Discussion paper

Media Firm Strategy and Advertising Taxes

BY
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Abstract

Empirical evidence suggests that people dislike ads in TV programs and other media products. In such situations standard economic theory prescribes that the advertising volume can be optimally reduced by levying a tax on ads. However, making use of recent advances in the theory of firm behavior in two-sided markets, we show that taxation of ads may be counterproductive. In particular, we identify a number of situations in which ad-adverse consumers are negatively affected by the tax, and we even show that the tax may lead to higher ad volumes. This unorthodox reaction to a tax may arise when consumers significantly dislike ads, i.e. in situations where traditional arguments for corrective taxes are strongest.

Keywords: Two-sided markets, media market, pricing strategy, taxation.

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1 Introduction

Media industries such as radio, TV, internet, newspapers, and magazines are major drivers in popular culture, and they take up the lion’s share of peoples’ leisure time. The average American, for example, watches over four hours of TV per day, whilst European viewers on average spend close to 3 hours and thirty minutes in front of their television sets.\(^1\) It is also a fact that most media firms rely partly or fully on advertising to provide funding for their business activities. However, empirical evidence suggests that people dislike ads in media products, at least on the margin. This has prompted worries about possible excessive advertising from the society’s point of view, and has lead European countries to restrict the amount of TV commercials.\(^2\) US states have also in the past imposed a tax on advertising in printed media, whilst a tax on ads based on a nuisance argument has been voiced in New Zealand (Allen et al., 2002).\(^3\)

It is surprising, given the importance media products play in people’s lives and the controversy over the use of advertising, that there hardly exists any formal analysis of how taxes on ads affect managerial behavior. Do managers respond to a tax on ads by increasing the price of ad inverts? If they do, conventional wisdom indicates that the content of ads should fall. Such a policy, therefore, seems well directed. Managers in media firms, however, have the complex task of serving

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\(^1\)See Anderson and Gabszewicz (2006) for further empirical documentation of media usage.
\(^2\)It is well documented that viewers try to avoid advertising breaks on TV, see Moriarty and Everett (1994), Danaher (1995), and Wilbur (2008). For printed newspapers there are some indications that the extent to which people consider commercials as bad varies across countries (Gabszewicz et al., 2004).
\(^3\)For a review of the continuing discussion of introducing taxes on ads in US states see, e.g., ANA (2005) and the webpage by the American Advertising Federation (AAF): http://www.aaf.org/ – government affairs.
different customers groups at the same time. In particular, they sell their products to two customer groups; advertisers and consumers.\footnote{Evans (2003) defines a two-sided market as one where we have (a) two distinct groups of customers, (b) positive network externalities (at least from one of the customer groups to the other), and (c) an intermediary that internalizes the externalities between the groups. See Rochet and Tirole (2006) for a more formal definition.} This two-sidedness in their business model suggests certain important trade-offs, which after taking into account the externalities that may exists between customer groups are less clear cut.

To see how the two-sideness affect management decisions, take the example of a newspaper that is financed partly by readers and partly by ad inlets. Advertisers naturally prefer a large readership making it optimal for the newspaper to set low subscription fees in order to increase the number of readers, since more readers allows the firm to derive higher advertising revenue. The newspaper, however, must take into account that readers might dislike ads, at least on the margin. If so, there are negative externalities from advertisers to readers. A newspaper that derives a substantial part of its revenue from the reader side of the market, will - if it behaves in an optimal way - have a low advertising volume compared to freesheets. If not, the willingness to pay for the newspaper will be excessively low. If, in contrast, readers perceive ads as a good even on the margin, a profit maximizing media firm will sell more advertising space than the quantity that maximizes advertising revenue, since the larger the advertising volume, the higher the readers’ willingness to pay for the newspaper. Arguably this might be the case for some specialized magazines, but does not seem to hold for the media industry in general. We shall therefore focus on the case where the public perceives ads as a nuisance, though our formal analysis also allows us to consider the case of ad-lovers.

Standard tax theory prescribes a corrective tax on a good that imposes negative externalities. We show, however, that even if ads produce negative externalities (readers dislike ads), taxing ads may not correct the externality. The reason is that a tax on ads reduces the profitability of selling eyeballs to the advertising market.
This means that a newspaper has less incentives to attract a large readership through low subscription fees. Introducing a tax on ads, therefore, is likely to hurt readers because the higher subscription fee may outweigh the benefit of viewing less ads. Rather surprisingly, it might not even be optimal for media firms to reduce the advertising volume. We show that the advertising volume actually may go up. To see why, note that the lower profitability of selling eyeballs to the advertising market discourages the media firms from attracting a large audience through a small advertising volume. If this effect is sufficiently strong, the advertising volume is (locally) increasing in the tax rate. Interestingly, this is most likely to be the case if the audience strongly dislikes ads, i.e. in a situation where the traditional arguments for imposing a corrective tax are strongest.

Our work is related to the recent development of theories of firm behavior in two-sided markets - see for instance Anderson and Coate (2005), Armstrong (2006), Caillaud and Jullien (2001, 2003), Crampes et al. (2009), Gabszewicz et al. (2002), and the review by Rochet and Tirole (2006). The focus of these contributions is how the two-sidedness of markets influences the pricing decision of firms. The effects of taxation are masked out in these papers. Kind et al. (2008) discuss the issue of taxation in two-sided markets but do not consider a tax on ads. Allen et al. (2002), in contrast, consider a tax on advertising, but resort to a one-sided market structure.5

The paper proceeds as follows: Section 2 introduces the model of a two-sided media market, followed by an analysis of the effects of ad taxes in section 3. Section 4 summarizes the results and offers some concluding remarks.

2 The model

We consider a firm which sells a media product - labelled newspaper (good $N$), for simplicity - to consumers at price $p^N$ and ad space (good $A$) to producers at

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5See Fullerton and Metcalf, 2002 for a survey on tax incidence in one-sided markets.
price $p^A$. Let $n$ and $a$ denote the respective quantities of the two goods. Both newspaper readers and advertisers are price takers, with inverse demand functions being downward-sloping in own quantity; $p^N_n \equiv \partial p^N/\partial n < 0; p^A_a \equiv \partial p^A/\partial a < 0$. We shall in order to capture essential features of the media market assume that the willingness to pay for an ad is increasing in the number of newspaper readers, that is, $\partial p^A/\partial n \equiv p^A_n(a, n) > 0$. Furthermore, we shall assume that $\partial p^N/\partial a \equiv p^N_a < 0$, which means that the readers’ willingness to pay for the newspaper is decreasing in the ad-level and thus, that the audience dislike ads. We summarize the two latter assumptions in Assumption 1:

**Assumption 1:** $\partial p^A/\partial n > 0$ and $\partial p^N/\partial a < 0$.

We would like to emphasize that the assumption above should not be confused with standard theory of complements. Complements are used to describe a situation where an increase in the price of one good causes a decline in consumption of both goods, measured by the change in the compensated demand by a single consumer (see e.g., Kreps 1990, p. 61). This is different from a two-sided market, where there are two distinct groups of customers that may respond differently to changes in prices. If a media firm reduces the price of advertising in order to sell more copies of a newspaper, say, it will have to accept lower sales of the newspaper, since the ad volume will be higher, other things being equal.

We shall let $t$ be the ad-valorem tax on ads so that the newspaper receives the net price $p^A/(1 + t)$ per advertisement. The tax rate $t$ may deviate from the general VAT rate, which for simplicity is set equal to zero. The profit level of the newspaper is given by

$$
\Pi = \frac{p^A(a, n)a}{1 + t} + p^N(a, n)n - k(a, n),
$$

where $k(a, n)$ is the cost function, with $k_i \geq 0$ ($i = a, n$) and $k_{ij} \geq 0$ ($i \neq j$).\(^6\)

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\(^6\)All the equations that follow go through independently of the sign of $p^N_a$.

\(^7\)Intuitively, one might expect that the marginal cost of printed newspapers is increasing in the
The media firm maximizes profit with respect to sales of newspapers and advertising space. We presuppose that the second-order conditions for profit maximization hold; \( \Pi_{aa} < 0 \), \( \Pi_{nn} < 0 \), and \( H \equiv \Pi_{aa} \Pi_{nn} - \Pi_{an}^2 > 0 \).

From (1) we find that the first-order condition for the newspaper’s advertising volume (\( \Pi_a = 0 \)) reads
\[
\frac{p_A^a + p_A^a a}{1 + t} = k_a - p_a^N n
\]
\[\equiv MR_a \]
\[\equiv PMC_a \tag{2}\]

The left-hand side of equation (2) measures the marginal revenue on the advertising side of the market of selling ads (\( MR_a \)), and this term should be set equal to marginal cost (\( k_a \)) in a standard one-sided market. However, a one-unit increase in the ad-level means that the willingness to pay for the newspaper falls by \( p_a^N \) units. With \( n \) newspaper readers, this represents a loss equal to \( p_a^N n \) for the media firm. We may therefore interpret the sum of the actual marginal costs \( k_a \) and the externality term \( -p_a^N n > 0 \) as the newspaper’s perceived marginal costs of advertising (\( PMC_a \)), that is, \( PMC_a \equiv k_a - p_a^N n \). Equation (2) simply says that these perceived marginal costs are equal to marginal revenue in optimum. Since \( PMC_a > k_a \) if the newspaper readers dislike ads, the first-order condition implies that the media firm sells a lower ad-volume than what maximizes profits on the ad-side of the market.

Setting \( \Pi_n = 0 \) we further find that
\[
\frac{p_n^N + p_n^N n}{1 + t} = k_n - \frac{p_a^A a}{1 + t}, \tag{3}
\]
which has a similar interpretation to that of equation (2): the marginal revenue on the newspaper side of the market (\( MR_n \)) should be set equal to the perceived marginal costs of selling a newspaper (\( PMC_n \)). These perceived costs will be smaller than the actual marginal costs (\( PMC_n < k_n \)) if a larger newspaper circulation ad-volume, and vice versa (so that \( k_{an} > 0 \)). However, there may also exist some cost synergies, which means that \( k_{an} < 0 \). Since our theoretical results go through in either case, we leave the sign of \( k_{an} \) unspecified.
increases the willingness to pay for ads. This is captured by the term $p_n^A a/(1 + t) \geq 0$.

From (2) and (3) it follows that:

**Lemma 1:** Ceteris paribus, an increase in the ad-valorem tax on ads reduces the marginal revenue of selling ads ($\partial MR_a/\partial t < 0$) and increases the perceived marginal costs of selling newspapers ($\partial PMC_n/\partial t > 0$).

Note that $PMC_n < 0$ if $k_n$ is sufficiently small compared to $p_n^A a$. This may for instance be the case with television and electronic newspapers, where marginal costs are approximately equal to zero. However, $PMC_a$ must certainly be positive if consumers dislike ads, even in cases where $k_n = 0$.

The interrelationship between the two sides of the market is illustrated in Figure 1, where for simplicity marginal costs are set equal to zero. The left-hand side panel shows the profits in the reader market from selling newspapers, $\Pi^N = p^N n$, while the right-hand panel shows the profits in the advertising market from selling ads, $\Pi^A = p^A a/(1 + t)$. If the advertisers did not care about the number of readers and the readers did not care about the number of ads, the newspaper would maximize profit by setting $n^* = \arg \max N \Pi^N$ and $a^* = \arg \max A \Pi^A$. However, with $p_n^A > 0$ and $p_a^N < 0$, first-order conditions (2) and (3) imply, other things equal, that we have $n^{opt} > n^*$ and $a^{opt} < a^*$. So the media firm sell more copies of the newspaper, but place less ad inverts than in a conventional market with no network externalities.
Figure 1: Implications of the first-order conditions.

3 Tax responses

Standard welfare economics prescribes to tax a good which imposes a negative externality.\(^8\) By assuming \(p_a^N < 0\) we have tilted the model so that according to standard theory, levying a tax on ads should have a positive welfare effect. Below, we show that this does not necessarily hold in a two-sided market.

First-order conditions (2) and (3) make it clear that equilibrium prices and quantities on both sides of the market depend on the tax rate on ads. Differentiating \(p^A = p^A(a(t), n(t))\) and \(p^N = p^N(a(t), n(t))\) with respect to \(t\) we find that the price changes subsequent to a tax increase are given by

\[
\frac{dp^A}{dt} = p^A_a \frac{da}{dt} + p^A_n \frac{dn}{dt} \quad \text{and} \quad \frac{dp^N}{dt} = p^N_n \frac{dn}{dt} + p^N_a \frac{da}{dt}.
\]

By totally differentiating first order conditions (2) and (3) we further have

\[
\frac{da}{dt} = \frac{1}{H(1+t)} \left[ MR_a \Pi_{an} + \frac{p^A_a}{1+t} (-\Pi_{an}) \right]
\]

and

\[
\frac{dn}{dt} = \frac{1}{H(1+t)} \left[ \frac{p^N_a}{1+t} \Pi_{aa} + MR_a \Pi_{an} \right].
\]

The sign of \(\Pi_{an} \equiv \partial^2 \Pi / (\partial a \partial n)\) turns out to be of particular relevance for the tax analysis, and by using equations (1) - (3) we find

\[
\Pi_{an} = p^N_a [1 + \varepsilon_n] + p^A_n (1 + t)^{-1} [1 + \varepsilon_a] - k_{an},
\]

where \(\varepsilon_n \equiv \frac{n}{p^A_n} \partial p^N_a / \partial n\) and \(\varepsilon_a \equiv \frac{a}{p^A_n} \partial p^A_n / \partial a\).

The cross derivative \(\Pi_{an}\) measures how the marginal profitability of selling newspapers, \(\Pi_n\), changes if the advertising volume increases. One might think that \(\Pi_{an}\)

\(^8\)If \(p^A_n\) and/or \(p^N_a\) are different from zero we have externalities between the customer groups. The reason is that price-taking producers and consumers do not take into account the effect of their actions on the demand in either side of the market.
is negative, given the assumption that the willingness to pay for the newspaper is decreasing in the advertising volume \( p_n^A < 0 \). However, if the elasticity of \( p_n^A \) with respect to \( n \) is smaller than minus one \( (\varepsilon_n < -1) \), the first term in (7) is positive. The interpretation of the second term in (7) is similar; this term is positive for \( p_n^A > 0 \) if \( \varepsilon_a > -1 \). Clearly, we might therefore have \( \Pi_{an} > 0 \), and we are not aware of any empirical studies which can help us determine the sign. We shall therefore consider both the case \( \Pi_{an} \geq 0 \) and \( \Pi_{an} < 0 \).

4 A tax on ads when \( \Pi_{an} \geq 0 \)

When \( \Pi_{an} \geq 0 \), the marginal profitability of newspaper sales is weakly increasing in the ad-volume. We shall start this section by assuming that \( \Pi_{an} = 0 \). In this case an increase in \( t \) unambiguously leads to a lower advertising volume \( (da/dt < 0) \), since the media firm’s marginal revenue of selling ads falls. Formally, this can be seen from equation (5), which now simplifies to

\[
\frac{da}{dt} \bigg|_{\Pi_{an}=0} = \frac{\Pi_{an}}{H(1+t)} M R_a < 0. \tag{8}
\]

By taxing ads, the government is able to reduce the ad volume in the newspaper. Other things equal, this makes the newspaper more attractive for the consumers. However, this does not imply that output of newspapers increases. On the contrary, from equation (6) we find

\[
\frac{dn}{dt} \bigg|_{\Pi_{an}=0} = \frac{\Pi_{aa}}{H(1+t)^2} p_n^A a < 0. \tag{9}
\]

The intuition for why \( dn/dt < 0 \) is clear from Lemma 1: a higher tax rate on ads increases the perceived marginal cost of selling newspapers.\(^9\) Thus, it is optimal to reduce output.

\(^9\)From (3) we have \( k_n - P M C_n = \frac{p_n^A}{1+t} > 0 \). Substituting for \( \frac{p_n^A}{1+t} \) into (9) we can write

\[
\frac{dn}{dt} \bigg|_{\Pi_{an}=0} = \frac{\Pi_{aa}}{H(1+t)^2} (k_n - P M C_n) < 0.
\]
The negative quantity effects of a higher tax on ads are magnified if \( \Pi_{an} > 0 \), since a smaller newspaper circulation then reduces the marginal profitability of selling ads and vice versa. This can be verified by noting that the last terms in the square bracket of (5) and (6) are negative when \( \Pi_{an} > 0 \). We can therefore state:

**Proposition 1:** Suppose that \( \Pi_{an} \geq 0 \). A higher ad-valorem tax on ads reduces sales of both ads and newspapers \( (da/dt < 0 \text{ and } dn/dt < 0) \).

Next, consider how an increase in \( t \) affects the end-user prices on the two sides of the market. The direct effect of a smaller sale of newspapers is to increase the price of newspapers (since the demand curve is assumed to be downward-sloping). Additionally, the willingness to pay for newspapers increases since the ad-volume is reduced. From equation (4) we therefore find \( dp^N/dt > 0 \).

The effect on the price of ads is ambiguous. The own-price effect suggests that the price increases, while the fact that newspaper sales fall suggests a lower price. The net effect depends on which of these effects dominates, such that \( dp^A/dt \lesssim 0 \).

We can state:

**Proposition 2:** Suppose that \( \Pi_{an} \geq 0 \). A higher ad-valorem tax on ads increases the price of newspapers \( (dp^N/dt > 0) \), while the effect on the price of ads is ambiguous \( (dp^A/dt \lesssim 0) \).

Somewhat surprisingly, and in sharp contrast to results in one-sided markets, Proposition 2 shows that the end-user price of the more heavily taxed good might fall. The end-user price of the good where the tax rate is unchanged, on the other hand, increases. This goes to show that managerial responses to a tax may be opposite of what the policy intends to achieve: it may lead to more advertising and may also have the unintended side-effect of making the media product (newspaper, say) more expensive to buy.
4.1 Example 1 (illustration of the case $\Pi_{an} \geq 0$)

In this section we illustrate the paradoxical results above by a specific example. This example carry merit value on its own since it also reveals that media firms may argue against a tax on ads for welfare reasons because we show that even though the consumers by assumption perceive ads as a bad, both the consumers and the society as a whole might be harmed if ads are taxed. In the main text we limit attention to a monopoly newspaper, but in the appendix we show that the qualitative results hold also under duopolistic competition.

We follow Godes et al. (2009) and Kind et al. (2007, 2009) in assuming that consumer demand for the newspaper is given by the inverse demand function

$$p^N = 1 - n - \gamma a,$$

where $\gamma$ is a positive parameter which measures the readers’ dislike for ads; the higher $\gamma$, the greater the consumers’ disutility of ads.

Consumer-good producers advertise in the newspaper if the benefit of doing so is larger than the cost. A producer’s gross gain from advertising in the newspaper is naturally increasing in its advertising level ($a$) and in the number of readers exposed to its advertising ($n$). We make it simple by assuming that the gross gain equals $an$. With a price per ad equal to $p^A$, the net gain from advertising is

$$\pi = an - p^A a.$$  \hspace{1cm} (11)

Without affecting the qualitative results, we assume that there is only one advertiser. Solving $a = \arg \max \pi$ subject to (10), we find that the inverse demand curve for ads equals

$$p^A = 1 - (p^N + 2\gamma a).$$  \hspace{1cm} (12)

The willingness to pay for an ad in newspaper $i$ is thus decreasing in its advertising volume ($\partial p^A / \partial a < 0$) and in the consumer price of the newspaper ($\partial p^A / \partial p^N < 0$). The reason for the latter is that a higher newspaper price tends to reduce newspaper circulation, thereby making advertising less attractive.
Analogously to equation (1), the newspaper’s profit level equals

\[ \Pi = \frac{p^A}{1 + t} + p^N n - k(a, n). \] (13)

Since the purpose of this example is to illustrate the consequences of taxing ads when the marginal profitability of newspaper sales is increasing in the ad level \((\Pi_{an} = \frac{\partial^2 \Pi}{\partial n \partial a} > 0)\), we shall for simplicity set \(k = 0\). We then have

\[ \Pi_{an} = \frac{1}{1 + t} - \gamma > 0. \] (14)

The assumption that \(k = 0\) is not critical, as long as the costs are not so high as to make \(\Pi_{an} < 0\).

Solving \(\{a, n\} = \text{arg max } \Pi\) we find that

\[ n = \frac{8\gamma}{D_1} \text{ and } a = \frac{4(1 - \gamma(1 + t))}{D_1}. \] (15)

In the Appendix we show that the denominator \(D_1\) is positive when the second-order conditions and the non-negativity constraints are satisfied. Non-negative prices require that \(\gamma \in (1/3, 1)\). If \(\gamma \leq 1/3\), consumers have so little aversion against ads that the media firms prefer to give the newspapers away for free to the consumers. In this case their whole profit originates from the ad market. Conversely, if \(\gamma \geq 1\), consumers have such a negative attitude towards ads that the media firm maximizes profits by setting \(a = 0\). In this case its entire revenue is derived from the reader market.

When analyzing the tax responses in this example, we confine ourselves to considering the consequences of a small tax increase from \(t = 0\). Differentiating (15) with respect to \(t\) we thus find that the quantity changes are given by

\[ \left. \frac{da}{dt} \right|_{t=0} = -\frac{1 + 5\gamma^2 - 2\gamma}{D_1^2} < 0 \text{ and } \left. \frac{dn}{dt} \right|_{t=0} = -\frac{2\gamma(1 - \gamma^2)}{D_1^2} < 0. \] (16)

By inserting for (15) into (10) and (12) we further have

\[ \left. \frac{dp^A}{dt} \right|_{t=0} = -\gamma \frac{2\gamma - 7\gamma^2 + 1}{D_1^2} < 0 \text{ for } \gamma < \gamma^* \equiv \left(1 + 2\sqrt{2}\right)/7 \] (17)

and \(\left. \frac{dp^N}{dt} \right|_{t=0} = \frac{3(1 + \gamma^2) - 2\gamma}{D_1^2} > 0\).
Figure 2 illustrates equations (16) and (17) graphically. Consistent with Proposition 1, sales of both advertising and newspapers fall subsequent to a higher tax. Note also that if $\gamma < \gamma^* \approx 0.55$, the end-user price of newspapers, where the tax rate is unchanged, increases, while the end-user price of advertising, where the tax rate has increased, falls. This is consistent with Proposition 2.

The reason why $dp^A/dt|_{t=0} < 0$ for $\gamma < \gamma^*$ is that if the readers do not care much about the ad-volume, the media firm will sell a large amount of newspaper copies in order to generate a high income from the ad-market. This incentive is significantly reduced if ads are taxed. Thus, there will be a big drop in newspaper sales. This reduces the willingness to pay for ads, leading to a fall in the ad price. Only for $\gamma > \gamma^*$ is the own-price effect so strong that the reduced supply of ad space increases the price of ads.

![Graphs showing price and quantity responses.](image)

**Figure 2:** Price and quantity responses.

Figure 2 verifies that price and quantity responses to higher taxes in two-sided markets may differ qualitatively from those we find in one-sided market. A second deviation from standard results in one-sided markets, is that even a small tax on a good with negative externalities (advertising) may have negative welfare consequences. To see this, we define welfare in the usual way as the sum of consumer surplus, profit, and tax revenue ($T$):

$$W = CS + 2\Pi + \pi + T,$$
where \( T = \frac{t_{1+t}}{1+t} \left(2p^Aa\right) \).

From the envelope theorem it follows that the tax revenue of increasing the tax rate marginally from \( t = 0 \) is equal to the profit losses of the media firms;

\[
\frac{d(\Pi_1+\Pi_2)}{dt}|_{t=0} = - \frac{d\pi}{dt}|_{t=0} .
\]

This means that \( \frac{dW}{dt}|_{t=0} = \frac{dCS}{dt}|_{t=0} + \frac{d\pi}{dt}|_{t=0} . \) By using equations (10), (11) and (12) we find the following simple expressions for consumer surplus and profit for the advertiser: \( ^{10} \)

\[
CS = n^2 \quad \text{and} \quad \pi = 2\gamma a^2 .
\]

From this we immediately see that

\[
\left. \frac{dCS}{dt} \right|_{t=0} = 2n \left. \frac{dn}{dt} \right|_{t=0} < 0 \quad \text{and} \quad \left. \frac{d\pi}{dt} \right|_{t=0} = 4\gamma a \left. \frac{da}{dt} \right|_{t=0} < 0 . \quad (18)
\]

It thus follows that for all \( \gamma \in (1/3, 1) \) we have

\[
\left. \frac{dW}{dt} \right|_{t=0} = -2\gamma (1-\gamma)(1+7\gamma^2) \frac{D_1^2}{D_1^3} < 0 .
\]

Even though advertising imposes a negative externality on newspaper readers, a tax on ads has a negative effect on consumer surplus and welfare. There are two reasons for this somewhat paradoxical result. First, the tax increases the perceived marginal costs of selling newspapers, as stated in Lemma 1. This effect is present independently of the sign of \( \Pi_{an} \). Second, if \( \Pi_{an} > 0 \), the lower output of newspapers reduces the marginal profitability of selling ads, which again reduces the marginal profitability of selling newspapers. In this sense a tax on ads leads to a vicious circle where output contractions of newspapers and ads mutually reinforce each other.

### 5 A tax on ads when \( \Pi_{an} < 0 \)

When \( \Pi_{an} < 0 \), the marginal profitability of newspaper sales is decreasing in the ad-volume. Contrary to the results above, it is then not necessarily true that a higher

\[\text{level is increasing in } \gamma. \] However, this is not correct, since the ad volume is decreasing in the reader’s disutility of ads. We consequently find \( \left. \frac{d\pi}{d\gamma} \right|_{\gamma} = -\frac{2(1-\gamma(1+t))(1+\gamma(1+t))^3}{D_1^3(1+t)^{-2}} < 0 . \)

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\( ^{10} \)The equation \( \pi = \gamma a^2 \) might leave the counterintuitive impression that the advertiser’s profit level is increasing in \( \gamma \). However, this is not correct, since the ad volume is decreasing in the reader’s disutility of ads. We consequently find \( \left. \frac{d\pi}{d\gamma} \right|_{\gamma} = -\frac{2(1-\gamma(1+t))(1+\gamma(1+t))^3}{D_1^3(1+t)^{-2}} < 0 . \)
ad-valorem tax on ads reduces sales on both sides of the market. It may actually be the case that output of either ads or newspapers increases. Equations (5) and (6), which for the sake of convenience we repeat here, make this clear:

\[
\frac{da}{dt} = \frac{1}{H(1+t)} \left[ MR_a \Pi_{an} + \frac{p_n^A}{1+t} (-\Pi_{an}) \right] \\
\frac{dn}{dt} = \frac{1}{H(1+t)} \left[ \frac{p_n^A}{1+t} \Pi_{na} + MR_a (-\Pi_{an}) \right]
\]

The first term in the square brackets of (19) is always negative, but the second term is positive if \(\Pi_{an} < 0\). The total effect is thus ambiguous. However, in the Appendix we prove the following result:

**Proposition 3.** Suppose \(\Pi_{an} < 0\). A higher ad-valorem tax on ads reduces sales on one side of the market, and may increase sales on the other side. The following combinations are possible:

- (i) \(da/dt \leq 0\) and \(dn/dt \geq 0\).
- (ii) \(da/dt > 0\) and \(dn/dt < 0\).

If sales of one good drop, the marginal profitability of selling the other good increases when \(\Pi_{na} < 0\). This explains why output of the two goods may move in opposite directions, as stated in Proposition 3. Due to the ambiguity of the quantity effects, it is clear that also the price responses (4) are ambiguous.

The last part of Proposition 3 is surprising, as it states that the ad-volume may increase following a rise in the ad tax. We shall now present an illustrative example with a monopoly newspaper to demonstrate that this result occurs when the readers’ disutility from ads is sufficiently high (see Appendix for a duopoly version).
5.1 Example 2 (illustration of the case $\Pi_{an} < 0$)

As in Example 1 we assume that the inverse demand function for newspapers equals

$$p^N = 1 - \gamma a - n. \quad (20)$$

For simplicity we further assume that we can linearize demand for ads around the equilibrium point to

$$p^A = 1 - a_i + n. \quad (21)$$

The willingness to pay for an ad is thus decreasing in the ad volume and increasing in the size of the readership.

The media firms’ profit functions are the same as in Example 1 (c.f. equation (11)), but to ensure as simple as possible that $\Pi_{an} < 0$, we specify the cost function as $k = an + n/2$. We then have

$$\Pi_{an} = -\frac{t(1 + \gamma) + \gamma}{1 + t} < 0. \quad (22)$$

The newspaper solves \( \{a, n\} = \arg \max \Pi \), implying that the first-order conditions for profit maximization are given by

$$a = (1 + t) \frac{4 - t - \gamma(1 + t)}{2D_2} \quad \text{and} \quad n = 2 \frac{1 - (1 + t)\gamma}{2D_2}. \quad (23)$$

The denominator $D_2$ is positive whenever the second-order conditions and non-negativity constraints hold; this is true for $\gamma \in (0, 1)$ (see Appendix).

To find the effects of a tax on ads we differentiate (23):

$$\left. \frac{dn}{dt} \right|_{t=0} = -\frac{2 - 2\gamma + \gamma^2}{D_2^2(\gamma + 2)^{-1}} < 0 \quad \text{and} \quad \left. \frac{da}{dt} \right|_{t=0} = \frac{3\gamma - 2}{2D_2^2(\gamma + 2)^{-1}} \leq 0. \quad (24)$$

The reason why newspaper sales fall, is that a tax on ads increases the perceived marginal costs of selling newspapers (c.f. Lemma 1). The drop in newspaper sales in turn raises the marginal profitability of selling ads, and (24) shows that $da/dt > 0$ if $\gamma > 2/3$. It is thus when the readers’ disutility from ads is sufficiently large that a higher tax on ads leads to more advertising. This is illustrated in Figure 4, which
also shows that the advertising price (inclusive of taxes) falls when $t$ increases. This is due to the fact that the willingness to pay for ads is reduced because the newspaper circulation falls ($dn/dt < 0$).

The intuition behind the sign of the quantity change ($da/dt$) in Figure 4 is as follows. In order to exploit the profitability of the advertising market, it is optimal for the newspaper to have a relatively low advertising volume (and high advertising price), and more so the larger is $\gamma$. This ensures that the newspaper will have a large readership. However, if the advertising revenue is taxed, it will be less profitable to have a high circulation. The lower circulation in turn increases the marginal profits of selling ads if $\Pi_{an} < 0$. It follows that the media firm has stronger incentives to increase the advertising volume subsequent to a higher advertising tax the larger is $\gamma$.\footnote{Mathematically, this can be seen by using equation (7) to find $\Pi_{an}|_{t=0} = p_a^A - \gamma - k_{an}$. Since $d \Pi_{an}|_{t=0}/d\gamma < 0$, a given reduction of newspaper sales leads to a larger increase in the marginal profitability of selling ads the higher $\gamma$ is.} This explains why $da/dt > 0$ for sufficiently high values of $\gamma$.

Also in this example newspaper readers are adversely affected by a tax on ads, but interestingly the advertisers might benefit. This is true if $da/dt > 0$. It can

Figure 4: Taxing ads. Consequences for advertising prices and sales volume.
further be shown that

\[ \frac{dW}{dt} \bigg|_{t=0} = -\frac{16 - 30\gamma + 15\gamma^2 - 4\gamma^3}{2D^2_2(2 - \gamma)} \]

is positive for \( \gamma \in (0.77, 1.0) \). For sufficiently high values of \( \gamma \) we thus find that a small tax on ads increases welfare. However, this is not because the tax leads to reduced output of the good which imposes a negative externality, but on the contrary because output of that good increases. This turns standard insight of how taxes work upside-down. Understanding the business model of media firms is decisive when assessing the effects of public policies.

6 Conclusion

In this paper we have made use of recent advances in the theory of management behavior in two-sided markets to analyze how a tax on advertising affect management decisions. Managers in two-sided platforms coordinate the demand of two distinct groups of customers. When devising a pricing strategy, management must account for the interaction between advertisers and consumer. This interaction starts in our analysis with the assumption that consumers (readers/viewers) perceive ads as a nuisance. Standard theory would in this case prescribe a tax on ads that makes management internalize the negative externalities from ads. However, standard theory neglects the linkages that exist between a platform firm’s customer groups. Including these linkages in the analysis, we find that a tax on ads may have adverse effects and therefore lead to undesirable policy outcomes.

In particular, we show that taxes on ads reduce the media firms’ incentives to attract a large audience that can be sold on the advertising market. Such taxes are therefore likely to harm the consumers, e.g. through higher prices and possibly even through higher advertising levels. This suggests that targeting market failure directly as standard theory prescribes, may not be the solution in markets where there are network externalities. Given the leisure time media products occupy in
peoples life, and the importance media products has in popular culture, one should therefore be careful when devising policy directed at this industry. Our study, then, shows that in a two-sided market such as the media market conventional use of taxes to correct for externalities is likely to produce unintended results. This is so because managers rationally takes into account that their model of business must trade off revenue from readers and advertisers. This trade off leads to a particular behavior by managers not found in businesses with only one customer group.

7 Appendix

7.1 Calculation of Example 1

Define $D_1 \equiv 24\gamma - 4(\gamma^2 + (1 + t)^{-2})(1 + t)$. Using equations (10), (12) and (13) we find $\frac{\partial^2 \Pi_1}{\partial n_1^2} < 0$, $\frac{\partial^2 \Pi_1}{\partial a_1^2} < 0$ and

$$H \equiv \left( \frac{\partial^2 \Pi_1}{\partial n_1^2} \right) \left( \frac{\partial^2 \Pi_1}{\partial a_1^2} \right) - \left( \frac{\partial^2 \Pi_1}{\partial n_1 \partial a_1} \right)^2 = \frac{D_1}{4(1 + t)}.$$

A sufficient condition for $H$ to be positive, and thus for the second-order conditions to hold, is that $D_1 > 0$.

Inserting for (10) and (12) into (15) we have

$$p^N = \frac{4^{3\gamma}(t + 1) - 1}{D_1(1 + t)} \quad \text{and}$$

$$p^A = \frac{4\gamma(1 + \gamma(t + 1))}{D_1}.$$

From (25) we find

$$\left. \frac{dp^N}{dt} \right|_{t=0} = 16\gamma \frac{3 - 2\gamma + 3\gamma^2}{D_1^2} > 0$$

and

$$\left. \frac{dp^A}{dt} \right|_{t=0} = -4\gamma \frac{4(2\gamma - 7\gamma^2 + 1)}{D_1^2} > 0.$$
The upward-sloping curve in Figure 3 is found by setting \( \frac{dp^A}{dt} \big|_{t=0} = 0 \).

Note from (25) that both \( p^A \) and \( p^N \) are non-negative for \( t = 0 \) iff \( \gamma \in [1/3, 1] \). Q.E.D.

### 7.2 Proof of Proposition 3

Note that \( H \equiv \Pi_{aa} \Pi_{nn} - \Pi_{an}^2 > 0 \) which, when \( \Pi_{an} < 0 \), implies

\[
\frac{\Pi_{aa}}{\Pi_{an}} > \frac{\Pi_{an}}{\Pi_{nn}} > 0.
\]

Rearranging both derivatives in (19), while using the above inequality, proves both statements in Proposition 3. Q.E.D.

### 7.3 Calculation of Example 2

Define \( D_2 = 4(1 + t) - (\gamma (1 + t) + t)^2 \). Using equations (13), (10), and (21) we find \( \frac{\partial^2 H}{\partial n^2} < 0, \frac{\partial^2 H}{\partial a^2} < 0 \) and

\[
H \equiv \left( \frac{\partial^2 \Pi_1}{\partial n^2} \right) \left( \frac{\partial^2 \Pi_1}{\partial a^2} \right) - \left( \frac{\partial^2 \Pi_1}{\partial n \partial a} \right)^2 = \frac{D_2}{4(1 + t)}.
\]

A sufficient condition for \( H \) to be positive, and thus for the second-order conditions to hold, is that \( D_2 > 0 \). This is ensured in the numerical example.

From (23) we have the following quantity responses to a VAT on ads:

\[
\begin{align*}
\frac{da}{dt} \bigg|_{t=0} &= \frac{3\gamma - 2}{2D_2 (\gamma + 2)^{-1}} \quad \text{and} \\
\frac{dn}{dt} \bigg|_{t=0} &= -\frac{2 - 2\gamma + \gamma^2}{D_2 (\gamma + 2)^{-1}}.
\end{align*}
\]

Inserting for the equilibrium quantities into the demand functions and differentiating we further have:

\[
\begin{align*}
\frac{dp^N}{dt} \bigg|_{t=0} &= \frac{4 - 2\gamma - \gamma^2}{2D_2 (\gamma + 2)^{-1}} \quad \text{and} \\
\frac{dp^A}{dt} \bigg|_{t=0} &= -\frac{2 - \gamma + 2\gamma^2}{2D_2 (\gamma + 2)^{-1}}.
\end{align*}
\]

Q.E.D.
7.4 Example 1 with duopolistic competition

With duopolistic competition (two newspapers) we follow Kind et al (2007) and assume that the consumers have the following utility function:

\[ U = \sum_{i=1}^{2} n_i - \left[ (1-s) \sum_{i=1}^{2} \frac{n_i^2}{2} + s \left( \sum_{i=1}^{2} \frac{n_i}{2} \right)^2 \right] ; i = 1, 2. \]  

(26)

The variable \( n_i \) in equation (26) denotes consumption of newspaper \( i = 1, 2 \), while the parameter \( s \in [0, 1] \) measures how differentiated the newspapers are; from the readers’ point of view they are completely unrelated if \( s = 0 \) (so that each newspaper behaves as a monopoly), while they are considered as perfect substitutes if \( s = 1 \). More generally, the readers perceive the newspapers as closer substitutes the higher \( s \) is.\(^{12}\)

With two newspapers consumer surplus equals

\[ CS = U - \sum_{i=1}^{2} \left( p_i^N + \gamma a_i \right) n_i. \]

Maximizing consumer surplus with respect to consumption of the two newspapers generates the inverse demand function

\[ p_i^N = 1 - \left( 2 - s \right) n_i/2 - \gamma a_i - sn_j/2 \quad (i, j = 1, 2; i \neq j). \]  

(27)

The advertiser’s profit function equals

\[ \pi = \left( \sum_{i=1}^{2} a_i n_i \right) - \left( \sum_{i=1}^{2} p_i^A a_i \right). \]  

(28)

Solving \( \{a_1, a_2\} = \arg \max \pi \) subject to (27) yields the following inverse demand curve for ads in newspaper \( i \):

\[ p_i^A = 1 - \frac{(2-s) \left( p_i^N + 2\gamma a_i \right) - s \left( p_j^N + 2\gamma a_j \right)}{2 \left( 1-s \right)} \quad (i, j = 1, 2; i \neq j). \]  

(29)

\(^{12}\)The Shubik-Levitan (1980) formulation in equation (26) ensures that the parameter \( s \) only captures product differentiation and not the size of the market. This is in contrast to the standard quadratic utility function, where one and the same parameter measures both product differentiation and market size. See Motta (2004) for details.
Since the two newspapers compete in the reader market if \( s > 0 \), equation (29) shows that the willingness to pay for ads in newspaper \( i \) is increasing in the advertising level and price of newspaper \( j \).

Analogously to the monopoly case considered in the main text, the profit level of newspaper \( i \) equals

\[
\Pi_i = \frac{p_i^A a_i}{1 + t} + p_i^N n_i,
\]

where we have maintained the assumption that \( k = 0 \).

Solving \( \{a_i, n_i\} = \operatorname{arg \ max} \Pi_i \) simultaneously for the two media firms, we find a unique symmetric equilibrium. Omitting subscripts, output of newspapers and advertising is given by

\[
n = \frac{2\gamma (4 - 3s)}{D_1} \quad \text{and} \quad a = \frac{4 (1 - \gamma (1 + t)) (1 - s)}{D_1}.
\]

The denominator is equal to \( D_1 \equiv 3 (8 (1 - s) + s^2) \gamma - 4 (\gamma^2 + (1 + t)^2) (1 + t) (1 - s) \).

In the main text we assumed that \( s = 0 \), which means that each media firm has monopoly power in its own market segment. All the qualitative results above survive as long as the consumers perceive the media products as imperfect substitutes. In particular, the firms will use their market power to shift part of the tax burden over to the consumers and the advertisers if \( s < 1 \) (contrary to what the profitably would be able to do in a one-sided market with \( k = 0 \)). The ability to do so is smaller the more fiercely the firms compete, though. This is most obvious if we use equation (30) and consider the consequences of a small tax increase from \( t = 0 \) on output:

\[
\begin{align*}
\frac{dn}{dt}igg|_{t=0} &= -8 (1 - s) \frac{\gamma (1 - \gamma^2) (4 - 3s)}{D_1^2} < 0 \\
\frac{da}{dt}igg|_{t=0} &= -4 (1 - s) \frac{4 (1 + 5\gamma^2 - 2\gamma) (1 - s) + 3s^2\gamma^2}{D_1^2} < 0.
\end{align*}
\]

Equation (31) shows that sales of both newspapers and advertising space fall subsequent to an increase in \( t \) as long as there is imperfect competition between the firms. However, as \( s \to 1 \) we have \( dn/dt = da/dt \to 0 \). The reason for this is that the consumers perceive the newspapers as perfect substitutes at \( s = 1 \), implying that the
media firms have no market power. Then the advertising tax works as a pure surplus tax. Thus, it is only in the limit case where the firms produce perfect substitutes that the qualitative results differ between a monopoly and duopoly setting.

7.5 Example 2 with duopolistic competition

In the duopolistic version of Example 1 we showed that the media firms' possibility of shifting the tax burden over to consumers and advertisers is smaller the less differentiated the consumers perceive the media products to be (as measured by the parameter $s$). It can be shown that the effects of an increase in $s$ (reduced newspaper differentiation) are the same in a duopolistic version of Example 2. For simplicity we therefore set $s = 0$. This means that we can simplify equation (27), which expresses consumer demand for the two media products, to

$$p^N_i = 1 - \gamma a_i - n_i.$$  

We now let advertising demand be equal to

$$p^A_i = 1 - a_i + n_i - ha_j.$$  

The inclusion of the parameter $h \in [0, 1]$ in equation (33) is inspired by Godes et al (2008), and measures to what extent the two newspapers compete in the advertising market. If $h = 0$ each newspaper has monopoly power in the advertising market, while they are perceived as perfect substitutes if $h = 1$. More generally, we have duopolistic competition between the newspapers in the ad market if $h > 0$.

Maintaining the assumption that $k_i = a_i n_i + n_i/2$, we have $\Pi_{an} = -\frac{t(1+\gamma)+\gamma}{1+t} < 0$.

The newspapers solve $\{a_i, n_i\} = \arg \max \Pi_i$ simultaneously. Omitting subscripts, the first-order conditions for a symmetric equilibrium are given by

$$a = (1 + t) \frac{4 - t - \gamma (1 + t)}{2D_2}$$

and

$$n = \frac{2 - (1 + t) (2 \gamma - h)}{2D_2}.$$  

The denominator equals $D_2 = 2 (2 + h) (1 + t) - (\gamma (1 + t) + t)^2$. We further have $\frac{\partial^2 \Pi}{\partial n_i^2} < $
\[ 0, \frac{\partial^2 \Pi}{\partial n_1^2} < 0 \text{ and} \]
\[ H \equiv \left( \frac{\partial^2 \Pi}{\partial n_1^2} \right) \left( \frac{\partial^2 \Pi}{\partial a_1^2} \right) - \left( \frac{\partial^2 \Pi}{\partial n_1 \partial a_1} \right)^2 = \frac{D_2 - 2h (1 + t)}{4 (1 - s) (1 + t)}. \]

A sufficient condition for \( H \) to be positive, and thus for the second-order conditions to hold, is that \( D_2 - 2h (1 + t) > 0 \).

From (34) we have the following quantity responses to a higher VAT on ads:
\[ \frac{da}{dt} \bigg|_{t=0} = \frac{3\gamma - 2}{2D_2^2 (\gamma + 2)^{-1}} - h \frac{2 (\gamma + 1)}{2D_2^2} \quad \text{and} \]
\[ \frac{dn}{dt} \bigg|_{t=0} = -\frac{2 - 2\gamma + \gamma^2}{D_2^2 (\gamma + 2)^{-1}} - h \frac{(5 - (\gamma + 1)^2)}{2D_2^2}. \]

Inserting for the equilibrium quantities into the demand functions and differentiating implies that:
\[ \frac{dp^N}{dt} \bigg|_{t=0} = \frac{4 - 2\gamma - \gamma^2}{2D_2^2 (\gamma + 2)^{-1}} + h \frac{\gamma^2 + 4}{2D_2^2} \quad \text{and} \]
\[ \frac{dp^A}{dt} \bigg|_{t=0} = -\frac{2 - \gamma + 2\gamma^2}{2D_2^2 (\gamma + 2)^{-1}} + h \frac{(\gamma + 1) (h - \gamma + 1)}{D_2^2}. \]

The first terms on the right-hand-side of equations (35) and (36) are the same as when we considered a monopoly newspaper. Since the second terms are continuous in \( h \), it follows that the results do not critically depend on whether there is competition. Indeed, it can be verified that the signs on \( dn/dt, dp^A/dt \) and \( dp^N/dt \) are the same for monopolists as for duopolists. It can further be shown that a sufficient condition for the monopoly result \( da/dt > 0 \) to hold is that \( h < 3/4 \), and that it holds for any value of \( h \) if \( \gamma < 2/3 \). The reason why we otherwise may have \( da/dt < 0 \), is that the larger \( h \) is, the less market power each newspaper will have in the advertising market, and the less profitable it is to sell more advertising space if ad revenue become taxed.

References


