1}{\alpha + \beta \gamma} \) has been applied. If, for example, \( r + M = 0.09, \ r = 0.05 \text{ and } \beta = 5.4, \ \gamma = 0.03, \ \alpha = 0.115 \) the optimal time \( t^* \) is reduced from \( t^* = 2.13 \) without rotation to \( t^* = 1.47 \) years with rotation and infinite time horizon.

![Figure 5: Optimal slaughtering time with and without rotation](image)

As a general conclusion we should be aware that the effect from rotation implies that the slaughtering time is earlier than without this effect.
4. SEA TEMPERATURE, GLOBAL WARMING AND ECONOMIC EFFECTS

4.1 Temperature dependent growth

Temperature plays an important role for the metabolism of the fish. In this section we will analyze more closely the relationship between temperature and the growth of the fish. Based on raw data from controlled experiments organized by producers of feed for the aquaculture industry we have estimated the growth and time paths for weight increase for salmon (see appendix A where the raw data are presented).

![Graph showing the relationship between temperature and growth time](image)

Figure 6: Temperature regimes and growth time to reach different weight classes

Figure 6 shows how many years it takes to reach different weight targets in different temperature regimes. The labels T1, T2, …, T16 refer to temperature measured in degrees centigrade. The figure clearly indicates that temperature plays an important role for the growth of the fish. The uppermost curve, T1, shows the trajectory for a constant seawater temperature equal to 1 degree. Under this condition it takes over 5 years for a fish to reach 5 kilogram or more. On the other hand, the flat curves at the bottom show the fastest growth
trajectories. The most advantageous environment is sea temperature in the interval from 7 to 18 degrees. The 1-5 degrees interval is the environment which has the worst growth conditions.

Figure 7: Temperature dependent growth for salmon reaching 3500 grams

Figure 7 shows how many years it takes to produce a 3.5 kilogram salmon, given different temperature regimes. The curve is based on the data reported in Appendix A. According to the figure it takes 4.5 years to produce the fish if the sea temperature is just 1 degree. In practice farmers do not produce salmon under such extreme conditions; this result is, as said before, based on laboratory experiments. If the temperature is 14 degrees, it takes only about half a year to produce a 4.5 kilogram salmon. The 14 degrees producer can deliver the fish after about half a year, but the 7 degrees producer can sell fish of the same size only after about one and a half year. Hence, during a period of one and a half years, the 14 degrees producer has supplied three times more than the 7 degrees producer. The greatest productivity gain will be realized if seawater temperature increases in areas with temperatures below 6 degrees. The highest productivity is realized in seawater environment with 16 degrees. Note that higher temperature than 16 degrees results in a physiological dysfunction which reduces the productivity. The convexity (in the right part of the curve) and the
existence of maximum productivity temperature level, imply that an increase in sea temperature induced by global warming will be counterproductive if the actual temperature exceeds the optimal level. It is an empirical question whether some geographical areas along the coast are exposed to this problem. If the sea temperature today is close to this critical level in the summer months (see Figure 1), further increase will periodically reduce productivity or even make it impossible to farm salmon.

Although most of the conclusions drawn so far are based on laboratory experiments, we must not uncritically apply it to reality. The actual temperature fluctuates, and if we are discussing growth conditions between northern and southern part of Norway, we must also take into consideration the exposure to daylight (see Lorentzen and Hannesson 2005).

4.2 Estimation of coefficients in the growth function

In the following we present the results from the estimation of each growth function, that is, the von Bertalanffy’s growth function, the logistic function, and the exponential function. We apply nonlinear regression and use a Broyden-Fletcher-Goldfarb-Shanno algorithm (Belsley 1980) for estimation. ESS in the table stands for error sum of squares.
4.2.1 Estimation of Bertalanffy’s growth function

Table 2 shows the results:

<table>
<thead>
<tr>
<th>TEMP. REGIME</th>
<th>THE VON BERTALANFFY’S GROWTH FUNCTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>CENTIGRADE</td>
<td>$w(t) = w_m(1 - \beta e^{-\alpha t})$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$w_m$</th>
<th>$\beta$</th>
<th>$\alpha$</th>
<th>$\text{ESS} = 0.0057$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>273.9</td>
<td>0.94112</td>
<td>0.04476</td>
<td>1.47</td>
</tr>
<tr>
<td>2</td>
<td>268.1</td>
<td>0.93982</td>
<td>0.051613</td>
<td>1.97</td>
</tr>
<tr>
<td>3</td>
<td>274.8</td>
<td>0.94150</td>
<td>0.058105</td>
<td>1.81</td>
</tr>
<tr>
<td>4</td>
<td>224.3</td>
<td>0.93766</td>
<td>0.078647</td>
<td>2.53</td>
</tr>
<tr>
<td>5</td>
<td>253.6</td>
<td>0.93949</td>
<td>0.10678</td>
<td>2.26</td>
</tr>
<tr>
<td>6</td>
<td>231.3</td>
<td>0.93843</td>
<td>0.15864</td>
<td>3.23</td>
</tr>
<tr>
<td>7</td>
<td>274.2</td>
<td>0.94065</td>
<td>0.17673</td>
<td>3.29</td>
</tr>
<tr>
<td>8</td>
<td>292.0</td>
<td>0.94208</td>
<td>0.19179</td>
<td>2.84</td>
</tr>
<tr>
<td>9</td>
<td>230.1</td>
<td>0.93804</td>
<td>0.23231</td>
<td>4.23</td>
</tr>
<tr>
<td>10</td>
<td>232.3</td>
<td>0.93779</td>
<td>0.24590</td>
<td>4.56</td>
</tr>
<tr>
<td>11</td>
<td>249.9</td>
<td>0.93902</td>
<td>0.25932</td>
<td>3.18</td>
</tr>
<tr>
<td>12</td>
<td>234.4</td>
<td>0.93827</td>
<td>0.29591</td>
<td>4.21</td>
</tr>
<tr>
<td>13</td>
<td>242.2</td>
<td>0.93906</td>
<td>0.31404</td>
<td>4.55</td>
</tr>
<tr>
<td>14</td>
<td>250.3</td>
<td>0.93922</td>
<td>0.33585</td>
<td>4.63</td>
</tr>
<tr>
<td>15</td>
<td>269.6</td>
<td>0.94024</td>
<td>0.35356</td>
<td>5.39</td>
</tr>
<tr>
<td>16</td>
<td>251.4</td>
<td>0.93968</td>
<td>0.38306</td>
<td>5.29</td>
</tr>
<tr>
<td>17</td>
<td>269.6</td>
<td>0.94024</td>
<td>0.35356</td>
<td>5.39</td>
</tr>
<tr>
<td>18</td>
<td>248.9</td>
<td>0.93917</td>
<td>0.32977</td>
<td>4.03</td>
</tr>
</tbody>
</table>

The result from the regressions shows that the model explains almost all of the variation in the dependent variable. From the table we can see that the parameters $w_m$ and $\beta$ are independent of temperature, which is consistent with what we already concluded in the theoretical part of the analysis. Statistical $t$-values are shown below the coefficient estimates. The average values of $w_m$ and $\beta$ are $\overline{w_m} = 254$ and $\overline{\beta} = 0.94$. Under the presentation of the von Bertalanffy
growth function we argued that the parameter \( \beta = 1 - \sqrt[3]{\frac{w(0)}{w_\infty}} \). If we substitute the average values of the parameters, we get an estimate of initial weight of the fish, i.e. \( 0.94 = 1 - \sqrt[3]{\frac{w(0)}{254}} \), which gives \( w(0) = 0.055 \) kilogram and is consistent with the raw data. The parameter \( w_\infty \) is the weight the fish reaches asymptotically when time is infinite. It is difficult to compare the estimated value with reality, because no salmon lives for ever. Nevertheless, this value seems way too high, as it implies a monstrous salmon of about 250 kg. Therefore, even if the regression looks nice enough, there is reason not to believe too much in this estimation. As will be shown below, this estimation implies a far too long growth period until the salmon are slaughtered. The reason why the regression is not to be believed despite good diagnostics is that the period covered by the growth data is very short (less than a year), and even if the von Bertalanffy function describes the growth of the fish very well over that interval, it is not necessarily valid when it is extrapolated over several years, Figure 8 shows the observed and estimated growth function for the fish for a temperature of 14 degrees centigrade.
Table 2 shows that the coefficient $\alpha$ is an increasing function of temperature. The coefficient reaches a maximum at 16 degrees, and then it decreases with higher temperature. We have estimated the relationship $\alpha = f(\text{temperature})$, and the estimation is based on data given in table 2. We estimated the following model:

$$\hat{\alpha} = 0.024107x - 0.056259D1 - 0.10416D2,$$

where $x$ is temperature ($x = 1, 2, 3, \ldots, 16$) and $D1$ is dummy variable for 17 degrees and $D2$ for 18 degrees. Statistical $t$-values are given in brackets. $R^2 = 0.98$ and DW=1.26 which indicates positive autocorrelation. There is also heteroscedasticity. Figure 9 shows the estimated point values of $\alpha$ conditioned on temperature.
We can conclude that the growth and selected parameters depend on temperature. By including the effects from temperature, the von Bertalanffy growth function can be expressed as follows:

\[ w(t : x, D) = 254[1 - 0.94e^{-(0.024107x-0.056259D1-0.10416D2)t}^3 \]

4.2.2 Estimation of the logistic growth function

Next we estimate the logistic growth function. The results are presented in table 3.
Table 3: Coefficient estimates of the logistic growth function

<table>
<thead>
<tr>
<th>TEMP. REGIME</th>
<th>THE LOGISTIC GROWTH FUNCTION $w(t) = \frac{1}{\alpha + \beta e^t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\alpha = 0.11472$  $\beta = 5.4111$  $\gamma = 0.46937$  ESS = 0.032</td>
</tr>
<tr>
<td>2</td>
<td>$\alpha = 0.11492$  $\beta = 5.3079$  $\gamma = 0.42141$  ESS = 0.044</td>
</tr>
<tr>
<td>3</td>
<td>$\alpha = 0.11336$  $\beta = 5.3734$  $\gamma = 0.37653$  ESS = 0.038</td>
</tr>
<tr>
<td>4</td>
<td>$\alpha = 0.11460$  $\beta = 5.3309$  $\gamma = 0.29592$  ESS = 0.038</td>
</tr>
<tr>
<td>5</td>
<td>$\alpha = 0.11426$  $\beta = 5.3203$  $\gamma = 0.17543$  ESS = 0.046</td>
</tr>
<tr>
<td>6</td>
<td>$\alpha = 0.11407$  $\beta = 5.3422$  $\gamma = 0.083317$  ESS = 0.038</td>
</tr>
<tr>
<td>7</td>
<td>$\alpha = 0.11367$  $\beta = 5.3122$  $\gamma = 0.051148$  ESS = 0.038</td>
</tr>
<tr>
<td>8</td>
<td>$\alpha = 0.11321$  $\beta = 5.3329$  $\gamma = 0.036739$  ESS = 0.039</td>
</tr>
<tr>
<td>9</td>
<td>$\alpha = 0.11390$  $\beta = 5.3240$  $\gamma = 0.026484$  ESS = 0.039</td>
</tr>
<tr>
<td>10</td>
<td>$\alpha = 0.11441$  $\beta = 5.3004$  $\gamma = 0.020928$  ESS = 0.039</td>
</tr>
<tr>
<td>11</td>
<td>$\alpha = 0.11248$  $\beta = 5.2061$  $\gamma = 0.015554$  ESS = 0.033</td>
</tr>
<tr>
<td>12</td>
<td>$\alpha = 0.11269$  $\beta = 5.2261$  $\gamma = 0.0097780$  ESS = 0.033</td>
</tr>
<tr>
<td>13</td>
<td>$\alpha = 0.11258$  $\beta = 5.2504$  $\gamma = 0.0068871$  ESS = 0.032</td>
</tr>
<tr>
<td>14</td>
<td>$\alpha = 0.11244$  $\beta = 5.2239$  $\gamma = 0.0045267$  ESS = 0.034</td>
</tr>
<tr>
<td>15</td>
<td>$\alpha = 0.11234$  $\beta = 5.2162$  $\gamma = 0.0028437$  ESS = 0.033</td>
</tr>
<tr>
<td>16</td>
<td>$\alpha = 0.11208$  $\beta = 5.2314$  $\gamma = 0.0021252$  ESS = 0.033</td>
</tr>
<tr>
<td>17</td>
<td>$\alpha = 0.11234$  $\beta = 5.2162$  $\gamma = 0.0028437$  ESS = 0.033</td>
</tr>
<tr>
<td>18</td>
<td>$\alpha = 0.11240$  $\beta = 5.2165$  $\gamma = 0.0050712$  ESS = 0.033</td>
</tr>
</tbody>
</table>

The estimation shows that the coefficients $\alpha$ and $\beta$ are independent of temperature. The average value of $\alpha$ and $\beta$ are, respectively; $\bar{\alpha} = 0.11335944$ and $\bar{\beta} = 5.2856722$. From the said estimated coefficients we can estimate the starting weight of the salmon, i.e., given that $t = 0$, we get $w(0) = 185$ grams ($0.185$ kilograms). The fish will asymptotically reach the maximum weight $1/\alpha = 8.82$ kilogram when $t \to \infty$. The estimated coefficients are significantly different from zero. The statistical $t$-values are in small fonts under the estimated value of the coefficient. Figure 10 shows the estimated logistic function and observed growth of salmon given 14 degrees centigrade sea temperature.
Figure 10: Estimated logistic growth and observed growth path for salmon given 14 degrees centigrade sea temperature

The nonlinear estimation gives almost a perfect fit to the observed values. Table 3 shows that the value of the coefficient $\gamma$ in the logistic function depends on temperature. We have estimated the relationship, and we found the best fit by using the following function:

$$\ln \hat{y} = -0.38804x + 0.73399D1 + 1.7005D2$$

$$(t\text{-statistics is given in brackets). R}^2 = 0.99 \text{ and DW}=0.79, \text{which indicates positive autocorrelation. Positive autocorrelation inflates R-square and the t-values Gleijser and Koenker tests indicate heteroscedasticity. Goldfeld-Quandt test indicates that the error variance is not constant between the subset of the three}$$
first observations and the rest of the sample. Figure 11 shows how the observed and estimated gamma in the logistic growth varies with temperature.

The estimated function deviates from the observed values for low temperature levels. The model is still applicable in a scenario analysis because farming of salmon does not take place in regions where the temperature is that low. The analysis shows that the value of $\gamma$ depends on temperature. Varying $\gamma$ will result in different growth trajectories. A logistic weight function which integrates the effect from temperature can be expressed in the following way:

$$w(t; x, D) = \frac{1}{0.11335944 + 5.2856722e^{[-0.38804x + 0.73399D1 + 1.7005D2]t}}$$

4.2.3 Estimation of the exponential growth function

Finally, we estimated the coefficients in the growth function $w(t) = e^{x-\beta t}$. The result is presented in Table 4. This specification gave the largest sum of squared
errors, and the clearest element of autocorrelation. The coefficient estimates are significantly different from zero, but positive autocorrelation inflates the $t$-statistics so we do not present them in the table.

Table 4: Estimated coefficients for the exponential growth function

<table>
<thead>
<tr>
<th>TEMP. REGIME</th>
<th>THE EXPONENTIAL GROWTH FUNCTION $w(t) = e^{\alpha - \beta t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\alpha = 2.9501$ $\beta = -7.5974$</td>
</tr>
<tr>
<td>2</td>
<td>$\alpha = 2.9355$ $\beta = -6.5546$</td>
</tr>
<tr>
<td>3</td>
<td>$\alpha = 2.9543$ $\beta = -5.8700$</td>
</tr>
<tr>
<td>4</td>
<td>$\alpha = 2.9428$ $\beta = -4.6760$</td>
</tr>
<tr>
<td>5</td>
<td>$\alpha = 2.9425$ $\beta = -3.2666$</td>
</tr>
<tr>
<td>6</td>
<td>$\alpha = 2.9467$ $\beta = -2.2956$</td>
</tr>
<tr>
<td>7</td>
<td>$\alpha = 2.9447$ $\beta = -1.9119$</td>
</tr>
<tr>
<td>8</td>
<td>$\alpha = 2.9499$ $\beta = -1.7264$</td>
</tr>
<tr>
<td>9</td>
<td>$\alpha = 2.9447$ $\beta = -1.5670$</td>
</tr>
<tr>
<td>10</td>
<td>$\alpha = 2.9391$ $\beta = -1.4661$</td>
</tr>
<tr>
<td>11</td>
<td>$\alpha = 2.9408$ $\beta = -1.3507$</td>
</tr>
<tr>
<td>12</td>
<td>$\alpha = 2.9425$ $\beta = -1.2183$</td>
</tr>
<tr>
<td>13</td>
<td>$\alpha = 2.9467$ $\beta = -1.1368$</td>
</tr>
<tr>
<td>14</td>
<td>$\alpha = 2.9433$ $\beta = -1.0443$</td>
</tr>
<tr>
<td>15</td>
<td>$\alpha = 2.9430$ $\beta = -0.96080$</td>
</tr>
<tr>
<td>16</td>
<td>$\alpha = 2.9464$ $\beta = -0.91758$</td>
</tr>
<tr>
<td>17</td>
<td>$\alpha = 2.9430$ $\beta = -0.96080$</td>
</tr>
<tr>
<td>18</td>
<td>$\alpha = 2.9427$ $\beta = -1.0659$</td>
</tr>
</tbody>
</table>

The table shows that $\alpha$ is independent of temperature. The average value is $\bar{\alpha} = 2.94437222$. The coefficient $\beta$ depends on temperature, and the following function shows how $\beta$ is determined by temperature:

$$\ln \hat{\beta} = 2.4363 - 0.885601 \ln x + 0.034006 D1 + 0.18846 D2$$

(20.99) (−15.75) (0.1806) (0.9957)

Statistical tests show that there is a positive autocorrelation (DW=0.82), which inflates the $t$-value and $R^2$. Hansen’s test indicates unstable variance and
coefficient value for the $\ln x$ variable. The observed and estimated values of $\beta$ are shown in Figure 12. Note that the figure shows a positive $\beta$.

![Figure 12: Estimated and plotted beta as a function of temperature](image)

Figure 12 shows that the model has high validity for temperature from 5-6 degrees and higher. The growth function which integrates the effects from temperature can be expressed in the following way:

$$w(t:x,D) = \exp\left[2.9444 - \frac{\left(e^{2.4363 - 0.88601\ln x + 0.03406D + 0.18846D^2} - 1\right)}{t}\right]$$

The variable $x$ is temperature, and $D$ is the dummy variable which absorbs dysfunctions when temperature is higher than 16 degrees. The time variable $t$ must be different from zero.

The analysis shows that the rate of growth of the fish is closely related to the level of the water temperature. The statistical analysis shows that the growth related coefficient is nonlinear with respect to temperature, except for the von Bertalanffy growth function.
4.3 Optimal slaughtering time and temperature

If we ignore various types of costs (feeding, insurance, etc.), the net discounted profit for a single year class of fish is maximized if the fish are slaughtered when the relative growth rate of the fish is equal to the opportunity cost of keeping the fish in the cages, i.e.

\[
\frac{\dot{w}(t^*)}{w(t^*)} = r + M
\]

In case we apply the Faustmann’s expression for rotation, we apply the following optimal rule

\[
\frac{\dot{w}(t^*)}{w(t^*)} = r + M + \frac{r}{(e^{rt} - 1)}
\]

Since we are interested in the qualitative effect of temperature on optimal slaughtering time, we will for simplicity ignore costs. For the single year class case the optimal slaughtering time \( t^*_B \) for the Bertalanffy growth function (\( t^*_B \)), the logistic growth (\( t^*_L \)) function, and the exponential function (\( t^*_E \)) is as follows:

\[
t^*_B = -\frac{1}{\alpha_B} \ln \left[ \frac{r + M}{\beta_B (3\alpha_B + r + M)} \right]
\]

\[
t^*_L = \left[ \frac{1}{\ln \gamma_L} \right] \ln \left[ \frac{-\alpha_L (r + M)}{\beta_L (r + M + \ln \gamma_L)} \right]
\]

\[
t^*_E = \sqrt{\frac{\beta_E}{r + M}}
\]
By substituting the estimated temperature effects on the temperature-related parameters $\alpha, \gamma_L$ and $\beta_E$, the effect from sea temperature on optimal slaughtering time can be plotted for each growth function. In one of the preceding paragraphs we showed that the rotation principle, based on Faustmann, entailed a shorter optimal slaughtering time compared to no rotation. It is not possible to endogenize the optimal slaughtering time if we apply the Faustmann formula as a part of the differential equation. Optimal slaughtering time must be derived numerically for each temperature level, fitting a curve to the optimal points. Figure 13 shows optimal slaughtering time for exponential growth (and no rotation), logistic growth with no rotation, and a logistic growth with rotation. We assume that the real interest rate and mortality rate are $r = 0.05$ and $M = 0.10$:

Figure 13: Optimal slaughtering time and sea temperature

The figure shows that in general the optimal slaughtering time is reduced if the average sea temperature increases. Notice that the curve for the rotation case shows that the length of the rotation period is continuously being shortened with increasing temperature. If we evaluate the curves against actual slaughtering
time, we can conclude that the logistic curve with rotation has the highest validity. Farmers are known to keep the fish in the cages for a period of one to two years, depending on geographical location and average sea temperature, and the fish is on average supplied when it is about 4-5 kilograms. We will therefore apply the logistic growth function in the remaining part of the analysis. We have assumed that the mortality rate is not affected by temperature. Above it was shown that the growth rate slows down when the temperature exceeds 16 degrees, but it is also possible that the mortality rate will be affected by temperature.

5. ECONOMIC EFFECTS OF A TEMPERATURE CHANGE – SINGLE COHORT CASE

5.1 Comparative study between Lista and Skrova

In Lorentzen and Hannesson (2005) we argue that the coast of Nordland will be one of the most productive areas for farming of salmon in the future. What is the economic effect if the temperature increases in the coastal waters off Norway? A scenario can be illustrated by comparing the environment in the north and southwest of Norway. We shall compare the sea water temperature off Lista in Vest-Agder and Skrova in Nordland county. Figure 14 shows where Lista and Skrova are located in Norway. Skrova is located about 1180 km north of Lista.
Table 5 shows the difference in temperature between these areas. This difference will definitely result in different productivity in the said geographical areas.

Table 5: Temperature level off Lista, Vest-Agder county for selected years.

<table>
<thead>
<tr>
<th>Month</th>
<th>Average Skrova</th>
<th>Average Lista</th>
<th>Difference in temp.</th>
<th>Standard deviation Skrova</th>
<th>Standard deviation Lista</th>
<th>95% confidence interval Skrova</th>
<th>95% confidence interval Lista</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>4.73</td>
<td>6.22</td>
<td>1.49</td>
<td>0.75</td>
<td>0.75</td>
<td>3.26-6.20</td>
<td>4.75-7.69</td>
</tr>
<tr>
<td>February</td>
<td>3.77</td>
<td>4.86</td>
<td>1.08</td>
<td>0.58</td>
<td>0.93</td>
<td>2.63-4.91</td>
<td>3.04-6.68</td>
</tr>
<tr>
<td>March</td>
<td>3.20</td>
<td>4.44</td>
<td>1.24</td>
<td>0.52</td>
<td>0.76</td>
<td>2.18-4.22</td>
<td>2.96-5.93</td>
</tr>
<tr>
<td>April</td>
<td>3.50</td>
<td>5.14</td>
<td>1.64</td>
<td>0.58</td>
<td>0.73</td>
<td>2.36-4.64</td>
<td>3.71-6.57</td>
</tr>
<tr>
<td>May</td>
<td>4.85</td>
<td>6.69</td>
<td>1.84</td>
<td>0.53</td>
<td>0.94</td>
<td>3.81-5.89</td>
<td>4.85-8.53</td>
</tr>
<tr>
<td>June</td>
<td>6.76</td>
<td>8.29</td>
<td>1.52</td>
<td>0.59</td>
<td>1.13</td>
<td>5.60-7.92</td>
<td>6.08-10.5</td>
</tr>
<tr>
<td>July</td>
<td>9.09</td>
<td>10.91</td>
<td>1.83</td>
<td>1.04</td>
<td>1.67</td>
<td>7.05-11.13</td>
<td>7.64-14.18</td>
</tr>
<tr>
<td>August</td>
<td>10.27</td>
<td>13.76</td>
<td>3.49</td>
<td>1.18</td>
<td>0.98</td>
<td>7.96-12.60</td>
<td>11.84-15.68</td>
</tr>
<tr>
<td>September</td>
<td>10.62</td>
<td>13.91</td>
<td>3.29</td>
<td>1.07</td>
<td>1.00</td>
<td>8.52-12.72</td>
<td>11.95-15.87</td>
</tr>
<tr>
<td>October</td>
<td>9.61</td>
<td>12.41</td>
<td>2.80</td>
<td>0.91</td>
<td>0.76</td>
<td>7.83-11.40</td>
<td>10.92-13.90</td>
</tr>
<tr>
<td>November</td>
<td>7.79</td>
<td>9.89</td>
<td>2.10</td>
<td>0.90</td>
<td>1.15</td>
<td>6.03-9.55</td>
<td>7.64-12.23</td>
</tr>
<tr>
<td>December</td>
<td>6.05</td>
<td>7.82</td>
<td>1.77</td>
<td>0.80</td>
<td>0.84</td>
<td>4.48-7.62</td>
<td>6.17-9.47</td>
</tr>
<tr>
<td>Yearly average</td>
<td>6.69</td>
<td>8.70</td>
<td>2.01</td>
<td>0.23</td>
<td>0.28</td>
<td>6.24-7.14</td>
<td>8.15-9.25</td>
</tr>
</tbody>
</table>

Source: The calculations are based on data from Institute for Marine Resources (IMR) in Bergen, Norway.
Table 5 shows that the temperature off Lista is on average about two degrees higher than at Skrova; the sample average shows a minimum of about 4 degrees in March and a maximum of between 13 and 14 degrees in August or September. Skrova is about 1500 km north of Lista, and the sea temperature off Lista is higher for all 12 months. The variance of the monthly temperature is also higher off Lista. As Table 5 shows, the difference each month is not constant.

5.1.1 Temperature and optimal slaughtering weight

Assume that the growth of the salmon follows a logistic growth path, and that $M = 0.1$ and $r = 0.05$. If the temperature increases, growth will accelerate. Today the average temperature off Skrova is about 7 degrees. At this temperature the optimal slaughtering time will be two years and six months (2.46 years), given no rotation. If the sea temperature increases to 8 degrees, the optimal slaughtering time will be two years and two and a half months (2.20 years). If we look at the rotation case, the optimal slaughtering time is 1.82 years for 7 degrees, and 1.62 for 8 degrees. The slaughtered fish is 8.3 kilogram in the no-rotation case, and 6.7 kilogram with rotation. Figure 15 shows the optimal slaughtering weight of the fish with and without rotation.
Figure 15 shows that the optimal slaughtering weight is almost independent of temperature in the non-rotation case. In the rotation case the optimal slaughtering weight increases with temperature, but at a diminishing rate. Figure 16 shows a rough weight distribution of slaughtered salmon in Norway during 2005. According to the figure, 26.5% and 28.8% of the fish is, respectively, in the weight group 3-4 kilograms and 4-5 kilograms. Above we have shown that farmers mainly slaughter the fish when it is 4-5 kilograms. In this analysis the slaughtering weight is several kilograms higher. This can probably be explained by the fact that the model does not take into account what size or weight of the fish the market prefers. In this paper we apply a constant price, independent on the size of the fish, and so the optimal slaughtering time depends only on the relative growth rate, the real interest rate, and the mortality rate. Clearly, if the price of the fish depends on the weight or seasonal demand, it will influence the optimal slaughtering time. Feed, insurance, and slaughtering costs have also some influence on the optimal slaughtering time.
5.1.2 Changes in quantity and frequency in slaughtering

The difference in slaughtering time means that the quantity produced and frequency of slaughtering will also be affected. The quantity could change if a geographical area moves into another temperature regime, for example due to climate change or due to natural, long run fluctuations in temperature.

Figure 17 shows the percentage change in quantity with and without rotation if the temperature increases from 7 degrees, which represents the initial, average temperature in the sea water off Skrova.
Figure 17: Percentage change in quantity due to temperature increase off Skrova

The figure shows that the productivity increases with higher temperature, but the gain decreases with increased temperature level. There is no difference between the rotation and no-rotation case for changes in temperature close to the initial level. Let us focus on the rotation case because it is most realistic scenario. Then, if the average temperature increases from 7 to 10 degrees, the production will increase by about 7 percent. If the temperature increases from 7 to 8 or from 7 to 9, the production increases by about 3% and 5%, respectively. If value added is on average a fixed share of quantity produced, than the value added would increase by the same percentage as the quantity. The calculations do not include temperature over 16 degrees.

The model calculations show that a climate change enhances the growth rate and generates a positive economic effect. A climate change which increases the growth rate implies that the value of the site increases, i.e., the economic rent of the site increases.
5.1.3 Change in gross present value due to changes in average temperature

It is also possible to analyse how a change in average temperature changes the discounted gross value of fish farming. We analyse the economic effect if the temperature \((t)\) increases from 7 to \(t^*\) degrees. Because of the nonlinearity in the growth function and the relationship between the value of the coefficients and temperature, the percentage change in gross revenue depends on the starting value of the temperature level. The argument for applying 7 degrees as the starting point is that this is close to the average yearly sea temperature off the coast of Nordland county, which is likely to become one of the most productive salmon counties in Norway in the future as a result of ocean warming (Lorentzen and Hannesson 2005). The mathematics behind the calculations and the curve in Figure 18 is as follows (we only present the expression based on the logistic growth function): Firstly we estimate the optimal slaughtering \(t^*_L\) time, given rotation and average sea temperature of 7 degrees. Secondly we estimate the gross present value \((GPV)\) for 7 degrees by applying the following expression:

\[
GPV_7 = \sum_{n=1}^{\infty} \left( p_0 w N(0) e^{-M(t^*_L)} e^{-r(n^*_L)} \right) = \frac{1}{\left( e^{r(t^*_L)} - 1 \right)} \left[ \frac{p_0 N(0) e^{-M(t^*_L)}}{\alpha + \beta e^{-0.38x(t^*_L)}} \right]
\]

The expression for the change in gross present value, given that the average sea temperature is greater or equal 7 degrees \(x \geq 7\) can be expressed in the following way:
\[
\frac{\Delta GPV}{GPV} = \frac{GPV_{x2} - GPV_7}{GPV_7}
\]

We continue to assume that the real interest rate and mortality rate are \( r = 0.05 \) and \( M = 0.10 \) and that the price \( p_0 \) of the salmon is constant and independent of weight. The calculations are independent of the price level and how many juvenile salmon are released at time \( t = 0 \). Figure 18 shows the percentage change in the gross present value, given an infinite chain of rotations in each temperature regime.

The figure shows that the discounted gross value increases linearly with temperature. If the sea temperature increases by 1 degree from 7 to 8 in the seawater off Skrova, the gross present value increases by 17.6%. If the temperature increases by 2 degrees, the gross present value increases by about 35%. The value in each regime is based on an infinite time horizon, so we are not looking at a problem where for example the farmer “rotates” so to speak for 10 years in one temperature regime and than 5 or 10 years in another
temperature zone etc. The result in Figure 18 is due to a number of effects. Higher temperature reduces the optimal slaughtering time, but increases the growth rate of the fish. Above it was shown that the optimal slaughtering weight increases slightly with higher temperature. The rotation period $t^*_L$ (optimal slaughtering time) is shorter the higher the temperature, and so the factor $(e^{t^*_L} - 1)^{-1}$ increases with increasing temperature. The natural mortality $N(t)$ of fish depends on the optimal slaughtering time, so the higher temperature, the lower number of wasted fish.

Figure 19 shows the percentage change in discounted gross value for a single cohort. We present the result for the logistic and the exponential growth function. Note that the origin represents 7 degrees and is close to the yearly average sea water temperature off Skrova.

![Figure 19: Change in temperature level and the effect on discounted gross value in the single cohort case](image)

The figure shows that the temperature increase raises the value of the salmon industry off Skrova, but at a diminishing rate. The exponential growth function
shows the largest positive effect, and the gap between the two functions increases with temperature. An increase in temperature from 7 to 8 degrees increases the gross discounted value by about 5%. An increase from 7 to 10 degrees increases the gross present value between 10 and 15%, depending on the growth function used. The positive economic effect is due to higher productivity and that the slaughtering takes place at an earlier point in time, and it gives a positive discounting effect when the biomass is capitalized at an earlier point in time.

5.2 Concluding remarks

The analysis shows that an increase in sea temperature reduces the optimal slaughtering time, and the optimal slaughtering weight is almost independent of an increase in the temperature level. In the rotation case the optimal slaughtering weight increases with higher average temperature.

The analysis shows that a 1 degree increase in average temperature level in the sea water off Skrova (compared to the status quo level) increases the production by 3%, and a two centigrade increase, increases the production by 5% per year.

In the rotation case (infinite time horizon) the gross present value (GPV) or the value of the firm increases by 17.6% for each degree increase in temperature level. In the single cohort case the gross present value (GPV) increases by about 5% if the temperature increases by 1 centigrade. The effect on GPV is positive but diminishes with increasing temperature.

6. GROWTH PATTERN AND SEASONALITY IN THE TEMPERATURE LEVEL

6.1 Descriptive statistics

Up to now we have assumed that the temperature is constant over the year. In reality the temperature varies cyclically over the year, and so will the growth of
salmon. Figure 20 shows the monthly sea temperature in the 1-50m water column off Skrova for selected years during the period 1937 to 2003. The figure shows that temperature is seasonal. If we look at the sample average, the temperature varies from a low point of about 3 degrees in March to slightly above 10 degrees in August or September. Figure 21 shows the variation in the monthly sea temperature for the southernmost part of the west coast, Lista. See also Table 5 in the previous section which describes the statistical properties of the seasonal temperature pattern for respectively Lista and Skrova.

Figure 20: Monthly sea temperature off Skrova, Nordland county for selected years. Source: Institute of Marine Resources, Bergen.

Figure 21: Monthly sea temperature off Lista, Vest-Agder county for selected years. Source: Institute of Marine Resources in Norway.
We estimated the stochastic process behind the temperature fluctuations in the sea water off respectively Skrova and Lista (for descriptive statistics, see Table 5). The following seasonal differenced second order differential model was estimated for Skrova. Superscripts refer to geographical area, i.e. ‘L’ refers to Lista and ‘S’ refers to Skrova, $B$ is the backward operator, $\epsilon_t^S$ and $\epsilon_t^L$ are the stochastic, white noise residuals for respectively Skrova and Lista.

\[
(-0.66468 B^1 + 0.17923 B^2)(1 - B^{12})y_t^S = \epsilon_t^S
\]

\[
AIC = -0.069727
\]
\[
SC = -0.034113
\]

Ljung-Box $Q$-statistics: $Q_{11} = 9.95$

The following model was estimated for Lista:

\[
(-0.95823 B^1 + 0.49983 B^2 - 0.45823 B^{13})(1 - B^{12})y_t^L = \epsilon_t^L
\]

\[
AIC = -18.266
\]
\[
SC = 18.320
\]

Ljung-Box $Q$-statistics: $Q_4 = 2.08, Q_7 = 6.69, Q_8 = 15.54, Q_{12} = 24.38$

Both models have imaginary roots, which indicate sinusoidal oscillations in temperature.

Figure 22 shows estimated and observed monthly temperature for Lista. The estimated curve is based on the presented ARIMA model for Lista.
The seasonality in temperature can be reproduced by calibrating trigonometric functions. The following functions reproduce the seasonality in temperature for respectively Skrova and Lista:

\[
y_S = 6.69 + 3.71 \sin \left( \frac{x}{2} \right)
\]

\[
y_L = 8.70 + 4.74 \sin \left( \frac{x}{2} \right)
\]

where \( x \): months, \( x > 0 \) and \( x_0 = 1 \) is the starting value. Below we present the trigonometric functions applied in a scenario analysis of how seasonal temperature oscillations affect farming of salmon in the sea water off Lista and Skrova. In the following we assume no shifts or structural changes in the temperature. The objective in this section is to show how the growth of the fish changes during a year because of recurrent cyclical movement in the temperature during a year.
6.2 Growth and seasonality

By applying the logistic growth function we can analyse the weight increase per month due to variation in the temperature per month. Figure 23 shows the weight increase for juvenile fish released in each of the months of the year, i.e., the temperature in each month is applied as the starting value. The figure shows the cyclical pattern of the weight increase due to differences in monthly temperature. The weight increase is similar to the cyclical movements in temperature. Above it was shown that a particular coefficient in the growth function is determined by temperature, and if the temperature oscillates, then the growth coefficient will also oscillate. Figure 23 is based on release of juvenile fish in each month so the figure shows the growth of the fish during one month. In practice farmers mainly release juvenile fish in April-May (“spring release”) and in the period August-October (“autumn release”). A minor release is done in the other months of the year. We can so far conclude that the growth path of the fish is a nonlinear hybrid of a stationary and nonstationary process with seasonal (sinusoidal) varying coefficients.

Figure 23: Increase weight per month due to temperature change
By integrating the seasonal oscillating temperature into the model, we obviously have a more valid approach to the actual growth of the fish compared to a growth function which assumes a constant temperature through the year. Figure 24 shows the growth path of the salmon off Skrova estimated with, respectively, constant and seasonal variation in temperature. The figure shows that the growth of the fish is influenced by the seasonality in the temperature. These fluctuations are not present in the model with a constant environmental parameter, i.e., a growth function based on average yearly temperature. Note that the growth of the fish is lower in periods with low temperature compared to the months with relatively higher temperature. The growth also is reduced because the growth rate decreases with age.

![Growth Paths for Salmon](image)

Figure 24: Growth of salmon with and without coefficient conditioned on seasonal variation in temperature

Figure 25 shows the relative growth rate of salmon in the water off Skrova with a seasonal, temperature-dependent growth parameter and a model with a constant growth parameter. Note that the constant parameter function has only one inflection point but the model with the seasonal variation in the growth
parameter has multiple inflection points. The figure shows that the relative growth based on a seasonal temperature-dependent parameter oscillates while the relative growth rate in the constant parameter growth function does not. Both functions are characterized by a declining growth rate over time. The temperature and the fluctuations of the temperature obviously have a significant impact on the growth of the fish.

![Relative Growth Rate](image)

**Figure 25:** Relative growth rate for respectively constant and seasonal temperature

![Growth Path](image)

**Figure 26:** Temperature dependent growth path for Lista and Skrova
Figure 26 shows the growth functions for Lista and Skrova. The figure shows that the growth path off Lista is steeper than the trajectory for Skrova. Increased temperature due to global warming will change the growth trajectories for both geographical regions. The Skrova path will probably shift and over time look like today’s Lista trajectory if global warming continues. It should also be mentioned that the growth paths do not reflect the seasonal variations in exposure to daylight, which clearly is different between south and north of Norway. Global warming will not change that.

6.3 Seasonal temperature oscillations and economic effect of global warming

Above we have shown that optimal slaughtering time varies with temperature. The higher the temperature, the earlier the fish should be slaughtered. We have also shown how the gross discounted value changes if the temperature increases. A comparison between Lista and Skrova shows that a permanent increase in sea temperature off Skrova from about 7 degrees to 8 or 9 will increase the gross discounted value by 10 to 15 percent and increase the quantity produced per unit of time by 12 to 20 percent, depending on the growth function used. This scenario presupposes constant parameters in the growth function, i.e. that the environmental parameter is independent of seasonal variation in temperature.

When the temperature is sinusoidal the optimal slaughtering time cannot be found by uncritically applying the first order condition for profit maximization, which for the logistic model is

$$t^*_L = \frac{1}{\ln \gamma} \ln \left[ \frac{-\alpha (r + M)}{\beta (r + M + \ln \gamma)} \right]$$
As previously shown, the parameter $\gamma$ is temperature dependent, and so the value of $\gamma$ will oscillate with the oscillating temperature, and $I^*_L$ will also oscillate. But because in the long term the growth rate of fish is declining, it is possible to apply a numerical method (search method) to find the optimum. We will illustrate the solution by applying figures. The first order condition of maximizing the present value of a single cohort is:

$$\frac{\dot{w}}{w} = r + M$$

### 6.4 A comparative analysis between Lista and Skrova

We have assumed $r + M = 0.15$. Figure 27 shows the relative growth rate for two versions of the logistic function, one which takes into account the seasonal temperature oscillations and another which has a constant environmental parameter. The constant parameter is determined by applying the average yearly temperature in the sea water off Lista and Skrova respectively.
Figure 27: Relative growth rate with and without oscillating parameters

The figure and the numerical calculations show that it is optimal to slaughter the fish at time $t = 2.7$ at Lista and $t = 2.2$ at Skrova if the environmental parameter ($\gamma$) in the growth function is conditioned on average temperature, 6.69 and 8.70 degrees, respectively. The figure shows that an environment with variable temperature can give multiple solutions which satisfy the optimality criterion. There exist local optima which have to be compared before we can be assured that the solution is a global optimum. In this example the optimality criterion $\frac{\dot{w}}{w} = r + M$ is satisfied for Skrova and Lista, given the time span which the figure covers. The figure also shows that there are differences between the optimal slaughtering time in the single cohort case with and without a varying environment.
Figure 28 shows the development of the discounted gross present value of a given initial number of fish (a single cohort) for constant and oscillating temperature and growth parameter for Lista and Skrova.

![GROSS PRESENT VALUE WITH CONSTANT AND SEASONAL TEMPERATURE SKROVA AND LISTA MAY RELEASE](image)

**Figure 28: Discounted gross value with constant and seasonal temperature and one cohort**

Functions with a constant environmental parameter (based on average yearly temperature) show only one candidate for optimum. If we differentiate the constant coefficient function with respect to time in the single cohort case, we find (which also is indicated by visual inspection of the figure) that the discounted value is maximized after 2 years and eight months if the farming take place in the water off Skrova (see mathematical expression for optimum in section “Optimal slaughtering time given different weight functions”). For Lista, the discounted gross value is maximized if the fish is slaughtered and sold after 2 years and two and a half month. The time difference between Lista and Skrova in the constant coefficient case is about six months. According to the Norwegian Directorate of Fisheries, Department for Aquaculture, it takes one year and four months to produce a four kilogram salmon off Bodø (not far from Skrova) and
one year and three months to produce the fish off the west coast of Norway. A comparison between this reference and figures based on the model presented in this analysis shows a relative good correspondence, especially for growth figures from farming in the south of Norway.

The analysis also shows that the discounted gross value for the Lista farmer is about 10 percent higher than for the Skrova farmer in, respectively, the seasonal variation case and in the constant temperature case (see Figure 27). The differences between the constant coefficient case and seasonally dependent case are summarized in Table 6. The following assumptions are used in the simulations: discount rate: 0.05 and mortality rate: 0.10, average temperature: Lista: 8.7 degrees with amplitude 4.74 and Skrova: average 6.7 degrees and amplitude 3.71.
Table 6: Optimal slaughtering time, quantity, weight and gross present value with constant and seasonal temperature

<table>
<thead>
<tr>
<th></th>
<th>LISTA</th>
<th>SKROVA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Constant</td>
<td>Seasonal</td>
</tr>
<tr>
<td></td>
<td>coefficient</td>
<td>coefficient</td>
</tr>
<tr>
<td>Optimal</td>
<td>2.2</td>
<td>1.8</td>
</tr>
<tr>
<td>slaughtering time</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t^*(y)ears$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weight of the fish</td>
<td>8.6</td>
<td>8.3</td>
</tr>
<tr>
<td>$w(t^*)$ kilogram</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quantity produced</td>
<td>6.90</td>
<td>6.93</td>
</tr>
<tr>
<td>Gross present value</td>
<td>6.22</td>
<td>6.33</td>
</tr>
</tbody>
</table>

Figure 29 shows which month maximizes the gross present value at Lista versus Skrova. All values are discounted to February so they can be comparable. The figure shows that farmers of salmon at Lista maximize the gross present value if the release of fish takes place in April, but note that the differences in GPV in marginal between March, April, May, June and July. The vertical lines indicate which month the GPV is maximized. The Skrova farmers maximize the gross present value if they release fish in June. The theoretical result corresponds to what is observed in practice. The aquaculture industry in the south (west coast of Norway) releases most of the juvenile fish in April, while the farmers in the north of Stadt mainly release the fish in June. The main explanation is that the temperature is too low in the north to release fish as early as is done on the west coast.
MAXIMUM GPV AND SLAUGHTERING TIME FOR DIFFERENT RELEASE MONTHS FOR SKROVA AND LISTA
SINGLE COHORT CASE

Figure 29: Gross present values given different release month for Lista and Skrova

7. SEASONAL TEMPERATURE OSCILLATIONS AND ROTATION

7.1 Optimal fish farming and infinite time horizon

In practice the fish farmers in Norway release juvenile salmon and trout mainly twice a year, the so called “spring release” and the “autumn release”. The spring release takes place in the period from April to June and the autumn release takes place in the period from August to October. Suppose the fish farmer releases juvenile fish in April. The fish will be ready for the market, i.e. slaughtered and sold, in the period from April to August next year. The fish is therefore slaughtered and sold during a period of 12 to 16 months after the release the previous year. The farmers organize the production in such a way that they can release new cohorts every year in the period April to June and in the period August to October. As a new cohort is released at the same time every year while the last year cohort is still not slaughtered and sold, farmers have to see to
that they have enough available sea areas and cages for a small period of overlapping production.  

With an infinite rotation of cohorts, the farmer’s objective is to maximize the gross present value ($GPV$) of fish farming with respect to the rotation period $t$, and given an infinitely numbers of identical rotations $n$. We will analyze this problem using the logistic growth function, which seems to provide the best description of the growth of the salmon. With an infinite number of identical rotations with time length $t^*_M \in (0,t^M)$ where $t^M$ is the time the fish needs to reach the maximum weight. The objective function can be expressed in the following way:

$$GPV = \sum_{n=1}^{\infty} (p_0 w(t) N(0) e^{-M(t)} e^{-r(t)})$$

where

- $p_0$ : Price of the fish
- $w(t)$ : Weight of fish at time $t$
- $N(0)$ : Released number of juvenile fish
- $M$ : Mortality rate
- $r$ : Real interest rate
- $n$ : Number of rotations

Because of seasonal (cyclical) variation in temperature, it is necessary to split up the growth process to fixed intervals of one month each. The calculation is based

---

3 Note that the infinite recurrence process between release of juvenile fish and slaughtering does not require that the previous cohort has to be slaughtered and sold before a new cohort is released. In this version of the rotation problem in fish farming, we assume (which also is consistent with practice) that there is a possibility for overlapping production for a minor time interval between different cohorts.
on the following logistic growth function $w(t) = \frac{1}{\alpha + \beta \gamma^t}$. The initial weight is $w(0) = \frac{1}{\alpha + \beta}$, given that $t = 0$. Based on calculation in previous section in this report, the initial weight of the juvenile fish is 0.184 kilogram. The constant $\alpha$ is estimated to 0.1114. The initial value of $\beta$, i.e. the value of $\beta$, given $t = 0$, is as follows $\beta(0) = \frac{1}{w(0)} - \alpha$, and empirically we have $\beta(0) = \frac{1}{0.184} - 0.1114 = 5.32$. This is the starting value for $\beta$, but as time progresses $\beta$ is updated (see below).

The fish grows toward the maximum value $w_\infty = \lim_{t \to \infty} \frac{1}{\alpha + \beta \gamma^t} = \frac{1}{\alpha}$, given that $0 < \gamma < 1$. The maximum weight is $w_\infty = \frac{1}{0.1114} \approx 8.98$ kilogram. Define “$i$” as the release month $i = 1, 2, 3, \ldots, 12$ and define “$j$” as the number of months after the release of the juvenile fish. A generalization of the formulae for the $\beta$ for each period is as follows; $\beta_j = \frac{1}{w_j} - \alpha$. The value of the parameter $\gamma$ is a function of the monthly temperature level. In a previous section $\gamma$ is estimated and can be approximated by the function $\gamma_j = e^{-0.388z_j}$ where the variable $z_j$ is the temperature level in month $j$ after the release. As mentioned, the weight level is updated for each month, so the exponent time-variable for gamma is $t = 1/12$. The weight of the fish after $j$ periods (months) in the sea can be expressed in the following way, given the release month $i = 1, 2, 3, \ldots, 12$ and where $i = 1$ stands for January, $i = 2$ stands for February etc.;

$$w_j = \frac{1}{\alpha + \beta_j e^{-0.388(z_j)1/12}}$$
where \( j-1 \) means the month before month \( j \). \( z_{j-1} \) is the sea water temperature in month \( j-1 \). If there exists an optimal rotation period, the following first order condition must be satisfied:

\[
\frac{\dot{w}(t^*)}{w(t^*)} = r + M + \frac{r}{(e^{\gamma} - 1)}
\]

For the logistic growth function, the relative growth rate can be expressed in the following way

\[
\frac{\dot{w}}{w} = -\frac{\beta y' \ln \gamma}{\alpha + \beta y'}
\]

### 7.2 Optimal rotation and the value of the firm

By applying the approximation for the temperature processes, the expression for GPV and the first order condition, we have estimated the optimal rotation period for Skrova and Lista. Figure 30 shows the first order condition for Skrova, for juvenile salmon released in May. The figure shows that the first order condition is satisfied four times (but not necessarily the second order condition), and the figure cannot tell which point is the global optimum. Figure 31 maps the discounted present values for Lista and Skrova (with and without seasonal temperature), and the maximum values shows where the second order condition is fulfilled.
RELATIVE GROWTH RATE AND OPTIMAL ROTATION FOR LISTA AND SKROVA - MAY RELEASE

![Graph showing relative growth rate and optimal rotation](image)

Figure 30: Relative growth rate for Skrova and Lista and opportunity cost of postponing slaughtering

Figure 31 shows the gross present value for farmers at Skrova and Lista with constant (average) temperature and seasonal varying temperature.

![Graph showing gross present value](image)

Figure 31: Gross present value for Lista and Skrova given constant and seasonal variation in temperature

The figure shows that the gross present value is maximized, given seasonal temperature variation, if the rotation period is about one year and eight or nine
months (between 1.67 and 1.75 years) for Skrova, and about one year and six or seven months (between 1.5 and 1.58 years) for Lista. It is optimal for farmers at Lista to slaughter two months earlier, compared to farmers at Skrova. The slaughtering weight at Lista is 7-7.6 kilograms and 6-6.4 kilograms at Skrova. In the constant temperature case the GPV is maximized for Lista and Skrova if the rotation period is respectively 19 months (1.58 years) and 24 (two years). Note that the optimal slaughtering time for farming off Lista is equal for respectively seasonal variation and constant temperature. But this result is only valid for the May release. Another release month would probably give another result.

8. TEMPERATURE CHANGES IN THE SEA WATER OFF LISTA AND SKROVA – SINGLE COHORT AND ROTATION CASE

8.1 Introduction to the scenario analysis

In most of the analysis we have focused on the effects of global warming on salmon farming at Skrova. We analysed the effect on farming at Skrova if the temperature increased and became more like the temperature level at Lista. On the other hand the fact that Lista today is exposed to relatively high sea water temperature in the summer months raises the question what will happen if global warming increases the temperature? In the following we will present scenarios which show different patterns of temperature changes and possible effects on the farming of salmon in the sea water off Lista and Skrova. The scenario analysis covers the single cohort case and the infinite time horizon with identical rotations. We let Lista represent the farmers in the southernmost counties in Norway and analyse what will happen if the temperature continues to increase in the warmest part of the coast of Norway. The scenario is organized in three parts: (I) the seasonal amplitude of the temperature increases and the average is constant, (II) the average temperature increases and the amplitude is constant, and finally (III) a simultaneous change in amplitude and average temperature. In
the simulations we assume that the juveniles are released into cages in July. The scenarios do not cover temperatures below 1 degree. Personal contact with experts on farming of salmon at the Norwegian Directorate for Fisheries and Institute of Marine Resources in Bergen indicates that dysfunction is initiated when the temperature surpasses 16 degrees or is lower than 1-2 centigrade. The negative effect of high temperature on growth is caused by \( i. a. \) less density of oxygen in the water and higher density of bacteria and algae. The negative effect is not linear. It follows from this that the mortality rate \( M \) is a U-shaped function of temperature, i.e. too low (below 2 degrees) and too high (above 16 degrees) temperature increases the mortality rate. In the scenario analysis we treat the mortality rate as a constant independent of temperature. In the simulation model we assume that temperature between 16 and 17 degrees gives the same growth rate as 12 degrees, between 17 and 18 is equal to 10 degrees, between 18 and 19 equal to 3 degrees, temperature between 19 and 20 is similar to 1 degrees, and finally temperature over 20 degrees or below 1 degree is equal to physiological breakdown. We have no scientific documentation for these assumptions, except that people in the business indicate a significant, negative change in the growth process when the temperature exceeds 17-18 degrees or creeps below 1 or 2 degrees. It follows from the assumption that critical temperature levels have negative effect on the output from the model.

8.1.1 Scenario I: Increase and reduction in amplitude

\textbf{Scenario I:} Increase/reduction in amplitude by respectively \( \pm 0.5, \pm 1, \pm 1.5, \pm 2 \) and \( \pm 2.5 \) compared to benchmark or status quo. We apply a deterministic trigonometric function which is calibrated for mapping the temperature structure in the sea water off Lista and Skrova. Figure 32 shows the status quo situation at Lista (thickest curve). The thin lines map the temperature trajectories for which the amplitude is increased or decreased.
Figure 32: Temperature oscillations at Lista: Status quo (bold line) and set of curves reflecting increased and reduced amplitude

Figure 33: Change in amplitude and the effect on the growth process for fish off Lista

Figure 33 shows the effect on the growth path if the amplitude increases as mentioned. Increased amplitude results in higher maximum and minimum temperature levels, which significantly affects growth. If the temperature is lower than 1 degree, the fish will not grow at all and die, if the temperature becomes so low that ice crystals are formed in the water. If the amplitude
increases by 3.1 degrees, the growth will be close to zero in some months because the temperature is about 1 degree.

Figures 34 summarizes the effect changes in amplitude due to climate change will have on gross present value and slaughter weight for farming in the seawater off Lista and Skrova. The vertical line which passes through zero in each of the figures indicates benchmark. A change in amplitude has only marginal effect on gross present value (GPV) for farmers at Lista. The effect is greater for Skrova. The tendency is that a higher amplitude increases the GPV, but if the amplitude gets high enough the temperature reaches critical low levels and the fish will die. A breakdown (fish will die) will take place at Skrova if the amplitude increases by more than 2.5 degrees. Farming of fish off Lista will break down if the amplitude increases by about 3-3.5 degrees. The figure also shows that change in slaughtering weight at Skrova increases with amplitude and falls with lower amplitude. There is no change in slaughtering weight at Lista if the amplitude is reduced, but the slaughtering weight is reduced if the amplitude increases. Calculations show that optimal slaughtering time for Lista falls with increased amplitude while the slaughtering time is not changed for Skrova.
It should also be mentioned that the effect of a change in amplitude on GPV, slaughtering time and weight of the fish depends on in which month the fish is released. This follows from the fact that different starting months give different sequences of temperature and hence different growth paths. It is, however, not realistic to expect that a global climate change should only change the temperature amplitude. The next scenario analyses how a change in average temperature will affect GPV, slaughtering weight and slaughtering time.

8.1.2 Scenario II: Change in average temperature

**Scenario II**: This scenario is based on the assumption that the average temperature is changed by 0.5, 1, 1.5, 2, 2.5, 3 etc. degrees compared to the status quo case. These scenarios assume that the amplitude is not affected. The average temperature is increased by 11.5% if the temperature increases by 1 degree compared to the status quo. Figure 35 shows the oscillating trajectories...
for the status quo case (bold line) and an increase in average temperature by 0.5, 1, 1.5, 2, 2.5 and 3 centigrade.

We have assumed that a too high temperature will reduce dramatically the growth of the fish. In this scenario the average temperature was increased stepwise for testing at what temperature level the growth is about the same as in the status quo situation. The result from the experiment is presented in Figures 36 and 37. The juvenile fish is released into the cages during May. Figure 36 shows that an increase in the average sea temperature accelerates the growth of the fish, and the maximum weight is reached in a shorter time than with lower average temperature. The increase in the average temperature appears to smooth the seasonal variation in the growth path, which are more marked in the benchmark growth path.
Figure 36: Weight function for farmed fish at Lista for different average temperatures compared to status quo

Figure 37: Increase in average temperature and the percentage change in GPV and slaughtering time

Figure 37 shows the effect an increase in average sea temperature has on gross present value (GPV), optimal slaughtering time, and slaughtering weight. Zero value in the figure represents the benchmark, i.e., no change in average temperature.
temperature. The figure shows that an increase in average temperature has a positive effect on GPV. The percentage change in GPV for Lista is bell shaped, and a maximum of 10% increase is reached when average temperature increases by 2.5 degrees. The effect is diminishing, but still positive when temperature increases more than 2.5 degrees. The percentage increase in GPV for Skrova is linear and Lista follows the same path to 2.5 centigrade increase. Calculation shows that a 1% increase in average temperature increases the gross present value for farmers at Skrova by about 0.22%. If global warming increases the sea temperature by 1 degree, the gross present value for single cohort increases by 3.3% for Skrova and 2.5% for fish farmers off Lista. An increase in average temperature reduces the optimal slaughtering time. The reduction in slaughtering time is diminishing with increased temperature.

Figure 38 shows how increases in average temperature affect optimal slaughtering weight for respectively Lista and Skrova. There is a tendency that increased average temperature reduces the slaughtering weight at Lista, while the opposite effect can be detected for Skrova. Clearly, fish farming in both
geographical areas can bear an increase in average temperature which is beyond
the temperature increase due to climate change which is predicted by IPCC
(2001). The sensitivity analysis shows that change in average temperature will
not necessarily cause problems. It is rather changes in amplitude which can
cause a breakdown. If the amplitude of the temperature in the seawater off
Skrova increases by 2-2.5 degrees critical temperatures occur, and farming is
close to a breakdown. If the amplitude increases by 3-3.5 degrees in the water
off Lista, farming will be extreme risky. In general, as long as the temperature
fluctuates inside biologically sustainable limits it does not cause any serious
damages. On the other hand, if the temperature is close to the extreme values, it
induces devastating effects.

It should also be mentioned that the temperature structure which is applied in
this analysis is based on observation in the water column between 1 and 50
meters. The temperature at the surface is higher, however, than further down.
Farming takes place at the surface, so the critical values which are presented in
this report actually underestimate the effect temperature changes will have on
farming of salmon and trout. We therefore expect that dysfunctions as low
growth and higher mortality will show up earlier than these simulations indicate.

8.1.3 Scenario III: Simultaneous change in amplitude and average temperature

**Scenario III:** In the last scenario we assume simultaneous changes in amplitude
and average temperature. While IPCC (2001) has predicted that the average
temperature will increase in the future, it has not made any predictions about the
amplitude of the variations. Therefore we analyse both decreasing and
increasing amplitude, given increasing average temperature. Figure 39 shows
examples of oscillating temperatures in the seawater of Lista which cover the
benchmark (status quo bold curve) and different changes in both average
temperature and amplitude, respectively ±1, ±1.5 centigrade changes in
amplitude and average. The benchmark is characterized by amplitude of 4.74 and an average of 8.70 degrees.

Figure 39: Temperature oscillations given increased amplitude and average temperature

Figure 40: Increase in amplitude and average temperature and the effect on growth

Figure 40 shows what the growth path will look like if the amplitude and average temperature increase simultaneously in the sea water of Skrova. The growth process for the fish accelerates 3-4 months after the release. If the amplitude and average temperature simultaneously increases more than 3-4
degrees, the increase results in an environment which is similar to a 1 degree increase. Again we see that too high temperatures have devastating effects on the growth of the fish. Figure 41 shows how gross present value (GPV), slaughtering time and slaughter weight change due to a simultaneous increase in amplitude and average temperature for farming in the sea water off Skrova. The scenario presupposes a July release. The percentage change reaches its maximum at an increment of about 3 degrees. A further increase gives a positive but diminishing effect.

Figure 41: Increase in amplitude and average temperature and the effect on gross present value, slaughter weight and slaughtering time for Skrova

Figure 42: Increase in amplitude and average temperature and the effect on gross present value, slaughter weight and time for Lista
Figure 42 shows the same scenario for Lista. The gross present value at Lista is more sensitive for simultaneous increase in amplitude and average temperature compared to Skrova. The effect on GPV is bell shaped and the maximum increase in GPV is obtained after 1.5 centigrade increase compared to status quo. The optimal slaughtering weight varies but increases with increased amplitude and average temperature. The optimal slaughtering weight at Skrova is increasing in the interval (0, 1.5) and is reduced thereafter. The optimal slaughtering time decreases with increased amplitude and average temperature.

Figure 43: Percentage change in GPV due to an increase in average temperature and a reduction in amplitude in the sea water of Skrova

What will happen if the average temperature increases and the amplitude is reduced? Figure 43 shows the effect on GPV for Lista if average temperature increases and amplitude is reduced. An increase in average temperature and a simultaneous decrease in amplitude has a positive effect on gross present value and increases the value of the firm located at Skrova. A similar scenario for firms located in the sea water of Lista gives the result shown in Figure 44. The GPV increases with reduced amplitude and increased average temperature. The
effect is not as strong as with similar changes at Skrova. The effect on GPV is positive but diminishing.

Figure 44: Percentage change in GPV due to an increase in average temperature and a reduction in amplitude in the sea water of Lista

8.2 Climate change and the economic effect in the rotation case with infinite time horizon

This section extends the analysis by including a set of successive fish releases, with an infinite time horizon. This part of the analysis is built on the rotation principle and the additional assumptions are that the juvenile fish is released in July. The analysis is built on monthly, seasonal variation in temperature as described above. Climate change means (1) changes in average temperature, (2) change in the amplitude of the temperature and (3) a combination of changes is amplitude and average temperature.

Figures 45 and 46 summarize the effect on gross present value (GPV), optimal slaughtering time and optimal slaughtering weight of an increase in the average temperature for fish farmers located off Lista and Skrova. At Skrova (Figure
45), a higher average temperature increases the gross present value and the slaughtering weight, while the slaughtering time is reduced. Calculation shows that a one percentage increase in temperature increases the gross present value by 1.07%. If the average sea water temperature increases by 1 degree (about 15% increase) due to global warming, the GPV increases by about 16%. The calculation is valid for the average temperature range from status quo (6.69) to 11 centigrade. At Lista (Figure 46) the percentage change in gross present value is bell shaped with increasing average temperature. Calculations show that the value of the fish farming firms increases by 0.75% per percentage increase in average temperature. This relationship is valid in the average temperature interval from 8.70 to about 11 centigrade. If the average temperature increases by 1 centigrade (11.5% increase compared to status quo), GPV increases by about 9% (8.74%).

Figure 45: Increase in average temperature and the effect on farming in the sea water off Skrova
Figure 46: Increase in average temperature and the effect on farming in the sea water off Lista

Figure 47: Increase in amplitude and average and the effect on gross present value, slaughtering time and weight for farmers off Skrova

Figures 47 and 48 show how simultaneous changes in amplitude and average temperature affect gross present value, slaughtering time and weight at Skrova and Lista. At Skrova, a simultaneous and equal level of increase in amplitude and average temperature increases significantly the gross present value.
Calculation shows that in the temperature range of increase from 0 to 2.5 degrees, GPV increases by 21% per degree of increase. The scenario shows that the optimal slaughtering weight increases and the optimal slaughtering time are reduced.

At Lista (Figure 48) the effect of the simultaneous increase in amplitude and average temperature on gross present value and optimal slaughtering weight is bell shaped. A 1 degree simultaneous increase in amplitude and average temperature increases gross present value by 12-13%. The scenario shows not only that a small simultaneous increase has a positive economic effect, but also that farming has small safety margins if the temperature increases more than 2 degrees. A climate scenario where the average temperature increases and amplitude decreases (not presented here), results in a higher gross present value, shorter rotation, and a greater slaughtering weight (but bell shaped with reduced amplitude and increased average temperature).
Figure 49: Change in productivity due to increase in average sea temperature off Lista and Skrova

Figure 49 shows how climate change, i.e. an increase in average temperature affects the productivity in farming in the sea water off Skrova and Lista. The percentage change in productivity is positive, oscillating but diminishing for both geographical areas. A 1 degree increase in average temperature increases the production by about 20% at Skrova, while the effect is about about 10% productivity gain at Lista. Note that the percentage increase has local maxima/minima, for example that the percentage change in production off Skrova is higher given 2 centigrade increase compared to 2.5 centigrade increase. The percentage change in gross present value for Skrova ($y$) can be approximated by the function $y = 18.84 + 11.2 \ln(x)$ where $x$ is the increase in average temperature. Note that the percentage change in productivity at Skrova oscillates and has at least three local optima. The odd productivity path can be explained by the oscillating optimal slaughtering weight and time which is illustrated in a previous figure. The effect at Lista is also bell shaped but the concavity is much stronger than Skrova. The differences in concavity indicate two different responses and safety margins with respect to increase in
temperature. The percentage change in GPV for farmers at Lista can be approximated by the function: 

\[ y = 4.48 + 5.86x - 1.56x^2. \]

Figure 50: Simultaneous increase in amplitude and average temperature and the effect on productivity.

Figure 50 shows how productivity is affected by a simultaneous increase in amplitude and average temperature. The effect on productivity is positive, but diminishing with increased temperature. If we compare the simultaneous increase with increase in only average temperature, the gains from an increase in average temperature is higher. The main reason is that higher amplitude leads to minimum and maximum temperature, which in not favourable with respect to growth of the fish.

8.3 Conclusion

Table 7: Amplitude and average temperature for Lista and Skrova

<table>
<thead>
<tr>
<th></th>
<th>AVERAGE TEMPERATURE</th>
<th>AMPLITUDE</th>
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<tr>
<td>Lista</td>
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<td>1.03</td>
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</table>

Table 7 shows the status quo situation in the sea water off respectively Lista and Skrova. We analysed the following problem for a single cohort and for a
multiple cohort (rotation) case: If climate changes, what are the expected effects on respectively optimal slaughtering time (rotation period), slaughtering weight and gross present value (GPV) of the fish farming firm? We define ‘climate change’ as a change in amplitude and average temperature. We analysed the problem by changing:

(1) the amplitude,

(2) the average temperature and

(3) simultaneously the amplitude and average temperature

In the following we will summarize the results from the scenario analysis.

An increase in amplitude increases the GPV for farming off Skrova and Lista. The effect for Skrova is highest. A one degree increase in amplitude increases GPV by about 0.5% in the single cohort case, given a July release. A decrease in amplitude reduces the GPV and the value of salmon plants located off Skrova. The effect on GPV for firms located off Lista is close to zero. It will probably be impossible to farm salmon in the sea water off Skrova if the amplitude increases by 2.5 degrees or more. For Lista the critical value is 3-3.5 degrees.

An increase in average temperature has a positive effect on gross present value for firms located at Lista and Skrova. The effect is linear for temperature increases up to 5 degrees for Skrova. For additional temperature increase the effect on GPV is positive but diminishing. The effect is also linear at Lista for increase up to 2.5 degrees (see Figure 36). A further increase has a positive but diminishing effect on GPV. In the said linear interval (almost identical for both regions), a 1% increase in average temperature increases GPV by 0.22 percent. If the average temperature off Skrova increases by 1 degree, i.e., from 6.69 to 7.69, the GPV increases by 3.3% in the single cohort case. In the rotation case (infinite time horizon) a 1 degree increase in average temperature increases
gross present value (GPV) by respectively 15-16% for farming off Skrova and about 9% for farming off Lista. The analysis shows that an increase in average temperature by 1 degree increases the productivity in fish farming off respectively Skrova and Lista by 20 and 12-13%.

The analysis shows that there are relatively big differences in the percentage change in gross present value (GPV) and productivity between the single cohort and rotation case. The previous figures show that the paths for respectively optimal slaughtering time and weight are different between the single cohort and rotation case. The main cause to these differences is that optimal slaughtering in the rotation case takes place at an earlier stage in the growth process of the fish compared to the single cohort case, and it has important consequences for the effect of temperature changes. A closer look at the problem shows that the vertical difference between the weight paths before and after an increase in average temperature is significantly higher in the time interval for optimal slaughtering in the rotation case compared to the single cohort case (see Figure 36).

A simultaneous increase in amplitude and average temperature induces a bell shaped increase in gross present value (GPV) for farmers located at Skrova and Lista. An increase by 1.5 degrees increases the value of firms located in the said regions by more than 5% in the single cohort case. Maximum percentage increase is reached when amplitude and average temperature increase by about 3 degrees. The maximum for Lista is reached if average temperature and amplitude increase simultaneously by 1.5 degrees. The simulations show that slaughtering weight decreases at Lista and Skrova if amplitude and average temperature increases. In the rotation case the gross present value (GPV) increases by respectively 20% for farming off Skrova and about 12-13% for farming off Lista if the amplitude and average temperature simultaneously increased by 1 degrees. A simultaneous 1 degree increase in amplitude and
average temperature increases the productivity about 15% for farming off Skrova and about 12% for fish farming located off Lista. A simultaneous increase in average temperature and a decrease in amplitude increases the gross present value for farmers located off Skrova and Lista. The percentage change is strongest for fish farming located off Skrova.

The most likely scenario is that global warming will increase the sea temperature along the coast of Norway. The sensitivity analyses show that a change in temperature has economic consequences for the aquaculture industry. A general increase in temperature will accelerate the growth process of the fish and increase the productivity in the salmon fish farming industry. The analysis of the single cohort case with seasonal variation in temperature indicates that the gross present value (GPV) will increase by 0.22% for each percentage increase in average temperature. Corresponding numbers for the rotation case is 1.07% increase per centigrade increase in average temperature. If the average temperature in the sea water off Skrova increases by 1 centigrade, the gross present value for the single cohort will increase by at least 3%. The productivity is estimated to increase by 2.6%. The numbers for the rotation case are respectively 20 and 19.9%. In the single cohort case the productivity is estimated to increase by 4.5% for firms located off Lista. An increase by 1 degree in the sea water off Lista will increase the value of firm (increase in GPV) by about 4.5%. The corresponding value for the rotation case is 11%. The increase in GPV is bell shaped with increasing temperature.

A general temperature increase in the sea water due to global warming will have a positive effect on productivity and on the value of the fish farming firms located along the coast. The effect is positive but diminishing with increasing temperature. The analysis also shows that farmers located in the southernmost parts of the coast have a narrower safety margin with respect to temperature increase compared to farmers located further north. If amplitude and/or average
temperature increase to the level where normal physiology for the fish is put under pressure, the probability for a breakdown is increasing. Global warming is contra productive for the industry if the sea temperature increases too much.

We have shown that farmers in practice mainly slaughter the fish when it is 4-5 kilograms, and it takes between a 12-14 months in the south (Vest-Agder, Rogaland and Hordaland) and 14-17 months to produce a 4-5 kilograms salmon north of Stadt. The estimated growth function which we apply in the analysis is relatively consistent with the observable growth data. In this analysis the slaughtering weight is some kilograms higher. The slaughtering weights are predicted in the model, and are respectively 5.58 and 6.8 kilograms for Skrova and Lista, given that the fish is released in July. The said difference between theory and practice can be explained by the fact that the model does not take into account what size or weight of the fish the market prefers. In this paper we apply a constant price which implies that the optimal slaughtering time depends only on the relative growth rate, the real interest rate, and the mortality rate. Clearly, if the price of the fish depends on the weight or dependent on seasonal demand, it will influence the optimal slaughtering time. Feed, insurance, and slaughtering costs have also some influence on optimal slaughtering time. We have chosen to leave these effects aside in order to focus on the temperature change due to global warming.
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The attached table shows the daily percentage increase in weight for a juvenile salmon given different, constant temperature regimes.

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Appendix A: The attached table shows the daily percentage increase in weight for a juvenile salmon given different, constant temperature regimes.