Performance Sensitive Debt - Investment and Financing Incentives

BY
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Abstract

Performance sensitive debt (PSD) contracts link the paid coupon to a measure of firm performance. PSD contracts are widely used, especially as corporate bank loans. In a model where a firm has assets in place and the opportunity to invest in a growth option, I analyze how PSD affects equityholders’ investment and financing incentives. With no pre-existing debt I show that PSD reduces a given firm’s optimal leverage, indicating that in this case PSD partially solves potential future conflicts related to debt overhang. With debt in place I show that PSD financing magnifies equityholders’ risk-shifting incentives, proving that in this case PSD is an inefficient financing tool. My conclusion questions the hypothesis that PSD is used to prevent asset substitution. When debt overhang creates problems of underinvestment I show that PSD financing partially resolves these inefficiencies. My conclusions are partially based on numerical analysis, but they are robust to changes in input parameters.

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1 Introduction

The interplay between financing and investment decisions of firms has been widely studied in the corporate finance literature. In their seminal paper, Modigliani & Miller (1958) proves the famous capital structure irrelevance principle, which states that under the assumptions of perfect and frictionless capital markets and fixed investment decisions, the value of a firm is independent of its financing decisions. Two important market frictions question the validity of the irrelevance theorem. The first is the problem of agency costs identified by Jensen & Meckling (1976). They argue that in the presence of debt financing, equityholders might be tempted to engage in asset substitution or risk-shifting activities. The second is the debt overhang or underinvestment problem identified by Myers (1977). His argument is that equityholders of a leveraged firm will underinvest because part of the proceeds accrue to debtholders.

Recently, some attention has been given to the widespread use of performance sensitive debt contracts (PSD). PSD contracts link the coupon paid on a firm’s debt to a variable measuring its credit relevant performance. A typical PSD contract will trigger increased coupon payments when firm performance worsens, and reduced coupon payments when firm performance strengthens. The two most commonly used categories of credit performance measures are either based on firm cash-flows or firm credit ratings. Since the mid 1990’s performance sensitive features in both private and public debt are common. Market participants indicate that more than 50% of recently issued syndicated bank loans in Europe include such features.

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1 See, e.g., Mjøs, Myklebust, & Persson (2011) and Asquith, Beatty, & Weber (2005) for more detailed information.
To provide a rationale for the use of PSD, two main theoretical explanations have been given. The first explanation is that high quality firms use PSD to signal its quality to the market. They are able to do so since the threat of increased coupons make PSD financing too expensive for low quality firms. The second explanation is that PSD financing disciplines equityholders and thereby reduce the problems of asset substitution.

In this paper I focus on how the use of PSD might effect a firm’s investment and financing incentives. I do so by extending the model of Leland (1994) to include investment and PSD. In my model a firm has assets in place and a growth option to expand its operations. I allow for the possibility that the firm’s initial capital structure might consist of both debt and equity. Equityholders endogenously determine when to exercise the growth option, and how to finance this option. They do so by maximizing the sum of equity and the new debt used to finance the growth option, meaning that existing creditors face a risk of dilution. The new debt contract might be of PSD type. My analysis allows for three different priority structures; equal priority (Pari Passu), old debt is senior to new debt and new debt is senior to old debt. The two latter cases is commonly referred to as absolute priority rule (APR).

To briefly summarize, I find that with no pre-existing debt, using PSD reduces a given firm’s optimal leverage, indicating that in this case PSD partially solve potential future conflicts related to debt overhang. With debt in place (debt overhang) I show that equal priority or making new debt senior induces risk-shifting, since equityholders exercise the growth option too early in this case. Making the debt contract performance sensitive increases the problems of risk-shifting. Giving priority to the old debt induces underinvesting, since equityholders exercise the growth option too late in this case. Interestingly, making the debt contract performance sensitive partially resolves this underinvestment problem. My analysis disregard the existing hypothesis that PSD is used to prevent asset substitution. In these cases PSD is an inefficient financing tool compared to straight debt.
My paper is related to the literature on real options, which has provided a good theoretical framework for the study of the interaction between financing and investment choices\textsuperscript{2}. The broad consensus in this literature is that the use of debt financing leads to inefficient investment decisions, which in turn destroys firm value, cf. the discussion above. This value destruction is commonly referred to as agency cost of debt. Mauer & Ott (2000) study the problem of underinvestment. They conclude that the costs resulting from underinvesting incentives significantly reduce optimal leverage. The same conclusion is reached by Titman & Tsyplakov (2007) who construct a dynamic model allowing for continuous financing and investment choices. As an additional finding they show that the cost of underinvestment decreases when debt maturity shortens. Mauer & Sarkar (2005) focus on the cost of overinvesting or risk-shifting. They show that such costs could significantly reduce a firm’s optimal leverage as well as increase the credit spread paid on debt.

Sundaresan & Wang (2007) study a situation where a firm has multiple growth options that need to be exercised sequentially. They show that pre-existing debt may significantly distort future investment decisions. Hackbarth & Mauer (2010) use a similar model, but they focus on debt priority structures. Allowing the firm to choose an optimal priority structure, they show that suboptimal investment incentives can be virtually eliminated.

Other important papers in the dynamic investment and financing literature are, e.g., Mello & Parsons (1992), Mauer & Triantis (1994), Parrino & Weisbach (1999), Hennesy & Whited (2005), Hennesy & Whited (2007), Hackbarth et al. (2007), Tserlukevich (2008), Tsyplakov (2008), Morellec & Schürhoff (2010a) and Morellec & Schürhoff (2010b).

My paper contributes to the existing literature by being the first paper to introduce PSD contracts into a real options framework, and thereby being able to make sharp predictions on how such debt financing affect investment decisions.

\textsuperscript{2}Dixit & Pindyck (1994) provide an extensive survey of the real options literature
My paper also contributes to the growing literature on performance sensitive debt, and the papers most closely related to this one is Bhanot & Mello (2006) and Koziol & Lawrenz (2009). Manso et al. (2010) show that PSD might be used by firms as a way of signaling quality to the market, and also find empirical evidence supporting their conclusion. Bhanot & Mello (2006) study rating-triggered bonds and their ability to mitigate risk-shifting problems. They argue that rating-triggered bonds are not an attractive financing instrument, and that they cannot solve the asset substitution problem. Contrary to this result, Koziol & Lawrenz (2009) find that rating-triggered bonds can be designed to mitigate asset substitution or asymmetric information problems. They conclude that the optimal design and optimal use of step-up bonds are highly dependent on which of the two problems the bonds are intended to deal with. My paper extends the analysis made in the two latter papers by letting equityholders endogenously determine investment timing. Both Bhanot & Mello (2006) and Koziol & Lawrenz (2009) incorporate investment risk using the approach pioneered by Leland (1998), where a firm is allowed to, ex post, increase asset risk by replacing the firm’s current assets with a set of new ones with the exact same value, but different risk. Hence, agency costs in these papers only reflect the impact of an increase in risk on the expected values of interest tax shields and bankruptcy costs. Using a real options framework I am able to capture this effect, but I am also able to measures the potentially much larger loss of pure operating firm value due to suboptimal investment decisions. I argue that this extension provides a more thorough analysis, and a better understanding of PSD and its effect on corporate investments.

The remainder of this paper is organized as follows. Section 2 describes the general model set-up and provides solutions to the security values needed to perform the analysis. Here I also discuss debt priority structures and optimal investment and financing policies. Section 3 reports closed form solutions with no pre-existing debt and numerical solutions when the firm has pre-existing debt. Here I also examine the robustness of the results to changes in

3Other important papers in the PSD literature is Tchistyi (2006), Tchistyi et al. (2010), Manso et al. (2010), Lando & Mortensen (2004) and Asquith et al. (2005).
model parameters. Finally, Section 4 concludes.

2 The model

2.1 Model Set-Up

I assume that a firm has assets in place which generate a continuous pre-tax cash flow $X_t$, and a growth option to expand its operations. More specifically I assume that $X_t$ follows a stochastic process under the equivalent probability measure $Q$ specified by:

$$dX_t = \mu X_t dt + \sigma X_t dB_t, \quad X_0 > 0,$$

where $\mu$ and $\sigma$ denotes, respectively, the constant drift and volatility parameters. $dB_t$ represents the increment of a standard Brownian motion. The firm may exercise the growth option by paying an investment cost of $I$. Immediately upon exercise, the firm’s cash flows increases from $X$ to $QX$, where $Q > 1$. I assume that the option to exercise is irreversible.

The firm’s initial capital structure (prior to investing) consists of equity and a single class of debt, which has infinite maturity and pays an exogenously given interest of $c_0$. The firm may finance the growth option by issuing a mixture of new debt and equity. I assume that this new debt issue also has infinite maturity. Since the goal in this paper is to study the interactions of investment and performance sensitive debt I allow for the possibility that the firm may issue debt with a more general coupon scheme reflecting firm performance. In this model, the current cash flow $X_t$, is the only state variable. Any measure of a firm’s credit quality is, thus, determined solely by $X_t$, and so $X_t$ itself can be used as the performance measure. In other words, the coupon scheme of the PSD obligation is given by some function $C_1(X_t)$. The function $C_1(X_t)$ can in principle have any functional form, and, thus, this formulation is quite general. When solving the model I make the assumption that $C_1(X_t)$ is linear. This simplifies the procedure of solving the model and makes the analysis more
transparent and tractable. The PSD obligation specifies a linear coupon scheme given by the function

\[ C_1(X_t) = c_1 - \gamma X_t c_1, \]  

where \( c_1 > 0 \) is the initial coupon payment, \( X_t \) is the current cash flow level, and \( \gamma \) is some \textit{ex ante} determined constant that governs the \textit{performance adjustment rate} of the contract. A large \( \gamma \) implies the PSD obligation is more performance sensitive. A \( \gamma = 0 \) is equal to regular fixed coupon debt.

I further assume that the firm is entitled to a tax benefit of debt equal to \( \tau C_1(X_t) \). This is the only reason for issuing debt in this model. However, issuing debt also introduce some bankruptcy costs \( \alpha \), assumed to be proportional to the all-equity firm value at default.

I assume that the manager’s and the equityholders’ incentives align, so that the manager chooses the investment and financing policy to maximize the market value of equity. Since the firm might have pre-existing debt \( (c_0 > 0) \), equity value maximization might not coincide with total firm value maximization. Throughout the paper I refer to the former case as the \textit{second-best} solution, and the latter case as the \textit{first-best} solution. Assuming that the investment policy is non-verifiable and hence non-contractible, the pre-existing debt generates a debt overhang problem which potentially distorts the investment and financing decision of the firm. Since creditors are rational and foresee this behavior they will price debt accordingly, meaning that equityholders eventually bear the costs of the suboptimal behavior. As in Leland (1994) equityholders optimally decide when to stop servicing debt and thereby go default. If default occurs, equityholders receive nothing and creditors receive the value of the firm’s assets net of bankruptcy costs. How the recovery value is split among creditors is determined by the priority structure, which in my model, is exogenously given.

Finally, I assume that agents are risk neutral and discount future cash flows at a constant risk-free rate \( r \). Throughout the paper I use the subscripts 0 and 1 to denote values \textit{before}
and after option exercise, respectively.

### 2.2 All-Equity Benchmark

If the firm is all-equity financed the total firm value $v^u_0(x, 0)$ is given by

$$v^u_0(x) = \mathbb{E} \left[ \int_0^\infty e^{-rt}(1 - \tau)X_t dt | X_0 = x \right] = \frac{x}{r - \mu}(1 - \tau). \quad (3)$$

I impose the usual restriction $r > \mu$.

The firm has an option to expand its operations by paying a fixed investment cost $I$. The increased production capacity from the option exercise increase the firm’s cash flows from $x$ to $Qx$, where $Q > 1$. In the case where the option is financed solely by equity the unlevered firm value $v^u_1(x)$ is given by

$$v^u_1(x) = \mathbb{E} \left[ \int_0^\infty e^{-rt}(1 - \tau)QX_t dt | X_0 = x \right] = \frac{Qx}{r - \mu}(1 - \tau) = Qv^u_0(x). \quad (4)$$

In the following sections I derive security and firm values before and after the exercise of the growth option. These values, furthermore, determine the first- and second-best investment and financing policies.

### 2.3 After Growth Option Exercise

Since the firm finance the growth option by issuing PSD, the coupon paid on the total debt is given by $c_0 + (c_1 - \gamma xc_1)$. The general solutions for the market values of equity and debt after exercising the growth option are

$$E_1(x) = (1 - \tau) \left[ \left( \frac{Qx}{r - \mu} - \frac{c_0 + c_1}{r} + \frac{c_1 \gamma x}{r - \mu} \right) \right] + e_1^a x^{\xi_1} + e_2^a x^{\xi_2}, \quad (5)$$

$$D^s_1(x) = \frac{c_0}{r} + d_1^a x^{\xi_1} + d_2^a x^{\xi_2}, \quad (6)$$
and

\[ D^n_1(x) = \frac{c_1}{r} \frac{c_1 \gamma x}{r - \mu} + d^n_{1n} x^{\xi_1} + d^n_{2n} x^{\xi_2}. \]  

(7)

Here \( x^d_t \) denotes the default threshold, and \( \xi_2 \) is negative root of the equation \( \frac{1}{2} \sigma^2 x(x - 1) + x \mu - r = 0 \). The constants \( c_1, c_2, d_1, d_2, d_1, d_2 \) are determined by the following boundary conditions:

\[
\begin{align*}
\lim_{x \to \infty} E_1(x) &= (1 - \tau) \left[ \left( \frac{Q}{r - \mu} - \frac{c_0 + c_1}{r} \right) + \frac{c_1 \gamma x}{r - \mu} \right], \\
E_1(x^d_t) &= 0, \\
\lim_{x \to \infty} D^s_1(x) &= \frac{c_0}{r}, \\
D^s_1(x^d_t) &= D^s_1(x^d_t), \\
\lim_{x \to \infty} D^n_1(x) &= \frac{c_1}{r} \frac{c_1 \gamma x}{r - \mu}, \\
D^n_1(x^d_t) &= D^n_1(x^d_t),
\end{align*}
\]

(8, 9, 10, 11, 12, 13)

Conditions (8), (10), and (12) are the usual ‘no-bubble’ conditions. Condition (9) states that at the default boundary \( x^d_t \) equity should have zero value, whereas conditions (11) and (13) are some general value matching conditions at the default boundary \( x^d_t \). When the firm defaults, debt seniority structure gives the recovery values for the first and the second debt, denoted by \( D^s_1(x^d_t) \) and \( D^n_1(x^d_t) \), respectively. The superscripts \( s \) and \( n \) refers to the seasoned and new debt issues, respectively. For \( x > x^d_t \) the value of equity \( E_1(x) \) is found to be

\[
E_1(x) = (1 - \tau) \left[ \left( \frac{Q}{r - \mu} - \frac{c_0 + c_1}{r} + \frac{c_1 \gamma x}{r - \mu} \right) - \left( \frac{Q x^d_t}{r - \mu} - \frac{c_0 + c_1}{r} + \frac{c_1 \gamma x^d_t}{r - \mu} \right) \right] \left( \frac{x}{x^d_t} \right)^{\xi_2}.
\]

(14)

As usual \( x^d_t \) is endogenously determined from the standard smooth-pasting condition

\[
\frac{\partial E_1}{\partial x} \bigg|_{x=x^d_t} = 0.
\]

(15)
Using (15) the optimal default boundary is given by

\[ x_1^d = \frac{(c_0 + c_1)(r - \mu)\xi_2}{r(Q + c_1 \gamma)(\xi_2 - 1)}. \]  

(16)

For \( x > x_1^d \) the market values of the two debt issues is given by

\[ D_s^1(x) = \frac{c_0}{r} - \left[ \frac{c_0}{r} - D_s^1(x_1^d) \right] \left( \frac{x}{x_1^d} \right)^{\xi_2}, \]  

(17)

\[ D_n^1(x) = \frac{c_1}{r} - \frac{c_1 \gamma x}{r - \mu} \left[ \frac{c_1}{r} - \frac{c_1 \gamma x}{r - \mu} - D_n^1(x_1^d) \right] \left( \frac{x}{x_1^d} \right)^{\xi_2}. \]  

(18)

The total debt value is \( D_1(x) = D_s^1(x) + D_n^1(x) \). Summing \( D_1(x) \) and \( E_1(x) \) gives the total levered firm value \( v_1(x) \):

\[ v_1(x) = \frac{Q x (1 - \tau)}{(r - \mu)} + \frac{\tau (c_0 + c_1)}{r} - \frac{\tau c_1 \gamma x}{r - \mu} \left[ \frac{Q x_1^d (1 - \tau)}{(r - \mu)} + \frac{\tau (c_0 + c_1)}{r} - \frac{\tau c_1 \gamma x}{r - \mu} \left( \frac{x}{x_1^d} \right)^{\xi_2}. \]  

(19)

Firm value \( v_1(x) \) is given by the unlevered firm value \( \frac{Q x (1 - \tau)}{(r - \mu)} \), plus the tax benefits of debt \( \frac{\tau (c_0 + c_1)}{r} - \frac{\tau c_1 \gamma x}{r - \mu} \), and minus the expected loss given default \[ \left[ \frac{Q x_1^d (1 - \tau)}{(r - \mu)} + \frac{\tau (c_0 + c_1)}{r} - \frac{\tau c_1 \gamma x}{r - \mu} \right] \left( \frac{x}{x_1^d} \right)^{\xi_2}. \]

For the later analysis I find it convenient to define the function \( v_1^n(x) \) as the sum of equity value \( E_1(x) \) and the second debt issue \( D_n^1(x) \). Using (14) and (18) \( v_1^n(x) \) is equal to

\[ v_1^n(x) = \frac{Q x (1 - \tau)}{(r - \mu)} + \frac{\tau (c_0 + c_1) - c_0}{r} - \frac{\tau c_1 \gamma x}{r - \mu} \left( D_n^1(x_1^d) - \frac{Q x (1 - \tau)}{(r - \mu)} - \frac{\tau (c_0 + c_1) - c_0}{r} + \frac{\tau c_1 \gamma x}{r - \mu} \right) \left( \frac{x}{x_1^d} \right)^{\xi_2}. \]  

(20)

The difference between \( v_1(x) \) and \( v_1^n(x) \) is crucial for my subsequent analysis. Equityholders no longer care about the payoffs to the seasoned debt, and so they choose the investment trigger point \( x_i \) and the optimal amount of debt \( C^*(X_i) = c_i^* - \gamma c_i^* x \) to maximize \( v_1^n(x) \) rather than \( v_1(x) \).
2.4 Before Growth Option Exercise

The general solutions for the market values of equity and debt prior to exercising the growth option are

\[
E_0(x) = (1 - \tau) \left( \frac{x}{r - \mu} - \frac{c_0}{r} \right) + e_1 x^{\xi_1} + e_2 x^{\xi_2},
\]

(21)

and

\[
D_0(x) = \frac{c_0}{r} + d_1 x^{\xi_1} + d_2 x^{\xi_2},
\]

(22)

where \( \xi_1 \) is the positive root of the equation \( \frac{1}{2} \sigma^2 x(x-1) + x \mu - r = 0 \). The constants \( e_1, e_2, d_1 \) and \( d_2 \) are determined by the following boundary conditions:

\[
E_0(x_0) = 0,
\]

(23)

\[
E_0(x_i) = E_1(x_i) - (I - D_1^a(x_i)),
\]

(24)

\[
D_0(x_0) = (1 - \alpha) \frac{x(1 - \tau)}{r - \mu},
\]

(25)

\[
D_0(x_i) = D_1^a(x_i).
\]

(26)

Condition (23) states that at the default boundary \( x_0^d \) equity should have zero value, whereas condition (24) is the value matching condition at the investment trigger \( x_i \). Similarly, conditions (25) and (26) are the value matching conditions for debt at the default boundary and the investment trigger point, respectively. The default boundary \( x_0^d \) is again optimally determined using the smooth pasting condition

\[
\frac{\partial E_0(x)}{\partial x} \bigg|_{x=x_0^d} = 0.
\]

(27)

It turns out that the equity value before exercising the growth option \( E_0(x) \) is given by

\[
E_0(x) = (1 - \tau) \left[ \frac{x}{r - \mu} - \frac{c_0}{r} \right] + A(x_i) \Sigma(x) + B(x_0^d) \Delta(x),
\]

(28)
where

\[
A = v_1^0(x_i) - I - (1 - \tau) \left( \frac{x_i}{r - \mu} - \frac{c_0}{r} \right),
\]

\[
B = (1 - \tau) \left( \frac{c_0}{r} - \frac{x_0^d}{r - \mu} \right),
\]

\[
\Psi(x) = \frac{(x_0^d)^{\xi_1} x^{\xi_2} - (x_0^d)^{\xi_2} x^{\xi_1}}{(x_0^d)^{\xi_2} (x_i)^{\xi_2} - (x_0^d)^{\xi_2} (x_i)^{\xi_1}},
\]

\[
\Delta(x) = \frac{x^{\xi_1} (x_i)^{\xi_2} - x^{\xi_2} (x_i)^{\xi_1}}{(x_0^d)^{\xi_2} (x_i)^{\xi_2} - (x_0^d)^{\xi_2} (x_i)^{\xi_1}}.
\]

Similarly, the value of debt before exercising the growth option \(D_0(x)\) is given by

\[
D_0(x) = \frac{c_0}{r} - \left( \frac{c_0}{r} - (1 - \alpha)(1 - \tau) \frac{x}{r - \mu} \right) \Delta(x) - \left[ \left( \frac{c_0}{r} - D_1^i(x_i^d) \right) \left( \frac{x}{x_1^d} \right)^{\xi_2} \right] \Psi(x) \quad (29)
\]

Summing \(E_0(x)\) and \(D_0(x)\) now gives us the total levered firm value before the growth option is exercised \(v_0(x)\):

\[
v_0(x) = (1 - \tau) \frac{x}{r - \mu} + \frac{\tau c_0}{r} + G(x_i)\Sigma(x) + H(x_0^d)\Delta(x),
\]

where

\[
G(x_i) = v_1(x_i) - I - \left( 1 - \tau \right) \left( \frac{x_i}{r - \mu} + \frac{\tau c_0}{r} \right),
\]

\[
H(x_0^d) = - \left( \frac{\tau c_0}{r} + \alpha(1 - \tau) \frac{x_0^d}{r - \mu} \right).
\]

The firm value \(v_0(x)\) is given by the sum of the unlevered firm \((1 - \tau)\frac{x}{r - \mu}\) plus the net gain of exercising the growth option \(G(x_i)\) multiplied by \(\Psi(x)\), the present value of receiving a unit payoff when the firm’s cash flow reaches the investment trigger point \(x_i\), minus the loss given default \(H(x_0^d)\) multiplied by \(\Delta(x)\), the present value of receiving a unit payoff when the firm goes bankrupt.
2.5 Option Exercise and Financing Policies

The first-best exercise and financing policy is to choose both the investment trigger point \( x_i \) and the optimal coupon \( c_1^* \) so that the total firm value is maximized, i.e., \( x_i \) and \( c_1^* \) is determined from the following optimality conditions:

\[
\frac{\partial v_0(x)}{\partial x} \bigg|_{x=x_i} = \frac{\partial v_1(x)}{\partial x} \bigg|_{x=x_i}, \quad (31)
\]

\[
\frac{\partial v_1(x)}{\partial c_1} = 0. \quad (32)
\]

I, furthermore, make the standard assumption that the first-best investment trigger point is incontractible, and so both debt and equity will be priced under the assumption that equityholders choose an equity-maximizing investment and financing strategy, i.e., \( x_i \) and \( c_1^* \) is chosen to maximize the sum of equity and the second debt issuance, rather than the sum of equity and total debt. I refer to this strategy as the second-best exercise and financing policy. \( x_i \) and \( c_1^* \) is now determined from the following optimality conditions:

\[
\frac{\partial E_0(x)}{\partial x} \bigg|_{x=x_i} = \frac{\partial v^n_1(x)}{\partial x} \bigg|_{x=x_i}, \quad (33)
\]

\[
\frac{\partial v^n_1(x)}{\partial c_1} = 0. \quad (34)
\]

As pointed out earlier the essential difference between the first-best policy and the second-best policy is that \( v^n_1(x) \) enters the right side of the optimality conditions (33) and (34) in the second best case, whereas the total firm value \( v_1(x) \) enters the optimality conditions (31) and (32) in the first best case.

2.6 Debt Priority Structure

If the firm chooses to finance the growth option by issuing debt, and if the firm already has existing debt, debt priority structure plays an important role. In the subsequent analysis I will consider the three most important situations:

- Existing debt is senior and new debt is junior (APR).
• Existing debt is junior and new debt is senior (APR).

• Existing debt and new debt has the same seniority (Pari Passu).

More formally I assume that if the equityholders declare bankruptcy the recovery value of the firm is simply a fraction of the un-levered firm value given in (4), i.e, \((1 - \alpha)\nu(x)\). The seniority structure further determines how this recovery value is split among creditors. If absolute priority is enforced and existing debt is senior to new debt, the recovery value of existing debt \(D_s(x_d^1)\) is given by \(\min\left(\frac{c_0}{r}, (1 - \alpha)v(x)\right)\), whereas the recovery value of new debt \(D_n(x_d^1)\) is given by \((1 - \alpha)v(x) - \min\left(\frac{c_0}{r}, (1 - \alpha)v(x)\right)\).

If existing debt is junior and new debt is senior the recovery value of new debt \(D_n(x_d^1)\) is given by \(\min\left(\frac{c_1}{r - \mu}, (1 - \alpha)v(x)\right)\), whereas the recovery value of existing debt \(D_s(x_d^1)\) is given by \((1 - \alpha)v(x) - \min\left(\frac{c_1}{r - \mu}, (1 - \alpha)v(x)\right)\).

In the case of equal seniority between existing and new debt the recovery value of existing debt \(D_s(x_d^1)\) is given by \((1 - \alpha)v(x) \times \kappa_s\), where \(\kappa_s = \frac{c_0}{c_0 + c_1 - c_1\gamma x_d^1}\) is the fraction of the total recovery value that existing creditors receive. Similarly, the recovery value of new debt \(D_n(x_d^1)\) is equal to \((1 - \alpha)v(x) \times \kappa_n\), where \(\kappa_n = 1 - \kappa_s\), is the fraction of the total recovery value that new creditors receive.

3 Solving the Model

3.1 No Debt Overhang

If the firm has no pre-existing debt, clearly \(x_0^d = 0\). The optimal default boundary \(x_d^d\) is given by

\[
x_d^d = \frac{c_1(r - \mu)\xi_2}{r(Q + c_1\gamma)(\xi_2 - 1)},
\]

(35)
which clearly is decreasing in $\gamma$. With no pre-existing debt there is no difference between the first-best and second-best financing and investment policies. The optimal investment trigger point $x_i$ is equal to

$$x_i = \frac{c_1(r - \mu)\xi_2}{r(Q + c_1\gamma)(\xi_2 - 1)} \left( \frac{1 - Q - \tau + Q\tau + c_1\gamma\tau}{Q\xi_2\tau(\alpha\tau - \tau - \alpha) + Q\tau + c_1\gamma\tau} \right)^{\frac{1}{\xi_2 - 1}}. \quad (36)$$

There is no closed form solution for the optimal coupon $c_1^*$, so this needs to be solved for by numerical methods. Doing so I normalize the starting value of the cash flow process to $x = 1$ and assume the following base case parameter values: $I = 10$, $Q = 2$, $r = 6\%$, $\sigma = 25\%$, $\tau = 15\%$, $\mu = 1\%$, $\alpha = 25\%$. Figure 1 plots the optimal leverage $D_1/(D_1 + E_1)$ for different values of the performance sensitivity parameter $\gamma$. The plot clearly shows that the optimal leverage is decreasing in $\gamma$, meaning that the risk of having to pay increased coupons in times when cash flow is low reduce the firm’s appetite of risky debt. This observation implies that firms which are initially capitalized with PSD, will reduce future conflicts related to debt overhang, since problems of debt overhang is increasing in initial leverage\(^4\). To make sure that this is a valid conclusion I examine how optimal leverage relates to changes in input parameters. Figure 2 clearly shows that optimal leverage is decreasing regardless of input parameter values, implying that my conclusion is robust.

### 3.2 Debt Overhang

With debt overhang equityholders have an incentive to deviate from first-best financing and investment policies, and to dilute existing creditors. This behavior represents a cost which may reduce the total value of the firm. Costs from such suboptimal decisions are typically referred to as agency costs of debt. Denote the total time zero market value of the levered firm when equityholders choose firm value maximizing financing and investment policies by $v_0^{FB}(x)$. Correspondingly the time zero market value of the levered firm when equityholders pursue equity maximizing financing and investment policies is denoted $v_0^{SB}(x)$. The total

agency costs of debt (AC) are then defined as:

\[ AC = \frac{v_0^{FB}(x) - v_0^{SB}(x)}{v_0^{FB}(x)}. \]

Since my focus is on how PSD affects investment and financing incentives I am interested in how the performance sensitivity parameter \( \lambda \) affects the total agency costs. If it is the case that using PSD financing reduces problems related to debt overhang and closes the gap between first-best and second-best policies, the agency costs will decrease for \( \lambda > 0 \), and PSD financing is a more efficient financing tool than fixed coupon debt. If it is the case that PSD financing worsens the problems related to debt overhang and increase the gap between first-best and second-best policies, the agency costs will increase for \( \lambda > 0 \), and PSD financing is inefficient compared to fixed coupon debt.

It is well known that debt overhang might lead to two different investment inefficiencies; underinvestment and asset substitution (risk-shifting). Whether debt overhang causes underinvestment or asset substitution is strongly related to the chosen priority structure. Un-
derinvestment occurs when existing creditors is sufficiently protected, meaning that they are almost sure to benefit from the proceeds resulting from the growth option exercise. Since equityholders pay the cost of exercising the option, and since they have to split the proceeds with existing creditors, debt overhang causes equityholders to postpone investment, i.e., the result is underinvesting. Equityholders’ incentives to engage in asset substitution
increase when initial creditors do not have sufficient protection or when it becomes to costly to postpone growth option exercise. Conveniently, the model framework I use, generates underinvestment when initial debt is senior to the new debt issue, whereas asset substitution is generated when both existing and new debt have equal priority, or when new debt is senior to existing debt. I illustrate this point by numerically solving the model with the following input parameters: $I = 10, Q = 2, r = 6\%, \sigma = 25\%, \tau = 15\%, \mu = 1\%, \lambda = 0, c_0 = 0.5$. The results are reported in Table 1. Here I report the first best investment trigger $x_i^{FB}$, the first best financing policy $c_1^{FB}$, the second best investment trigger $x_i^{SB}$, the second best financing policy $c_1^{SB}$, the first best firm value $v_0^{FB}$, the second best firm value $v_0^{SB}$ and the agency cost of debt (AC) for the different debt priority structures. When both types of debt have equal priority we see that equityholders risk-shift by investing at a threshold value of 1.18, which is lower than the first best investment threshold of 1.27. They also deviate from first best financing by taking on more additional debt, as seen by the coupon of 1.53 which is larger than the first best coupon of 0.91. The deviations from first best leads to a total value reduction of 0.27\%, which is quite small. If new debt is senior to existing debt we clearly see that the incentives to engage in risk shifting increase substantially. The investment threshold is now 1.03, with an optimal coupon of 1.85. The value reduction is now equal to 3.95\%, which is large. When initial debt has seniority above new debt we see that equityholders underinvest, as the investment threshold increases to 1.33, which is larger than the first best threshold of 1.27. Equityholders again deviate from first best financing by taking on less additional debt, as seen from the coupon of 0.57, which is smaller than the first best coupon of 0.91. The value reduction from the policy deviations is equal to 0.76\% in this case.

In Table 1 I assumed that the growth option was financed by issuing fixed coupon debt. Assume now that the growth option could be financed by issuing PSD. To get a feeling of how using PSD financing changes equityholders’ investment and financing incentives, Table 2 report the agency costs of debt for different values of the sensitivity parameter $\lambda$. From the numbers it is clear that in the cases where debt overhang causes equityholders to engage
<table>
<thead>
<tr>
<th>Priority Structure</th>
<th>( x_{i}^{FB} )</th>
<th>( c_{1}^{FB} )</th>
<th>( x_{i}^{SB} )</th>
<th>( c_{1}^{SB} )</th>
<th>( v_{0}^{FB} )</th>
<th>( v_{0}^{SB} )</th>
<th>AC (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal Priority</td>
<td>1.27</td>
<td>0.91</td>
<td>1.18</td>
<td>1.53</td>
<td>26.29</td>
<td>26.22</td>
<td>0.27</td>
</tr>
<tr>
<td>Initial Debt Senior</td>
<td>1.27</td>
<td>0.91</td>
<td>1.33</td>
<td>0.57</td>
<td>26.29</td>
<td>26.09</td>
<td>0.76</td>
</tr>
<tr>
<td>New Debt Senior</td>
<td>1.27</td>
<td>0.91</td>
<td>1.03</td>
<td>1.85</td>
<td>26.29</td>
<td>25.26</td>
<td>3.95</td>
</tr>
</tbody>
</table>

**Table 1:** Table reports the first best investment trigger \( x_{i}^{FB} \), the first best financing policy \( c_{1}^{FB} \), the second best investment trigger \( x_{i}^{SB} \), the second best financing policy \( c_{1}^{SB} \), the first best firm value \( v_{0}^{FB} \), the second best firm value \( v_{0}^{SB} \) and the agency cost of debt (AC) for different debt priority structures; equal priority, initial debt has seniority above new debt and new debt has seniority above initial debt. Input parameters are fixed at base case values: \( I = 10, Q = 2, r = 6\%, \sigma = 25\%, \tau = 15\%, \mu = 1\%, \lambda = 0, c_0 = 0.5 \) in asset substitution, using PSD financing only enhances the problem. With \( \lambda = 0.3 \) the agency costs of debt increase approximately 1.5 and 2.2 percentage points in the cases of equal priority and new debt being senior, respectively. In the case where debt overhang causes underinvestment PSD financing reduces the agency costs of debt by 0.6 percentage points. The reason for these results is that issuing debt that makes the firm pay higher coupons when cash flow is low lead the equity option faster and further out of the money, and, hence, equityholders have larger incentives to invest earlier rather than later. Table 2 also illustrates the main point in this paper, namely that PSD financing worsens the problem of asset substitution, but may partially resolve problems of underinvestment.

<table>
<thead>
<tr>
<th>Priority Structure</th>
<th>( \lambda = 0 )</th>
<th>( \lambda = 0.05 )</th>
<th>( \lambda = 0.1 )</th>
<th>( \lambda = 0.15 )</th>
<th>( \lambda = 0.2 )</th>
<th>( \lambda = 0.25 )</th>
<th>( \lambda = 0.3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal Priority</td>
<td>0.27</td>
<td>0.39</td>
<td>0.54</td>
<td>0.73</td>
<td>0.98</td>
<td>1.33</td>
<td>1.79</td>
</tr>
<tr>
<td>Initial Debt Senior</td>
<td>0.74</td>
<td>0.68</td>
<td>0.61</td>
<td>0.53</td>
<td>0.43</td>
<td>0.33</td>
<td>0.14</td>
</tr>
<tr>
<td>New Debt Senior</td>
<td>3.95</td>
<td>4.12</td>
<td>4.36</td>
<td>4.66</td>
<td>5.05</td>
<td>5.54</td>
<td>6.18</td>
</tr>
</tbody>
</table>

**Table 2:** Table reports the agency costs of debt related to deviations from first best investment and financing policies for different values of the performance sensitivity parameter \( \lambda \). The increasing agency costs in row 2 and 4 indicates that PSD financing worsens the problem of asset substitution, whereas the decreasing agency costs in row 3 indicates that PSD financing partially resolves problems related to underinvestment.

### 3.3 Variation of Input Parameters

As I showed in subsection 3.2, equal priority or letting the new debt have seniority above existing debt both generated problems of asset substitution. When I examine how my results
are affected by changes in input parameters I focus only on the case where all creditors have equal priority\(^5\), or where existing debt is senior to new debt.

Figure 3 plots the agency costs of debt (AC) as a function of the performance sensitivity parameter \(\lambda\) under the assumption that existing debt and new debt have equal priority. It clearly shows that the agency costs of debt is monotonously increasing for all different parameters used, except for the drift parameter \(\mu\), where the agency costs starts to decline when \(\lambda\) becomes sufficiently large. The agency costs are, however, still smallest for \(\lambda = 0\). These results ensure that the conclusions made in section 3.2, that PSD financing worsens problems related to debt overhang, is valid for any input parameter values, and hence remains very robust.

Figure 4 also plots the agency costs of debt (AC) as a function of the performance sensitivity parameter \(\lambda\), but now under the assumption that initial debt has priority above new debt, i.e., the focus is on underinvestment. It shows that the agency costs of debt is monotonously decreasing in \(\mu, \tau\) and \(\alpha\). When the initial debt is high \(c_0 = 1\), we see that the cost of postponing investment is too big, and equityholders have incentives to risk-shift, leading agency costs of debt to increase with \(\lambda\). Also for large values of \(\sigma\) we see that having a too aggressive performance sensitivity parameter might lead equityholders to risk-shift. For low invest cost \(I\) equityholders also have incentives to invest early, rendering PSD inefficient in this case. The effect of changing the growth option component \(Q\) is also not unambiguous, but the valuation effects are so close to zero that these are negligible. Over all the analysis supports the conclusion that PSD financing reduce problems related to underinvestment.

\(^5\)In the case of new debt being senior, the graphs show exactly the same patterns as for equal priority. They are available upon request.
4 Conclusion

I examine interactions between investment and financing decisions using a dynamic model where a firm has assets in place, and an option to expand operations. My model allows for the possibility that the firm’s initial capital structure might consist of both debt and equity, and also the possibility that the growth option is financed by issuing performance sensitive debt (PSD). I specifically address the question whether PSD financing could solve inefficiencies related to asset substitution and underinvestment.

With no pre-existing debt I show that any firm would have lower optimal leverage when using PSD, compared to using regular fixed coupon debt. This observation suggests that firms which are initially capitalized by PSD would have less future problems related to debt overhang, since such problems is increasing in initial leverage.

With pre-existing debt my model clearly illustrates that PSD financing increases equity-holders’ incentives to engage in asset substitution, and that PSD is inefficient compared to fixed coupon debt in these cases. This conclusion questions the hypothesis that PSD is used to prevent asset substitution. Instead, the analysis suggests that PSD partially reduce agency costs related to underinvestment.
References


Figure 3: This figure plots the agency costs of debt as a function of the performance sensitivity parameter $\lambda$, for different input parameter values. It assumes that the creditors have equal priority in bankruptcy. When changing one parameter all others are kept at their base case values: $I = 10, Q = 2, r = 6\%, \sigma = 25\%, \tau = 15\%, \mu = 1\%, c_0 = 0.5$. 
Figure 4: This figure plots the agency costs of debt as a function of the performance sensitivity parameter $\lambda$, for different input parameter values. It assumes that existing debt has seniority in bankruptcy. When changing one parameter all others are kept at their base case values: $I = 10, Q = 2, r = 6\%, \sigma = 25\%, \tau = 15\%, \mu = 1\%, c_0 = 0.5$.