Incentive provision when contracting is costly

BY

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Abstract

We analyze optimal incentive contracts in a model where the probability of court enforcement is determined by the costs spent on contracting. We show that contract costs matter for incentive provision, both in static spot contracts and repeated game relational contracts. We find that social surplus may be higher under costly relational contracting than under costless verifiable contracting, and show that there is not a monotonic relationship between contracting costs and incentive intensity. In particular we show that an increase in contracting costs may lead to higher-powered incentives. Moreover we formulate hypotheses about the relationship between legal systems and incentive provision, specifically the model predicts higher-powered incentives in common law than in civil law systems.

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1 Introduction

Costly contracting and measurement problems are textbook explanations for why employment contracts often lack explicit statements regarding performance-related pay. Paul Milgrom and John Roberts (1992) state that (p. 330) "the incompleteness (...) and the shape of the employment contract are all responses to the impossibility of complete contracting. (...) Briefly, they involve the difficulties of foreseeing all the events that might possibly arise over time (...) the difficulties of unambiguously describing these events (...) and the costs of negotiating acceptable explicit agreements over these many terms even if they could be described"

Despite this insight, analyses of the relationship between contracting costs and the shape of the employment contract are scarce. In particular, we know little about the relationship between contracting costs and incentive intensity, except that contracting costs are generally regarded as an impediment to incentive pay.

In this paper we analyze optimal incentive provision in a simple principal-agent model with unobservable effort and costly contracting. We assume that the principal can write an incentive contract that specifies the desired quality of the agent’s output and contingent bonuses if the quality requirements are delivered. But writing such contracts are costly for the principal. Moreover, we assume that the probability that the incentive contract will be enforced by a court of law is determined by the costs spent on contracting. In particular we assume that contracting increases the probability that the court can verify the quality of the agent’s output and thus that the court can verify whether or not the principal has fulfilled her bonus obligations.\footnote{In the aftermath of the 2008 financial crisis, legal disputes about bonus payments have not been uncommon. As a recent example, seventy-two city bankers are suing Dresdner Kleinwort and Commerzbank for €33m ($47.8m) worth of unpaid bonuses in the biggest case of its kind in the UK, see Financial Times, September 8, 2009.} Due to incomplete legal enforcement, we also allow the parties to engage in relational contracting. A
relational contract relies on self-enforcement and is modelled as a repeated game between the parties.

We report on the following results: First, social surplus may be higher under costly contracting and imperfect enforcement than under costless contracting and perfect enforcement. If the parties have limited liability, then a (stationary) relational incentive contract may be higher-powered (and leave more rent to the agent) than a perfectly verifiable contract.

Second, there is not a monotonic relationship between contracting costs and incentive intensity. Since contracting costs are used to explain the lack of incentive pay, one might expect that higher contracting costs reduce the level of incentive pay. However, we show that an increase in contracting costs may in fact lead to higher-powered incentives.

Third, optimal incentive pay depends crucially on the shape of the contract cost function. And since the shape of this function is partly determined by the legal system, we can formulate hypotheses about the relationship between legal systems and optimal incentive pay. In particular, we argue that the model predicts higher-powered incentives in common law than in civil law systems.

Forth, we find that higher trust in the relationship i.e., better conditions for relational contracting, does not necessarily lead to higher-powered incentives. Rather, we find that higher trust, proxied by the discount factor in the repeated game, may lead to lower-powered incentives.

Related literature: Starting with the seminal papers of Townsend (1979) and Dye (1985), costly contracting and imperfect enforcement is increasingly recognized as an important vehicle to understand the nature of transactional relationships. One strand focuses on the formation of incomplete contracts (Anderlini and Felli, 1999), Battigalli and Maggi (2002) and Shavell (2006), while others focus on contract design problems, in particular the tension/trade-off between ex ante contract specifications and ex post renegotiations (Chakravarty and MacLeod (2009), Bajari and Tadelis (2001),
Our approach differs in that we analyze a model where ex ante contracting affects the court’s ability to verify whether the parties have fulfilled their contract obligations. This approach relates to Doornik (2010) who analyze a model where contracting affects the level of expected enforcement costs, and thus the probability of ending up in court. It also relates to Ishiguro (2002) and Bull and Watson (2004), who endogenize the probability of verification, but who unlike us consider ex post actions such as evidence disclosure instead of ex ante contracting.

In repeated game models of relational contracting, the common assumption is that verifiability is exogenously given, and that contracting is costless or prohibitively costly. In the first models dealing with the interaction between formal and informal contracting, such as Schmidt and Schnitzer (1995) and Baker, Gibbons and Murphy (1994), the level of contracting costs does not matter. Recently, however, Sobel (2006), MacLeod (2007), Battigalli and Maggi (2008) and Kvaløy and Olsen (2009) have introduced models where contracting costs influence the interaction between legal enforcement and relational contracting. But these papers assume symmetric information and do not deal with incentive problems due to unobservable effort and moral hazard.

The main contribution of the paper is to examine costly contracting and endogenous verifiability in an otherwise standard moral hazard model. In the classic moral hazard models (e.g. Holmström, 1979), perfect enforcement is assumed, while in models of incomplete contracting, it is commonly assumed that contracting is prohibitively costly so that legal enforcement is impossible (starting with Grossman and Hart, 1986). Moreover, the large majority of

\footnote{Schmidt and Schnitzer (1995) and Baker, Gibbons and Murphy (1994) analyze models with both verifiable and non-verifiable variables, but the verifiability of a given action or signal is exogenously given. Other models that address the relationship between verifiable and non-verifiable variables are Bernheim and Whinston (1998) and Pearce and Stacchetti (1998).}

\footnote{Our set up is closest to Kvaløy and Olsen (2009) who analyze a model where ex ante contracting-level affect verifiability.
models dealing with incomplete and/or relational contracting have generally focused on environments where the parties have symmetric information (see in particular MacLeod and Malcomson, 1989). Notable exceptions are Baker, Gibbons and Murphy (1994, 2002), MacLeod (2003), Fuchs (2007) and in particular Levin (2003) who makes a definite treatment of relational contracts with asymmetric information. But neither of these papers open for costly contracting and probabilistic enforcement, like we do. A reason for this gap in the literature might be that first best incentives can be achieved (under risk neutrality) if there is a positive probability of court enforcement, and if sufficiently large payments are feasible and enforceable. To make the model interesting and closer to reality, we thus adopt the assumption from Innes’ (1990) that the principal is financially (and legally) constrained and cannot offer wages above the value of output.

Our approach is then to extend the model in Kvaløy and Olsen (2009) to a situation with asymmetric information in terms of unobservable effort. We show how this extension to some extent complicates the analysis of the relationship between costly formal contracting and relational contracting. But we also show that the main qualitative results from the symmetric model apply to standard incentive problems with moral hazard.

The remainder of the paper is organized as follows: Section 2 presents the model and characterizes optimal contracts. In Sections 3 and 4 we analyze how optimal incentives in the relational contract varies with contracting costs and the discount factor, respectively. Section 5 concludes. Proofs not explicitly stated in the text are contained in an appendix.

2 Model

We consider a relationship between two risk neutral parties, a principal and an agent, where the agent produces either high ($q_H$) or low ($q_L$) value for the principal. The probability of producing $q_H$ depends on the agent’s effort, and
is for simplicity given by the effort level: $e = \text{prob}(q_H)$. Effort costs are given by $C(e)$, where $C'(e) > 0$, $C''(e) > 0$, $C(0) = 0$. We assume that output is observable to both parties, but that the agent’s effort level is unobservable to the principal, so the parties must contract on output: the principal pays a fixed salary $s$, and a contingent bonus $\beta_i$, $i \in (H, L)$ if the agent delivers quality $q_i$.

We assume that the agent is protected by limited liability, and hence that the fixed salary as well as net payments must be non-negative ($s \geq 0$ and $s + \beta_i \geq 0$). Note that this allows the contract to specify a ‘punishment’ in terms of a negative bonus for, say, bad performance ($\beta_L < 0$).

Following Kvaløy and Olsen (2009) we assume that there is a probability $v \in [0, 1)$ that the contracted quality can be verified.\(^4\) We follow the standard assumption from incomplete contract theory saying that if the variables in a contract are non-verifiable, then the contract is not enforceable by a court of law. Hence, the probability of verification, $v$, can thus be interpreted as the probability of legal enforcement of the bonus contract, $\beta_i$. If the court verifies quality, it can verify whether or not the parties have fulfilled their obligations regarding the contracted bonus payments.

The probability $v$ is assumed to depend on the level of contracting: the more the parties invest in specifying contract terms, the higher is the probability that the court can verify the realized quality. We let $K(v)$ be the cost that must be incurred to achieve verifiability level $v$, and we interpret $K$ as the costs associated with writing explicit contracts specifying the quality of the agent’s output.

To keep the model simple, we assume that values accrue directly to the principal in the process of production, so that the agent cannot hold up values ex post. The model then best describes situations where the agent provides ongoing services like consulting, maintenance, IT services, HR ser-

\(^4\)By not allowing for $v = 1$, we assume that perfect verifiability is prohibitively costly. This is in line with the standard assumption ($v = 0$) in the relational contract literature.
services, administrative services etc.

We analyze a repeated relationship where the following stage game ($\Gamma$) is played each period:

1. The principal makes an investment $K(v)$ in writing a contract with verifiability level $v$, where $v$ is common knowledge, and offers a contract $(s, \beta_L, \beta_H)$ to the agent. If the agent rejects the offer, the game ends. If he accepts, the game continues to stage 2.

2. The agent takes action $e$ and quality $q_i$ is realized.

3. The parties observe $q_i$. The principal is obligated to pay the fixed salary $s$, and then the parties choose whether or not to honor the contingent bonus contract $\beta_i$. The decision to honor or deviate (offer $\beta_i' \neq \beta_i$) belongs to the principal if $\beta_i > 0$ and to the agent if $\beta_i < 0$.

4. The parties choose whether or not to go to court. If at least one party goes to court and the court verifies quality, it rules according to a breach remedy that is ex ante common knowledge. If no party goes to court, or if the court does not verify quality, the agent and the principal obtain payoffs $s + \beta_i' - C(e)$ and $q_i - s - \beta_i' - K(v)$, respectively.

A spot contract is taken to be a perfect public equilibrium (PPE) of this stage game. We deduce the optimal spot contract below applying a standard breach remedy. We then move on to analyze the infinite repetition of the stage game $\Gamma$. A relational contract between the parties describes a PPE of this infinitely repeated game.

With respect to the breach remedy, we assume that the parties apply expectation damages (ED), which entail that the breacher has to compensate the victim so as to make her equally well off as under contract performance. ED is the most typical remedy, and is also regarded as the most efficient one in the seminal literature on optimal breach remedies (Steven Shavell,
1980; and William P. Rogerson, 1984). Given (UCC §2-718 (1987) and RESTATEMENT (SECOND) OF CONTRACTS §356, which prevents courts from enforcing terms stipulating damages that exceed the actual harm, no party-designed damage rule can do better than expectation damages in our model.

2.1 The spot contract

Our interpretation of the breach remedy ED is as follows: If the court verifies insufficient payments, it rules that the breacher is to comply with his/her part of the contract and pay $\beta'_i = \beta_i$ as specified in the contract. If the court verifies that the breacher has more than fulfilled the contract terms, it takes no action.

By backwards induction we start with stage 4, where the players simultaneously and independently choose whether to accept $\beta'_i$ or to go to court. If at least one player does not accept, but rather goes to court, the payoffs are given by the procedures defined above.

One sees that the court is avoided in stage 4 if and only if the parties have adhered to the contract. If $\beta_i > 0$ and the principal has deviated by offering $\beta'_i < \beta_i$, the agent is worse off accepting than taking the case to court (because the expected payment in court is here $v\beta_i + (1-v)\beta'_i > \beta'_i$). Similarly, if $\beta_i < 0$ (but $s + \beta_i \geq 0$) and the agent has deviated by offering to pay back less ($\beta'_i > \beta_i$), the principal will go to court in stage 4. Given these responses, we see that the party making the decision in stage 3 will optimally deviate from the contract and offer $\beta'_i = 0$, because his/her expected outlay in court will then be minimal and equal to $v\beta_i < \beta_i$.

In stage 2, the agent’s expected payoff will now be $s + v(\beta_L + e\Delta\beta) - C(e)$, where $\Delta\beta = \beta_H - \beta_L$. He will choose effort to maximize this payoff, which

\footnote{The principal will never offer $\beta'_i > \beta_i$ in this game.}
gives IC and participation (IR) constraints as follows

\[ v \Delta \beta = C'(e) \]  
\[ s + v(\beta_L + e\Delta \beta) - C(e) \geq 0, \]

where we have assumed that his reservation payoff is zero.

Without further constraints, the principal would in stage 1 then maximize her payoff \( q_L + e\Delta q - (s + v(\beta_L + e\Delta \beta)) - K(v) \) subject to IC and IR. Note that first best effort, given by \( \Delta q = C'(e) \), can be achieved with a bonus \( \Delta \beta = \frac{\Delta q}{v} \). With no restrictions on bonuses, the principal could then obtain the first best allocation asymptotically by increasing \( \Delta \beta \) and letting \( v \) and thus \( K(v) \) go to zero (assuming \( K(0) = 0 \)).

But as we have argued above, arbitrarily large bonuses are not realistic. Assume now restrictions on \( \beta_i \) such that

\[ s + \beta_i \leq q_i \]

The motivation behind this constraint is twofold. One is limited liability: the principal cannot commit to pay wages above the agent’s value added. This constraint resembles Innes (1990) who in a financial contracting setting assumes that the investor’s (principal’s) liability is limited to her investment in the agent. The other source relates to legal practice. Enforcing a payment \( s + \beta_i > q_i \) is equivalent to a breach remedy that stipulates damages that exceed the actual harm. And as noted, legal practice prevents courts from enforcing such rules.

Finally we have the constraints arising from the agent’s limited liability. As explained above these are

\[ s \geq 0 \quad \text{and} \quad s + \beta_i \geq 0 \]

Consider now the principal’s problem. Note that the agent’s participation
constraint (IR) will not bind, since \( s + v\beta_L \geq v(s + \beta_L) \geq 0 \) by LL, and the agent’s payoff therefore (by IC) will be no less than \( ve\Delta\beta - C'(e) = eC'(e) - C(e) > 0 \). (The inequality follows by strict convexity of \( C(e) \).) The agent will thus get a rent. The rent is costly to the principal, and it follows (from IR) that she will optimally choose \( s + v\beta_L = 0 \) and therefore \( s + \beta_L = 0 \). This implies that the constraint on bonuses (BR) is fulfilled for \( i = L \) (assuming \( q_L \geq 0 \)), and takes the form \( \Delta\beta \leq q_H \) for \( i = H \).

Substituting from IC we now see that the principal obtains a payoff given by \( q_L + e\Delta q - eC'(e) - K(v) \), and that she is subject to a bonus constraint (BR) that is equivalent to \( C'(e) = v\Delta\beta \leq vq_H \). Since no effort will be exerted if \( v = 0 \), the principal will invest in contract specifications if marginal and fixed contracting costs \( (K'(0) \) and \( K(0) \)) are not too large. Assuming this is the case we obtain the following

**Proposition 1** The spot equilibrium entails a contract \((s, \beta, v)\) with \( v > 0 \), \( \beta_L = s = 0 \) and \( \beta_H = \Delta\beta = q_H \), yielding effort less than the first best level \( (e = e_s < e_F) \) and given by

\[
\max_{e,v} [q_L + e\Delta q - eC'(e) - K(v)] \quad \text{s.t.} \quad C'(e) = vq_H
\]

The agent gets a rent \( u_A = e_s C'(e_s) - C(e_s) > 0 \). In equilibrium the principal deviates from the contract and pays no bonus if high output is realized. The case then ends in court, where the contracted bonus is enforced if quality is verified.

The constraint \( C'(e) \leq vq_H \) must clearly bind, otherwise \( v \) could be reduced and the payoff thereby increased. This implies that spot effort is also smaller than the effort level that is optimal for verifiable output and limited liability for the agent, i.e. the effort level that would be optimal if complete verifiability \( (v = 1) \) were costless for the principal \( (K(1) = 0) \). In that case the principal’s payoff would be \( q_L + e\Delta q - eC'(e) \) and the optimal
effort would be given by

\[ e^c = \arg \max_e \left[ q_L + e\Delta q - eC'(e) \right]. \] (1)

The principal’s bonus constraint \((C'(e) \leq q_H)\) would clearly not bind here (since the optimal effort ignoring this constraint satisfies \(C'(e^c) < \Delta q \leq q_H\)). Since on the other hand the constraint is binding when verifiability is costly, the effort levels corresponding to costly and free verifiability clearly satisfy \(e_s < e^c\).

### 2.2 Relational contract

Since verifiability is costly, and it is uncertain whether a legal court is able to enforce the contract, the parties may also rely on self-enforcement. Through repeated transactions the parties can make it costly for each other to breach the contract, by letting breach ruin future trade.

A self-enforcing relational contract is a perfect public equilibrium of the infinitely repeated game where the stage game \(\Gamma\) is played every period. In long-term relationships, ongoing investments in contract modifications are common. But contract modifications do not necessarily imply that equilibrium \(v\) is changed. In fact, we consider stationary contracts where the same verification equilibrium \(v\) and output \((q_L, q_H)\) is realized every period. Such a case arises when e.g., new technological developments or market demands imply that the content of \((q_L, q_H)\) changes, but the costs required to produce the object of value \((q_L, q_H)\), or the verification level \(v\), do not change. Then contract modifications are required even if costs \(C(q)\) and \(K(v)\) are unaffected.\(^6\)

We consider stationary trigger strategies, where the parties revert to the equilibrium of the stage game forever if a party deviated from the contract in

\(^6\)It can be shown that whether such costs are incurred every period, or just prior to the first stage game, is not crucial for the results we obtain.
any history of play. The conditions for implementing a *relational incentive contract* is then satisfied if the parties honor the contract for both high and low output $q_i, i \in \{L, H\}$.

First note that under a relational contract equilibrium, the agent trusts the principal to honor the contract, and hence chooses effort according to

$$\Delta \beta = C'(e) \quad \text{(ICR)}$$

Now, the principal will honor the contract if the net present value from honoring is greater than the net present value from reneging. This holds iff

$$q_i - s - \beta_i - K(v) + \frac{\delta}{1 - \delta} \pi_P \geq q_i - s - K(v) - \max\{v\beta_i, 0\} + \frac{\delta}{1 - \delta} u_p, \quad i = L, H, \quad \text{(EP)}$$

where $\delta$ is a common discount factor, $u_p$ is the principal’s spot payoff and $\pi_P = q_L + e\Delta q - K(v) - s - \beta_L - e\Delta \beta$ is the payoff (per period) under relational contracting. The RHS of the inequality captures the principal’s payoff after her two possible deviations. First, if $\beta_i > 0$ and the principal reneges on the bonus payment (and then optimally offers $\beta_i' = 0$), the agent will go to court, where he obtains $v\beta_i$, and he will then insist on spot contracting forever after. Second, if $\beta_i < 0$ (but $s + \beta_i \geq 0$, which may occur for $i = L$) the principal may renege by not accepting the payment from the agent, in which case there will also be spot contracting forever after.

Participation for the agent requires

$$\pi_A = s + \beta_L + e\Delta \beta - C(e) \geq u_A \quad \text{(IRR)}$$

where $u_A$ is the agents’s payoff in the spot contract. The enforceability constraints (EA) for the relational contract pertaining to the agent are

$$s + \beta_i + \frac{\delta}{1 - \delta} \pi_A \geq s + \min\{0, v\beta_i\} + \frac{\delta}{1 - \delta} u_A, \quad i = L, H, \quad \text{(EA)}$$

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If $\beta_i > 0$ the agent must be no worse off accepting than refusing the offered bonus payment. If $\beta_i < 0$ (but $s + \beta_i \geq 0$) he must be no worse off accepting to 'pay back' the specified 'bonus' rather than refuse and be taken to court, where he will in expectation obtain $s + v\beta_i$.

Since $\beta_H > \beta_L$ to provide incentives, the relevant enforceability constraints will be the EA constraint corresponding to $\beta_L$ for the agent, and the EP constraint corresponding $\beta_H$ for the principal. In addition we have BR-constraints $s + \beta_i \leq q_i, i = L, H$.

The optimal relational contract for the principal maximizes her (period) payoff $\pi_P = q_L + e\Delta q - K(v) - s - \beta_L - e\Delta \beta$ subject to all constraints.\(^7\) As shown in the appendix, it is optimal that EA binds (for $\beta_L$), and that consequently the relational enforceability conditions are equivalent to the following condition:

$$
\frac{\delta}{1 - \delta} \left[ q_L + e\Delta q - K(v) - C(e) - u \right] \geq C'(e)(1 - v), \quad (EC)
$$

where $u = u_A + u_P$ is the total spot surplus. The RHS here is the largest one-period gain that can be obtained by deviating from the bonus contract, namely $\Delta \beta(1 - v)$, where $\Delta \beta = C'(e)$ by ICR. The LHS is the future total loss incurred when the relational contract is broken. The condition says that, to deter deviations, this loss must be no smaller than the total temptation to deviate.

Since $s + \beta_L \geq 0$ by limited liability for the agent, we see from IRR and ICR that he will get a rent at least equal to $eC'(e) - C(e) > 0$. This exceeds the agent’s spot payoff $u_A$ if effort exceeds spot effort ($e > e_s$). In such a case IRR will clearly not bind, hence $s + \beta_L = 0$, and the principal will maximize her payoff $q_L + e\Delta q - K(v) - eC'(e)$ i.e. total surplus minus the agent’s

\(^7\)We consider only stationary contracts, which is not restrictive when parties are risk neutral and there is no limited liability (Levin, 2003). When parties have limited liability, stationary contracts are not necessarily optimal. Fong and Li (2009) show, however, that under limited liability the optimal contract reaches a stationary equilibrium after a "probation phase".
rent—subject to EC. Since social surplus also depends on contracting costs, we may have lower effort in the relational contract equilibrium than in the spot equilibrium even if the relational surplus exceeds spot surplus. If this is the case \((e < e_s)\) then clearly IRR will bind and \(s + \beta_L > 0\) (assuming \(q_L > 0\)), implying that the principal’s payoff will be \(q_L + e\Delta q - K(v) - C(e) - u_A\). She then maximizes this payoff subject to EC.

There are thus two cases, depending on whether the participation constraint IRR for the agent binds or not. We obtain the following result.

**Proposition 2** For given \(\Delta q\), there is \(q'_L > 0\) such that for \(q_L \geq q'_L\) we have:

(i) If the relational contract yields effort exceeding the spot level \((e > e_s)\), then \(s + \beta_L = 0\), IRR is not binding, and \((e, v)\) solves

\[
\max_{e,v} [q_L + e\Delta q - K(v) - eC'(e)] \quad \text{s.t.} \quad \text{EC}
\]

(ii) If the relational contract yields effort smaller than the spot level \((e < e_s)\), then \(s + \beta_L > 0\), IRR is binding, and \((e, v)\) solves

\[
\max_{e,v} [q_L + e\Delta q - K(v) - C(e) - u_A] \quad \text{s.t.} \quad \text{EC}
\]

We are here not primarily interested in a complete characterization of the optimal relational contract for all parameter values, but rather in being able to say something about how the contract will change in response to certain parameter changes. For this purpose the above characterization is sufficient.

Consider now the optimal relational contract. Such a contract can obviously not yield a higher payoff than the principal’s maximal payoff when output is verifiable (and the agent is protected by limited liability), i.e. the payoff given in (1). But we see that this payoff is attainable if the associated constraint IRR

\footnote{The BR constraints will typically not bind since \(s + \beta_L = 0 \leq q_L\) and \(s + \beta_H = \Delta \beta \leq \Delta q \leq q_H\) due to \(\Delta \beta = C'(e)\) and effort being no larger than first-best effort.}
optimal effort $e^c$ is implementable (satisfies the EC constraint) with zero contract investment ($v = 0 = K(0)$). From EC we see that this is indeed feasible only if the discount factor is sufficiently large ($\delta > \delta^c$ for some $\delta^c < 1$). For lower $\delta$ the payoff must be lower, but it cannot be smaller than the spot payoff. It is clear that for a range of discount factors (in $(0, \delta^c)$) the optimal relational contract will have higher payoff for the principal than the spot contract, and entail an interior solution ($0 < v < 1, e > 0$) if the contract cost function $K(v)$ has sufficiently small marginal and absolute (fixed) costs at $v = 0$.

It is worth noting that the optimal contract may have $e > e^c$, and thus entail a level of effort that exceeds the effort level that is optimal for verifiable output. In the appendix we show the following result.

**Proposition 3** For a class of cost functions (including quadratic ones) and parameters the following holds: There is an interval of discount factors $(\delta', \delta^c)$ such that for $\delta$ in this interval the relational contract entails positive contract investment ($v > 0$) and a level of effort that exceeds the level that is optimal for verifiable output ($e > e^c$).

An example illustrating such an outcome is depicted in Figure 1.
Constraint curves (convex) corresponding to two levels of \( \delta \),
and indifference curves (concave) for the principal.

The figure shows two (convex) 'constraint curves' that delineate the set of
implementable contracts \((e, v)\), i.e. contracts satisfying EC, for two levels of
the discount factor \( \delta \). These contracts are here in the regions north-east of the
respective curves. The other (concave) curves in the figure are indifference
curves for the principal, drawn for \( e > e^c = 0.5 \) in this example.\(^9\) The
outer (green) curve corresponds to the principal’s payoff being equal to her
spot surplus, the inner (blue) curve corresponds to a higher surplus. (The
'bliss point' for the principal is \( e = e^c, v = 0 \).) A higher \( \delta \) enlarges the
set of implementable contracts, and is illustrated by the shift (leftwards) of
the constraint curve in the figure. For the higher \( \delta \) the optimal relational
contract is defined by tangency of the constraint and indifference curves. For
the lower \( \delta \) there is no relational contract that yields a higher payoff to the
principal than her spot payoff.

\(^9\)The example has \( C(e) = e^2 / 2, K(v) = kv^2 / 2 \) and \( \Delta q = 1, q_H = 9 / 8, k = 1 / 4 \).
If the principal were constrained only by the agent’s limited liability, she would have chosen $e = e^c < e^{FB}$. Effort would be lower than first best because she must leave rents to the agent. But a contract with such an effort level may yield a relatively low social surplus. By increasing the level of effort she can increase the social surplus and hence ease implementation of the contract. This is the reason why it may optimal for the principal to choose a relational contract with $e > e^c$.

The analysis in this section shows that the optimal contract and hence the optimal bonus, $\Delta \beta$, will depend crucially on the form of the contract cost function, also under relational contracting. In the next sections we examine how the optimal bonus varies with the cost function $K(v)$, and the level of trust, represented by $\delta$.

3 Contract costs and optimal incentives

The necessary cost to achieve a given probability of legal enforcement will depend on the complexity of the transactions and the quality of the performance measures, as well as the strength of enforcement institutions and the practice of legal courts. We will in this section point out two relationships between contract costs and optimal incentives that we find particularly intriguing, one regarding the cost level and one regarding the form of the cost function.

First, we address the cost level issue. Since contracting costs are used to explain the lack of incentive pay, one might expect that higher contracting costs reduce the level of incentive pay. However, we can show that an increase in contracting costs may actually lead to higher-powered incentives. To show this we consider a function $K(v, \kappa)$ with $K_{\kappa} \geq 0$, and examine how incentives and effort vary with the parameter $\kappa$. It turns out that the elasticity of the marginal cost function is an important determinant for how variations in $\kappa$ affect incentive provision. This elasticity can be expressed as $(1 - v)K_{\kappa v}/K_v$, 17
since this expression measures the relative increase in marginal costs per percentage reduction in the probability of non-verification \((1 - v)\). We find the following.

**Proposition 4** Given a relational contract equilibrium \((v^*, e^*)\), consider a cost variation that leaves marginal costs unaltered at \(v^*\) \((K_{\text{vr}}(v^*, \kappa) = 0)\).

(i) If costs increase more at the relational equilibrium \(v = v^*\) than at the spot equilibrium \(v = v^s\) (so that \(K_r(v^*, \kappa) > K_r(v^s, \kappa))\), this will lead to higher-powered incentives iff the following condition holds:

\[
(1 - v)K_{\text{vr}}/K_v + \gamma \frac{\delta e^*}{1 - \delta} K_{\text{vr}}/K_v < 1
\]

where \(\gamma = 0\) if \(e^* < e_s\) and \(\gamma = 1\) if \(e^* > e_s\). In the former case the condition holds iff the marginal contract cost function is inelastic, in the sense that \((1 - v)K_{\text{vr}}/K_v < 1\).

(ii) If costs increase less at the relational equilibrium \(v = v^*\) than at the spot equilibrium \(v = v^s\) (so that \(K_r(v^*, \kappa) < K_r(v^s, \kappa)\)), the cost increase will lead to higher-powered incentives iff the opposite condition holds.

The proposition demonstrates that endogenous contracting costs and the opportunities for the parties to engage in relational contracting create a non-trivial relationship between contracting costs and incentive intensity. Under plausible assumptions an increase in contracting costs may lead to higher-powered incentives. Part (i) of the proposition is the most striking one since it shows that incentive intensity in the relational contract may increase even if contracting costs increase more at the relational equilibrium than at the spot equilibrium. Part (ii) of the proposition complements insights from previous literature (e.g. Baker, Gibbons and Murphy, 1994), that less attractive outside options (worse spot contracts) may benefit the relational contract.

An increase in the cost to achieve a given verifiability level can stem from higher job complexity. The costs associated with describing a job’s tasks and operational performance metrics are likely to be higher the more
complex the job is. The result in Proposition 2 then says that, under certain conditions, higher job complexity may generate higher-powered incentives. The intuition is that higher job complexity may lead the parties to increase the level of contracting such that the probability of verification increases. This in turn makes the parties able to implement higher-powered incentives. Interestingly, higher-powered incentives are more common in human capital intensive industries (see e.g. Long and Shields, 2005 and Barth et al, 2008), and one reason may be that knowledge-intensive jobs require more detailed contracts.

Consider now the form of the cost function. As argued in Kvaløy and Olsen (2009), the form of the cost function $K()$ may depend on the legal system. Differences in $K()$ may pertain to differences in contract enforcement between common law and civil law systems. The common law system is assumed to be more willing to enforce specific contract terms than civil law, which to a larger extent set party-designed contract terms aside if it conflicts with the civil codes. This indicates that the marginal effect on $v$ of investing in detailed contracts is higher in common law (see Djankov et al, 2003). On the other hand the civil codes assure that a minimum level of verifiability can be achieved at relatively low costs. This suggests that $K()$ as a function of $v$ will tend to be flatter, but have a higher intercept in common law compared to a civil law system. It further suggests that we may interpret a marginal change where $K'_v(v^*, \kappa) > 0$ and $K_v(v^*, \kappa) = 0$ as a marginal move from common to civil law practice. Interestingly, we find such a move from common law to civil law lead to lower-powered incentives. Formally,

**Proposition 5** Given a relational contract equilibrium $(v^*, e^*)$, consider a cost variation that (i) increases marginal costs at $v^*$ ($K_{vc}(v^*, \kappa) > 0$) and (ii) leaves absolute costs unaltered at $v = v^*$ and at the spot equilibrium $v = v^*$ (so that $K_v(v^*, \kappa) = K_v(v^*, \kappa) = 0$). This variation, which implies increased marginal contracting costs for the given $v^*$, will lead to lower-powered incentives.
Interestingly, empirical studies indicate a higher frequency of performance related pay in central common law countries like U.S., U.K. and Australia than in civil law countries such as France (see Brown and Heywood, 2002 for an international comparison on performance pay). In order to test our hypothesis, one could look at the relationship between performance pay and judicial formalism, as indexed by Djankov et al (2003), but unfortunately, one still lacks good international data on performance related pay.

4 Trust and optimal incentives

The discount factor can be seen as a proxy for trust, see e.g. Hart (2001), since in a repeated relationship between P and A, if A knows that P has a high discount factor, A knows that P values future trade with A. Hence, A trusts P and P is trustworthy. In this sense, the repeated game approach formalizes an economic concept of trust and trustworthiness.\(^\text{10}\) A common feature of the relational incentive contracts studied in the literature is that incentive intensity is positively related to the parties’ trust in the relationship, i.e. their discount factors. The higher the discount factor, the higher is the present value of the ongoing relationship relative to the present value of reneging on the contract. When this ‘punishment’ from reneging increases, the parties are able implement higher-powered incentives without running the risk of opportunism (see Levin, 2003).

We will here show that this relationship does not generally hold when the principal can invest in contracting in order to increase the probability of legal enforcement. We find,

\textbf{Proposition 6} Higher trust (higher \(\delta\)) leads to lower-powered incentives if\(f\)
\[(1 - v)K_{vv}/K_v + \gamma \frac{\delta e^s}{1-\delta} (K_{vv}/K_v - h(e^*, v, \delta)) \right\} < 1, \] where \( \gamma = 0 \) if \( e^* < e^s \) and \( \gamma = 1 \) if \( e^* > e^s \), and \( h(e, v, \delta) = \frac{1}{(1-\delta)} (1 - \frac{\delta}{1-\delta} \frac{K_v}{C_v^0}) > 0. \)

The intuition behind the proposition is that the parties realize the surplus from higher trust by reducing contracting costs, instead of by increasing the incentive intensity. We see that the elasticity of \( K_v() \) is important also here. The response in \( v \) to a change in \( \delta \) is larger, the less elastic is \( K_v() \). When \( K_v() \) is inelastic, the standard result that higher \( \delta \) leads to higher-powered incentives does not necessarily hold, since a higher \( \delta \) can make it optimal to reduce \( v \) so much that the principal finds it profitable to also reduce incentive provision.

5 Conclusion

In this paper we have endogenized the probability of legal enforcement in an otherwise standard moral hazard model with limited liability. We have assumed that the probability of contract enforcement is determined by the level of ex ante (costly) contracting, and have analyzed both a static and repeated game version of the model.

The main message from the paper is that contract costs matter for incentive provision, both in the static spot contract and in the repeated relational contract. Interestingly, there is not a monotonic relationship between contracting costs and incentive intensity. We show that if the marginal contract costs are inelastic, an increase in contracting costs may lead to higher-powered incentives. Moreover, we find that social surplus may be higher under costly relational contracting than under costless verifiable contracting.

Since the shape of the contract cost function is partly determined by the legal system, we can also formulate hypotheses about the relationship between legal systems and incentive provision. Specifically, we argue that the model predicts higher-powered incentives in common law than in civil law systems. Empirical studies indicate higher frequency of performance related
pay in common law countries than in civil law countries, but one needs better international data on performance pay in order to test this hypothesis.

Our paper (together with Kvaløy and Olsen, 2009) offers a simple framework that is well suited for analyzing the relationship between trust-based informal contracts and legal institutions. The model can be extended to incorporate other legal variables such as litigation costs and alternative breach remedies. Variations of this framework could also be applied to other topics where repeated games and legal institutions are important, such as optimal firm boundaries, public versus private ownership, and the sustainability of cartels and collusive agreements.
Appendix

Proof of Proposition 2

We first show that the enforceability conditions EA and EP can be replaced by EC. Given that $\beta_H > \beta_L$ is necessary to provide incentives, the relevant EP constraint will be the one corresponding to $\beta_H$, with $\beta_H \geq 0$. The constraint can then be written as

$$\frac{\delta}{1-\delta} [q_L + e\Delta q - K(v) - s - \beta_L - e\Delta \beta - u_P] \geq \beta_H(1 - v)$$

(2)

Given $\beta_H > \beta_L$, the relevant EA constraint will be the one corresponding to $\beta_L$, which can be written as

$$\beta_L - v \min\{0, \beta_L\} + \frac{\delta}{1-\delta} [s + \beta_L + e\Delta \beta - C(e) - u_A] \geq 0$$

(3)

If (3) doesn’t bind, then $\beta_L$ can be reduced, keeping $s + \beta_L$ and $\Delta \beta$ fixed, without violating any constraints. This will strictly relax (2), and then $v$ can be reduced, increasing the payoff $\pi_P$. Hence it is optimal to have (3) binding, and thus $\beta_L \leq 0$ by IRR. Substituting for $\beta_L(1 - v)$ from (3) and for $\Delta \beta$ from ICR into (2), we then see that the relational enforceability conditions are equivalent to condition EC.

To prove the proposition, note first that the agent’s spot payoff is

$$u_A = e_s C'(e_s) - C(e_s)$$

(4)

where spot effort satisfies $e_s < e^c = \arg \max_e [e\Delta q - eC'(e)]$, and hence $u_A < Q \equiv e^c C'(e^c) - C(e^c)$. Note that $e^c$ is, for fixed $\Delta q$ independent of $q_L$, and so is consequently $Q$.

Define $s_L = s + \beta_L$, and write EC as $G(e, v, \delta, u) \geq 0$ by defining

$$G(e, v, \delta, u) = q_L + e\Delta q - C(e) - K(v) - \frac{1-\delta}{\delta} C'(e)(1-v) - u$$

(5)
Substituting for $\Delta \beta = C'(e)$ from ICR and ignoring the agent’s LL constraints for the moment, the Lagrangean for the principal’s optimization problem can be written as

$$l = q_L + e\Delta q - K(v) - s_L - eC''(e) + \mu (s_L + eC''(e) - C(e) - u_A) + \lambda G(e, v, \delta, u) + \eta (q_L - s_L) + \varphi (q_H - s_L - C'(e))$$

Here $\mu, \lambda, \eta, \varphi$ are (non-negative) multipliers on the IRR, EC and (two) BR constraints, respectively.

The LL constraints are $s_L = s + \beta_L \geq 0$ and $s = s_L - \beta_L \geq 0$. Since we showed above that $\beta_L \leq 0$, the relevant LL constraint is $s_L \geq 0$. The optimality conditions include

$$\frac{\partial l}{\partial s_L} = -1 + \mu - \eta - \varphi \leq 0, \quad s_L \geq 0, \quad \text{(compl. slack)}$$

$$\frac{\partial l}{\partial e} = \Delta q - C'(e) - eC''(e)(1 - \mu) + \lambda G_e - \varphi C''(e) = 0$$

$$\frac{\partial l}{\partial v} = -K'(v) + \lambda G_v = 0$$

If $e > e_s$ then IRR doesn’t bind and hence $\mu = 0$ and $s_L = 0$. (IRR binding would yield $s_L < 0$, which is impossible.) Since $s_L = 0 < q_L$ we have $\eta = 0$. If now BR binds for $q_H$ we have $C'(e) = q_H \geq \Delta q$. This implies $G_e < 0$ and consequently $\frac{\partial l}{\partial e} < 0$, which is a contradiction. Hence we have $s_L = 0$ and all other constraints except EC being slack. This proves statement (i) in the proposition.

If $e < e_s$ then IRR requires $s_L > 0$, which implies $\mu = 1 + \eta + \varphi > 0$ (so IRR binds). If now BR binds for $q_H$ then $C'(e) = q_H - s_L \geq q_H - q_L = \Delta q$. This implies that effort exceeds first-best effort, which contradicts $e < e_s$. Hence BR for $q_H$ is slack and $\varphi = 0$.

Assuming $q_L \geq Q$ where $Q$ was defined in the paragraph following (4),
we can see that the BR constraint for \( q_L \) must also be slack. If not, we would have \( s_L = q_L \) and thus from IRR \( q_L + eC'(e) - C(e) = u_A \), which is impossible when \( q_L \geq Q > u_A \). Hence all constraints except IRR and EC must be slack in this case, and this proves statement (ii) in the proposition.

**Proof of Proposition 3.**

Consider the EC constraint for \( v = 0 \), given by by \( G(e,0,\delta) \geq 0 \), see (5). (To save notation we ignore the dependence on \( u \) here.) Let \( \delta^c \) be the minimal \( \delta \) for which \( v = 0, e = e^c \) can be implemented; it is given by

\[
G(e^c,0;\delta^c) = e^c \Delta q - C(e^c) - \tilde{u} - \frac{1 - \delta^c}{\delta^c} C'(e^c) = 0
\]  

(6)

where \( \tilde{u} \) denotes the spot surplus in excess of \( q_L \), i.e. \( \tilde{u} = u - q_L = e_s \Delta q - C(e_s) - K(v_s) \). Since \( e_s < e^c < e^{FB} \), we see that \( \delta^c \) is well defined.

Assuming \( C''(e) \geq 0 \) the function \( G(e,0;\delta) \) is strictly concave in \( e \), and it satisfies \( G(e,0;\delta) < 0 \) for \( e = 0 \) and for \( e \) sufficiently large. For given \( \delta \) the equation \( G(e,0;\delta) = 0 \) has thus generically two or none solutions for \( e \). If \( G_e(e^c,0;\delta^c) > 0 \), then \( G(e,0;\delta^c) > 0 \) for all \( e \) in some interval \( (e^c,e'_c) \), hence all these \( e \) can be implemented for \( v = 0 \) and \( \delta = \delta^c \). Since \( G_{\delta} > 0 \), there is then by continuity a \( \delta' < \delta_c \) such that for \( \delta \in (\delta',\delta_c) \) we have \( G(e,0;\delta) > 0 \) for all \( e \) in some interval \( (e_\delta,e'_\delta) \) with \( e'_\delta > e_\delta > e^c \), and \( G_e(e_\delta,0;\delta) > 0 \). For given such \( \delta \) all \( e \) in this interval can be implemented with \( v = 0 \). We may assume (if necessary by choosing \( \delta' \) closer to \( \delta^c \)) that the principal’s payoff at \( e = e_\delta, v = 0 \) exceeds his spot payoff.

Assume now \( G_e(e^c,0;\delta^c) > 0 \) (we verify that this is feasible below). Let \( \delta \in (\delta',\delta_c) \) and let \( e_\delta > e^c \) be the minimal effort that can be implemented with \( v = 0 \). We have here \( G_e(e_\delta,0;\delta) > 0 \), and since \( G_e(e,0;\delta) = -K'(0) + \frac{1 - \delta}{\delta} C'(e) > 0 \) for \( K'(0) = 0 \) (and \( e < e^{FB} \)) the feasible set defined by \( G(e,v;\delta) \geq 0 \) is here delineated by a curve \( v(e) \) with \( v(e_\delta) = 0 \) and slope \( \frac{dv}{de} = -\frac{G_e}{G_v} < 0 \).
An indifference curve for the principal (given by \( e \Delta q - eC'(e) - K(v) = \text{const} \)) has slope \( \frac{dv}{de} = -\frac{\Delta q - C'(e) - eC''}{K'(v)} < 0 \) for \( e > e^c \). Since this slope is infinite at \( v = 0 \) for \( K'(0) = 0 \), while the slope of the constraint curve is finite at this point, the principal is better off with some \( v > 0, e < e_\delta \) than with \( v = 0, e = e_\delta \), and hence better off than with any implementable \((e, v)\) with \( v = 0 \). It remains to show that the principal’s optimal \((e, v)\) has \( e > e^c \).

For quadratic cost functions the function \( G(e; v; \delta) \) is a quadratic form in \((e; v)\), and the feasible set defined by \( G(e; v; \delta) \geq 0 \) is then delineated by a curve that is either a parabola, a hyperbola or an ellipse. Since for the given \( \delta \) this curve intersects the \( e \)-axis at two points \((e_\delta, e_0)\) and has a negative slope at \( e_\delta \), the curve must be tangent to an indifference curve at some point \((e, v)\) with \( e > e^c \). (Indifference curves have slope equal to zero for \( e = e^c \).) This point of tangency is optimal, and satisfies \( e > e^c \) and \( v > 0 \).

It remains to verify that the assumption \( G_e(e^c, 0; \delta^c) > 0 \) can hold. Note that by definition of \( e^c \) (as \( \arg \max [e \Delta q - eC'(e)] \)) we have \( \Delta q - C'(e^c) - e^cC''(e^c) = 0 \) and hence

\[
G_e(e^c, 0; \delta^c) = \Delta q - C'(e^c) - \frac{1 - \delta^c}{\delta^c}C''(e^c) = (e^c - \frac{1 - \delta^c}{\delta^c})C''(e^c)
\]

Thus \( G_e(e^c, 0; \delta^c) > 0 \) if \( \delta^c \) is sufficiently large (\( \delta^c > \frac{1}{1+e^c} \)). By definition of \( \delta^c \) (see (5)) this will be the case if \( e^c \Delta q - C(e^c) - \bar{u} \) is sufficiently small, i.e. if the spot surplus \( \bar{u} = e_s \Delta q - C(e_s) - K(v_s) \) is sufficiently close to \( e^c \Delta q - C(e^c) \). This will hold e.g. if \( q_H \) is sufficiently large, since then \( v_s = C'(e_s)/q_H \) is small (and consequently \( K(v_s) \) is small, assuming e.g. \( K(0) = 0 \) and \( e_s \) is close to \( e^c \)). This shows that \( G_e(e^c, 0; \delta^c) > 0 \) for some parameter specifications, and thus completes the proof.

**Proof of Propositions 4 and 5.**

To simplify notation, we set \( q_L = 0 \) in this proof. Define \( f(e, v, \kappa) \) as the total per period surplus, and \( g(e) = eC''(e) - C'(e) \). It follows from Proposition 2 that the variables in the relational contract solve the following

\[ v_s = C'(e_s)/q_H \]
problem

\[
\max_{e,v} \left( f(e, v, \kappa) + \gamma g(e) \right) = e\Delta q - C(e) - K(v, \kappa) + \gamma [eC'(e) - C(e)] \quad \text{s.t.} \quad \text{EC} \quad \tag{7}
\]

where \( \gamma = 0 \) if \( e < e_s \), and \( \gamma = 1 \) if \( e > e_s \). Comparative statics (for local variations) can then be derived from this problem.\(^{11}\)

Note that the EC constraint can be written as (see (5))

\[
G(e, v, \kappa, \delta) = f(e, v, \kappa) - H(e, v, \kappa, \delta) \geq 0, \quad \text{where} \quad H(e, v, \kappa, \delta) = \frac{1 - \delta}{\delta} C'(e)(1 - v) + u(\kappa)
\]

Here \( u(\kappa) \) denotes the spot surplus, and we note from Proposition 1 that \( u'(\kappa) = -K_s(v^s, \kappa) \), where \( v^s \) is the equilibrium spot verification probability.

Let \( L = (f + \gamma g) + \lambda G \) be the Lagrangean for problem (7). Given sufficient second order conditions (SOC), standard comparative statics yield

\[
e'(\kappa) = \frac{1}{D} \left( [L_{\nu\nu} G_{e} - L_{\nu v} G_{v}] G_{\kappa} + [L_{\nu \kappa} G_{v} - L_{\nu \kappa} G_{e}] G_{v} \right), \quad \tag{8}
\]

where \( D > 0 \) is the determinant of the bordered Hessian of \( L \). (For completeness this is verified at the end of this proof).

Note that from \( L = (f + \gamma g) + \lambda G \), \( G = f - H \) and the first-order conditions (FOCs) \( f_k + \gamma g_k = -\lambda G_k \), \( k = e, v \), we have

\[
G_k L_{ij} = G_k (f_{ij} + \gamma g_{ij}) + G \lambda G_{ij}
\]

\[
= (f_k - H_k)(f_{ij} + \gamma g_{ij}) - (f_k + \gamma g_k)(f_{ij} - H_{ij})
\]

\[
= f_k H_{ij} - H_k f_{ij} + \gamma (G_k g_{ij} - g_k G_{ij})
\]

\(^{11}\)As stated the objective is not continuous at \( e = e_s \), but this can be amended by subtracting \( (1 - \gamma)u_A \) from \( f + \gamma g \). This has no bearing on the comparative statics formulae.
Substituting this in for $e'(\kappa)$ yields

$$e'(\kappa)D = [(f_eH_{vv} - H_vf_{vv}) + \gamma(G_ev_v - g_eG_{vv}) - (f_vH_{ev} - H_vf_{ev}) - \gamma(G_vg_{ev} - g_vG_{ev})]G_\kappa$$

$$+ [(f_eH_{ek} - H_vf_{ek}) + \gamma(G_ev_{ek} - g_eG_{ek}) - (f_vH_{ek} - H_vf_{ek}) - \gamma(G_vg_{ek} - g_vG_{ek})]G_v$$

Using $g_v = g_\kappa = 0$, $G_{v_j} = f_{v_j} - H_{v_j}$ and $H_{vv} = 0$ then yields

$$e'(\kappa)D = [-H_e f_{vv} - \gamma g_e f_{ev} - (f_v H_{ev} - H_v f_{ev})] G_\kappa$$

$$+ [(f_e H_{ek} - H_v f_{ek}) - (f_v H_{ek} - H_v f_{ek}) + \gamma g_e (f_v - H_v f_{ek})] G_v$$

Substituting the partials of $f(e, v, \kappa) = c\Delta q - C(e) - K(v, \kappa)$ and $H(e, v, \kappa) = \frac{1 - \delta}{\delta} C'(e)(1 - v) + u(\kappa)$ in (9), and noting that $f_{ev} = f_{ek} = H_{ek} = H_{ek} = 0$, we obtain

$$e'(\kappa)D = [H_e K_{vv} + \gamma g_e K_{vv} + K_v H_{ev}] [f_\kappa - H_\kappa] + [-H_e K_{vk} - \gamma g_e K_{vk}] G_v$$

$$= [H_e K_{vv} + \gamma g_e K_{vv} + K_v H_{ev}] [-K_\kappa - u'(\kappa)] - [H_e + \gamma g_e] K_{vk} G_v$$

$$= \left\{ \left[ -(1 - v) K_{vv} + K_v + \frac{\gamma g_e K_{vv}}{H_{ev}} \right] [K_\kappa + u'(\kappa)] - \left[ (1 - v) + \frac{\gamma g_e}{(-H_{ev})} \right] K_{vk} G_v \right\} (-H_{ev})$$

where the last equality follows from $H_e/H_{ev} = -(1 - v)$.

Since $H_{ev} = -\frac{1 - \delta}{\delta} C'' < 0$ and $g_e = e C''$ we then see that $e'(\kappa)$ has the same sign as

$$\left[ -(1 - v) K_{vv} - \gamma \frac{\delta e}{1 - \delta} K_{vv} + K_v \right] [K_\kappa + u'(\kappa)] - \left[ (1 - v) + \gamma \frac{\delta e}{1 - \delta} K_{vv} \right] K_{vk} G_v$$

Note that FOC implies $G_v = -f_v/\lambda = K_v/\lambda > 0$.

In Proposition 4 we have (i) $K_{vk} = 0$ and (ii) $K_\kappa > K_\kappa(v^*, \kappa) = -u'(\kappa)$, hence we see that $e'(\kappa)$ has the same sign as $-(1 - v)K_{vv} - \gamma \frac{\delta e}{1 - \delta} K_{vv} + K_v$.

From ICR ($\Delta \beta = C'(e)$) it follows that effort and incentives covary, and this proves the proposition.

In Proposition 5 we have (i) $K_{vk} > 0$ and (ii) $K_\kappa = K_\kappa(v^*, \kappa) = \frac{28}{\lambda}$. 

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In Proposition 5 we have (i) $K_{vk} > 0$ and (ii) $K_\kappa = K_\kappa(v^*, \kappa) = \frac{28}{\lambda}$.
$-u'(\kappa)$, and since $G_v > 0$ as noted above, we have from (10) that $e'(\kappa)$ has the same sign as $-K_{\kappa\kappa}$. This proves Proposition 4.

For completeness we finally verify the standard comparative statics formula (8). Differentiation of the FOCs ($L_e = L_v = G = 0$) yields

$$
\begin{bmatrix}
L_{ee} & L_{ev} & G_e \\
L_{ve} & L_{vv} & G_v \\
G_e & G_v & 0
\end{bmatrix}
\begin{bmatrix}
e'(\kappa) \\
v'(\kappa) \\
\lambda'(\kappa)
\end{bmatrix}
= 
\begin{bmatrix}
-L_{\kappa\kappa} \\
-L_{\kappa v} \\
-G_{\kappa}
\end{bmatrix}
$$

(11)

and hence

$$
e'(\kappa) = \frac{1}{D} \begin{vmatrix}
-L_{ee} & L_{ev} & G_e \\
-L_{ve} & L_{vv} & G_v \\
-G_{\kappa} & G_v & 0
\end{vmatrix}
= \frac{1}{D} \left[ L_{vv} G_e G_{\kappa} + L_{\kappa\kappa} G_v^2 - L_{\kappa v} G_v G_{\kappa} - L_{\kappa v} G_e G_v \right]
$$

where $D$ is the determinant of the Hessian in (11). The sufficient SOC for this problem is $D > 0$. (see e.g. Intriligator, M.D. (1981) Mathematical programming with applications to economics, Ch. 2 in Arrow and Intriligator (eds.) Handbook of Mathematical Economics, North Holland.). This verifies (8) and completes the proof.

**Proof of Proposition 6**

Applying the comparative statics formula (9) to variations wrt $\delta$, and noting that $f_{ev} = f_{\delta e} = f_{i\delta} = 0$, we obtain

$$
e'(\delta)D = \left[ -H_e f_{vv} - \gamma g_e f_{vv} - (f_v H_{ev} - H_v f_{ev}) \right] G_{\delta}
$$

(12)

$$
+ \left[ (f_v H_{\delta e} - H_v f_{\delta e}) - (f_e H_{v\delta} - H_e f_{v\delta}) + \gamma g_e (f_{v\delta} - H_{v\delta}) \right] G_v
$$

$$
= \left[ H_e K_{vv} + \gamma g_e K_{vv} + K_v H_{ev} \right] G_{\delta} + \left[ f_v H_{\delta e} - (f_e + \gamma g_e) H_{v\delta} \right] G_v
$$

As above (in the previous proof) the first term in the last line can be written
as

\[
[H_v K_{vv} + \gamma g_v K_{vv} + K_v H_{ev}] = \left[ - (1 - v) K_{vv} - \gamma \frac{\delta e}{1 - \delta} K_{vv} + K_v \right] H_{ev} \tag{13}
\]

where \( H_{ev} = -\frac{1 - \delta}{\delta} C'' < 0 \).

Next note that the FOCs \((f_k + \gamma g_k = -\lambda G_k)\) imply \((f_e + \gamma g_e)/f_v = G_e/G_v\), and that \(G = f - H\) then implies

\[
H_e/H_v = -\gamma g_e/H_v + (f_e + \gamma g_e - G_e)/(f_v - G_v) = -\gamma g_e/H_v + G_e/G_v
\]

Noting that \(H_{e\delta}/H_{v\delta} = H_e/H_v\), we see that the last parenthesis in (12) can be written as

\[
[f_e H_{e\delta} - (f_e + \gamma g_e) H_{v\delta}] = [H_e/H_v - G_e/G_v] f_v H_{v\delta} = [\gamma g_e/H_v] K_v H_{v\delta} \tag{14}
\]

From (12, 13, 14) we then have

\[
e' (\delta) D = \left[ - (1 - v) K_{vv} - \gamma \frac{\delta e}{1 - \delta} K_{vv} + K_v \right] H_{ev} G_\delta + \gamma g_v K_v (H_{v\delta}/H_v) G_v = \left\{ \left[ (1 - v) K_{vv} + \gamma \frac{\delta e}{1 - \delta} K_{vv} - K_v \right] + \gamma \frac{g_v}{-H_{ev}} K_v^2 H_{v\delta}/H_v \right\} (-H_{ev}) G_\delta
\]

We have \(G_\delta = -H_\delta\) so \(H_{v\delta}/G_\delta = -H_{v\delta}/H_\delta = (1 - v)^{-1}\). Since \(H_{ev} = -\frac{1 - \delta}{\delta} C'' < 0\) and \(G_\delta > 0\) we see that \(e'(\delta)\) has the same sign as

\[
\left[ (1 - v) K_{vv} + \gamma \frac{\delta e}{1 - \delta} K_{vv} - K_v \right] + \gamma \frac{g_v}{H_{ev}} K_v^2 (1 - v) H_v
\]

To verify the claim in the proposition, it remains to show that \(-G_v/H_v (1 - v) = h(e, v, \delta) > 0\), i.e. that

\[
-\frac{G_v}{H_v} = 1 - \frac{\delta}{1 - \delta} C'(e) > 0 \tag{15}
\]
We have $G_v/H_v = (f_v - H_v)/H_v = -K_v/H_v - 1$ and $H_v = -\frac{1-\delta}{\delta} C'(v)$. This proves the equality in (15). The inequality follows from FOC $G_v = -f_v/\lambda = K_v/\lambda > 0$ and $H_v < 0$. This completes the proof.

References


