Cost allocation in collaborative forest transportation

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Abstract

Transportation planning is an important part of the wood flow chain in forestry. There are often several forest companies operating in the same region and co-ordination between two or more companies is rare. However, there is an increasing interest in collaborative planning as the potential savings are large, often in the range 5-15%. A key question is how savings should be distributed among the participants. In this paper we investigate a number of possibilities based on economic models including Shapley value, the nucleolus, separable and non-separable costs, shadow prices and volume weights. We also propose a new allocation method based on finding as equal relative profits as possible among the participants. A case study including eight forest companies is described and analyzed.

Keyword: Transportation, OR in natural resources, Supply chain management, Economics, Group decisions and negotiations, Linear programming
1 Introduction

Transportation corresponds to a large proportion, about one third, of the total raw material cost for the forest industry in Sweden. Transportation is one part of the wood supply chain, see illustration in Figure 1, that begins at harvest areas in the forests. Here, trees are bucked i.e. cut into logs and there are often many possible types which depend on e.g. diameter, length and quality. The number of log types is in the range 5-15. Transportation can be done in one or two steps depending on the season. When the roads are in good condition it is possible to make the transportation directly to the industry. Here, sawlogs are taken to sawmills or ports for direct export, pulplogs to pulp- and/or paper mills and wood fuel to heating plants. The latter is done by special vehicles as it generally requires that the wood fuel is chipped. The main transportation is however done by logging trucks for saw- and pulplogs. The principal difference between sawlogs and pulplogs is that sawlogs have a larger diameter. It is also possible to include train and ship transportation to and from terminals.

Figure 1: An illustration of the wood supply chain.

There are several actors involved in the woodflow. The main ones are:

- industrial forest-enterprises, with large forest assets as well as their own pulp and paper
industries and sawmills. These can be either private or state owned

- forest owners associations, which represent the private entities and have their own pulp-and sawmills
- independent sawmills, without any larger forest asset
- independent forest owners not connected with any industry

In addition to these primary actors, representing the wood producers and wood consumers, there are also the loggers and the transporters, harvesting and carrying the wood from forest to mill. Management can be centralized or decentralized. Even though all actors involved recognize the importance of co-operation and integration along the woodflow chain it is easy to observe and explain why the different actors upon optimizing their individual short-term goals take decisions that can hinder integration and co-operation. One problem often encountered when trying to co-operate is that planning and decision-making becomes much more complicated. Planning becomes dependent on what the co-operating actors are planning to do, and both plans should be changed in order to account for the interrelationships.

Large volumes and relatively long transport distances together with increasing fuel prices and environmental concern makes it important to improve the transportation planning, see e.g. Epstein et al. (1999) and Weintraub et al. (1996). In many cases, volumes of the same assortment is transported in opposite directions due to a low level of interaction between the forest companies. Supply, demand and companies are geographically evenly dispersed for each company in the region and there is generally a high potential for coordination of the wood flow. Coordination creates opportunities to better utilize transport capacity. This can be done by wood bartering and/or backhauling. Wood bartering can be used in such a way that destination between supply and demand nodes are changed. Backhauling can be used to find better routes by combining two or several destinations (combination of a supply and demand point). In this way the unloaded distance can be lowered. Examples when wood bartering and backhauling have improved the transportation efficiency are found in Forsberg et al. (2005).

Wood bartering (or timber exchange) between forest companies in order to reduce transport cost is fairly common in Sweden. The largest proportion consists of pulp wood exchange since this assortment normally is cut into common lengths, for example 3 meters, and its qual-
ities are fairly equal no matter where in the geography the wood is harvested. Hence, this assortment is easy to exchange. Sawlogs are more rarely used for exchange since the logs are cut in specific lengths depending on which sawmill that will receive the logs. If exchanging sawlogs the planning for exchange has to be done before the logs are harvested and cut into specific lengths. Timber exchange seldom include more than two forest companies since the planning tends to be more complex the more companies that are involved.

In the case when coordination should be included in the operative planning there are a number of questions arising: How should the potential coordination be computed? How should the saving be divided among the participants? The first question can be approached by using a Decision Support System (DSS) based on Operations Research (OR) methods. The models and methods used in the system FlowOpt (Forsberg et al., 2005) can be used to find the actual saving if all participants co-operate as compared to no coordination. The second question is often not addressed. The reason for this is that wood bartering is often in practice only used between two companies and then only with a fixed volume. This means that each company uses its internal DSS to compute the saving for different volume levels without revealing it to the other company. This can be due to keeping sensitive information away from competitor. Also, it may be substantial differences in savings among the companies. However, to fully use the potential for coordination this question will be important.

In this paper we will address the second question. We will study and use a cost allocation method as a tool for allocating costs. That is, we do not split savings, instead we split the common cost among the participants. Further, we do not aim at identifying the “best” cost allocation, but instead, at suggesting and analyzing a number of alternatives. The most simple solution to a cost allocation problem would be to split the common cost equally, weighted with each participants volume. Most probably, this cost allocation will not be considered as fair, since – for example – a participant will pay, possibly, more than when operating alone. We will also study a number of different cost allocation methods that are, partly, based on solution concepts from co-operative game theory. The methods used are based on the Shapley value, the nucleolus, separable and non-separable costs, shadow prices and volume weights. When choosing among different solution concepts, we seek one that satisfies specific fairness criteria, called properties. There is a number of different properties presented in the literature, and an extensive list can be found in Tijs and Driessen (1986). In this context we will only discuss a
limited number of properties.

In a case study carried out in southern Sweden eight forest companies studied the potential savings of integrated or coordinated transportation planning. The potential saving was as high as 14%. The purpose of this paper is to develop a framework for participating companies to find values in how the savings should be distributed. In our study we have found that existing approaches do have some disadvantages when it comes to the relative savings. In this paper we therefore suggest a new approach motivated to get an allocation that provides an as equal relative profit as possible among the participants.

The outline of this paper is as follows. In Section 2 we describe shortly Operations Research (OR) models used in transportation planning. In Section 3 we describe a number of economic models used for cost allocation. Included is also a small test example. In Section 4 we describe the Decision Support System (DSS) used, the case study and the numerical results. To use the proposed methodology in practice there are a number of aspects to consider and these are discussed in Section 5. Finally, we make some concluding remarks.

2 Transportation planning

Transportation planning in forestry is done in several steps and is divided into strategic, tactical and operational planning. Decisions on a strategic level is often influenced by harvesting and road building/maintenance considerations. Tactical decisions often concerns planning from one week to one year. On an annual basis transportation is often integrated with harvesting planning, and then often to decide preliminary catchment areas for combinations of industries and assortments. This is also used as a basis to distribute areas to own or sub-contracted transport organizations/ hauliers. A problem which often ranges from one to several weeks is to decide the destination of logs, that is, which supply point should deliver to which demand point. This is used to define transport orders for transporters and is the one we focus on in this paper. The operational problem is to decide actual routes for individual trucks.
### 2.1 LP model

A planning tool often used for tactical planning is a Linear Programming (LP) model with variables \(w_{ij}\) representing the flow from supply point \(i\) (with supply \(s_i\)) to demand point \(j\) (with demand \(d_j\)). Corresponding unit cost is given by \(e_{ij}\) and depends essentially on the distance travelled and has a concave non-decreasing characteristic. The sets \(I\) and \(J\) represent the supply and demand points respectively. Constraints are expressed on supply and demand and the objective is to minimize the total cost. In forestry there are several assortments (depending on e.g. species and dimensions) and assortment groups can be used to specify the demands. A demand for an assortment group can then be fulfilled by one or several different assortments, depending on the definition of the assortment group. A supply point include one assortment and each demand point one assortment group. This means that not all combinations of supply and demand points are possible. We can define the set \(J_i\) as the demand points that supply point \(i\) can deliver to. In the same way we define set \(I_j\) as the supply points that can deliver to demand point \(j\). The basic LP model can be written as:

\[
[P1] \quad \min \quad z = \sum_{i \in I} \sum_{j \in J_i} e_{ij}w_{ij} \\
\text{s.t.} \quad \sum_{j \in J_i} w_{ij} \leq s_i, \quad \forall i \in I \quad \text{(supply at supply points)} \\
\quad \sum_{i \in I_j} w_{ij} = d_j, \quad \forall j \in J \quad \text{(demand at demand points)} \\
\quad w_{ij} \geq 0, \quad \forall i, j
\]

In the LP model above, the cost is based on the fact that the truck drives full from supply to demand point and empty in the other direction. This is the base of standard agreements. However, this gives an efficiency of just 50%. Efficiency would be improved if routes involving several loaded trips were used, i.e. backhauling. Backhauling refers to when a truck that has carried one load between two points, carries another load on its return. The geographical distribution of mills is important in this context. To use backhaulage tours can dramatically decrease the cost, savings between 2-20% are reported in different case studies, see Carlsson and Rönnqvist (1998) and Forsberg (2003). The modified LP model can be written as:
\[ \text{[P2]} \quad \min \quad z = \sum_{k \in K} c_k x_k \]
\[ \text{s.t.} \quad \sum_{k \in K} a_{ik} x_k \leq s_i, \quad \forall \ i \in I \quad \text{(supply at supply points)} \]
\[ \sum_{k \in K} d_{jk} x_k = d_j, \quad \forall \ j \in J \quad \text{(demand at demand points)} \]
\[ x_k \geq 0, \quad \forall k \]

Here, the variable \( x_k \) denotes the flow in route \( k \) and \( c_k \) the unit cost to use route \( k \). The set \( K \) represent all routes including direct flows in model [P1] and backhauling. The coefficients \( a_{ik} \) and \( d_{jk} \) have value 1 if they either pick up or deliver to that specific supply/demand point, otherwise 0. We note that the direct flow variables are represented using columns including only 2 nonzero elements (i.e. one pick up and one delivery). There is often a large number of potential routes and they can typically not be used explicit in a solver. Instead they are generated in a column generation approach. The restrictions imposed on the routes are e.g. driving time or the number of pick up and deliveries.

### 2.2 Coordination between companies

In our case we consider the problem to co-ordinate planning for several companies. It is common that transport costs can be decreased if companies apply bartering. However, this is difficult as planners not want to reveal supply, demand and cost information to competitors. In practice this is solved by deciding on wood bartering of specific volumes. Today this is done in an ad-hoc manner and is mostly dependent on personal relations. In Figure 2 we illustrate the potential benefits with wood bartering when two companies are involved. Here we have four mills at two companies (two mills each) together with a set of supply points for each company. In the left part each company operates by itself. The catchment areas are relatively large as compared to the right part where all supply and demand point are used on equal terms.
3 Economic models

With quantitative models we can coordinate transportation planning. With more than two companies it is difficult to find bartering volumes of different assortments (between all pairs of companies) and there is a need to find a more quantitative way how the savings should be distributed among the participants. In this section we describe a number of economic concepts or models that have been used to distribute costs in various industrial areas.

3.1 Basic definitions and Properties

Each solution concept that can provide us with a cost allocation is said to satisfy a number of properties, i.e. fairness criteria. There is, however, no concept that satisfies all criteria listed in the literature. Below we list some of the most commonly used properties. We denote by a coalition $S$ a subset of participants, and by the grand coalition $N$ all participants. It is assumed that all participants have the opportunity to form and cooperate in coalitions. When coalition $S$ co--operates, the total (or common) cost $c(S)$ is generated. In terms of co-operative game theory this cost function is called the characteristic cost function and each participant is called a player. We say that the cost allocation problem is formulated as a co-operative game.

A cost allocation method that splits the total cost, $c(N)$, among the participants $j \in N$ is said to be efficient, that is

$$\sum_{j\in N} y_j = c(N),$$
where $y_j$ is the cost allocated to participant $j$. A cost allocation is said to be *individual rational* if no participant pays more than its “stand alone cost”, which is the participant’s own cost, when no coalitions are formed. Mathematically, this property is expressed as

$$y_j \leq c(\{j\}).$$

The *core* of the game is defined as those cost allocations, $y$, that satisfy the conditions

$$\sum_{j \in S} y_j \leq c(S), \quad S \subset N$$

$$\sum_{j \in N} y_j = c(N) \quad (\text{efficiency})$$

That is, no single participant or coalition of participants should together be allocated a cost that is higher than if the individual or coalition would act alone. A cost allocation in the core is said to be *stable*.

For each coalition, $S$, and a given cost allocation, $y$, we can compute the *excess*

$$e(S,y) = c(S) - \sum_{j \in S} y_j,$$

which expresses the difference between the total cost of a coalition and the sum of the costs allocated to its members. For a given cost allocation, the vector of all excesses can be thought of as a measure of how far the cost allocation is from the core. If a cost allocation is not in the core, at least one excess is negative.

The cost function (or the game) is said to be *monotone* if

$$c(S) \leq c(T), \quad S \subset T \subset N.$$  

Note, that this means that if one new company is included in a coalition, the cost never decreases. The game is said to be *proper* if

$$c(S) + c(T) \geq c(S \cup T), \quad S \cap T = \emptyset.$$  

That is, the cost function is sub-additive. In such a game it is always profitable (or at least not unprofitable) to form larger coalitions.
3.2 The Shapley value

The Shapley value is a solution concept that provides us with a unique solution to the cost allocation problem. The computation formula stated below expresses the cost to be allocated to participant $j$, and is based on the assumption that the grand coalition is formed by entering the participants into this coalition one at a time. As each participant enters the coalition, he is allocated the marginal cost, by which his entry increases the total cost of the coalition he enters. The amount a participant receives by this scheme depends on the order in which the participants are entered. The Shapley value is just the average marginal cost of the participants, if the participants are entered in completely random order.

Thus, the cost allocated to participant $j$ is equal to

$$y_j = \sum_{S \subset N : j \in S, |S| \neq |N|} \frac{(|S| - 1)(|N| - |S|)}{|N|} [c(S) - c(S - \{j\})],$$

where $|.|$ denotes the number of participants in the considered coalition. The summation in this formula is the summation over all coalitions $S$ that contain participant $j$. The quantity, $c(S) - c(S - \{j\})$, is the amount by which the cost of coalition $S - \{j\}$ increases when participant $j$ joins it, here denoted by the marginal cost of participant $j$ with respect to the coalition $S - \{j\}$.

The Shapley value is based on four axioms formulated by Shapley in 1953. These axioms express that a cost allocation computed according to this solution concept satisfies the properties of efficiency, symmetry, dummy property and additivity. Symmetry means that if two arbitrary participants, $i$ and $j$, have the same marginal cost with respect to all coalitions not containing $i$ and $j$, the costs allocated to these two participants must be equal. The dummy property states that if participant is a dummy in the sense that he neither helps nor harms any coalition he may join, then his allocated cost should be zero. Finally, additivity expresses that, given three different characteristic cost functions $c_1$, $c_2$ and $c_1 + c_2$, for each participant the allocated cost based on $c_1 + c_2$ must be equal to the sum of the allocated costs based on $c_1$ and $c_2$, respectively. For an exact formulation of these axioms we refer to Shapley (1953).

The Shapley value provides us with a cost allocation that is unique, however there is no general guarantee that it is stable, e.g. it does not necessarily satisfy individual rationality. It can be proven that the Shapley value is the only value that fulfills the above four axioms.
3.3 The nucleolus

When computing the nucleolus of a game, we identify a cost allocation that minimizes the worst inequity, such that individual rationality is satisfied. That is, we ask each coalition $S$ how dissatisfied it is with the proposed allocation $y$ and we aim at minimizing the maximum dissatisfaction of any coalition. The dissatisfaction of a cost allocation $y$ for a coalition $S$ is expressed by the excess, which measures the amount by which coalition $S$ falls short of its potential $c(S)$ in the allocation $y$. The nucleolus is the cost allocation, $y$, that has the lexicographically greatest associated excess vector. For a more formal definition of this concept we refer to Schmeidler (1969).

The nucleolus exists and is unique. The nucleolus satisfies the symmetry axiom and the dummy axiom. If the core is non-empty, the nucleolus is in the core i.e. it represents a stable cost allocation. The pre-nucleolus is defined as the nucleolus, but it is not required that individual rationality is satisfied. If the game is monotone, it follows that the pre-nucleolus satisfies individual rationality. If the core is non-empty, also the pre-nucleolus is stable.

3.4 Other cost allocation principles

3.4.1 Allocations based on separable and non-separable costs

In Tijs and Driessen (1986) cost allocation methods are presented, based on that the total cost to be allocated is divided into two parts: The separable and the non-separable costs. Methods based on this idea first allocate to each participant his separable cost, then distribute the non-separable cost among the participants according to given weights. The separable cost is equal to $m_j = c(N) - c(N - \{j\})$, e.g., the marginal cost of participant $j$, with respect to the grand coalition. In the literature of this field, this is simply called participant $j$’s marginal cost. The non-separable cost that remains to be distributed is then

$$g(N) = c(N) - \sum_{j \in N} m_j.$$ 

Depending on which weights are chosen, we have different versions of the method; the two most straightforward methods are the Equal Charge Method, ECM, which distributes the non-separable cost equally, and the Alternative Cost Avoided Method, ACAM, that uses the weights.
\( w_j = c(\{j\}) - m_j \), expressing savings that are made for each participant by joining the grand coalition instead of operating alone.

Tijs and Driessen also describe the Cost Gap Method, CGM, where the weights are computed according to

\[
\bar{w}_j = \min_{S \in \mathcal{S}} g(S), \quad \text{where} \quad g(S) = c(S) - \sum_{j \in S} m_j.
\]

This choice of weights can be explained as follows. The separable cost, \( m_j \), is seen as a lower bound for the cost allocated to participant \( j \), when joining the grand coalition. The amount \( m_j + w_j \) can be seen as an upper bound for the cost allocated to participant \( j \), since it is what he will pay if all other participants pay their marginal cost, in the best coalition \( S \) from the view of \( j \). The use of this method assumes that

\[
g(S) \geq 0, \quad \forall S \quad \text{and that} \quad \sum_{j \in \mathcal{N}} w_j \geq g(\mathcal{N}).
\]

Thus, methods based on separable and non-separable costs allocate the costs according to

\[
y_j = m_j + \frac{w_j}{\sum_{i \in \mathcal{N}} w_i} g(\mathcal{N}).
\]

A cost allocation that is computed by the ECM or the ACAM satisfies efficiency and symmetry. When the CGM is used, also individual rationality and the dummy property are fulfilled.

### 3.4.2 Allocation based on shadow prices

In model [P2] described earlier we get dual or shadow prices for each of the supply and demand constraints. We define \( u_i \) and \( v_j \) as the shadow prices for the supply and demand constraints respectively. When we solve [P2] for the coalition \( S = \mathcal{N} \) we get \( c(\mathcal{N}) \). The optimal dual solution has the property

\[
c(\mathcal{N}) = \sum_{i \in \mathcal{I}} u_i s_i + \sum_{j \in \mathcal{J}} v_j d_j.
\]

Distribution of costs in linear production models, in where [P2] is a special case, has been proposed by Owen (1975) and Granot (1986). They show that the core is non-empty and that a solution can be obtained from the associated LP-problem. The solution is based on market prices which in the LP-model is represented by the shadow prices. Each company’s contribution can be found by computing its contribution to the dual objective function value. We assume that
company \( c \) has contribution \( s_i^c \) to supply constraint \( i \) and \( d_j^c \) to demand constraint \( j \). Then we can compute its contribution as

\[
y_c = \sum_{i \in I} u_is_i^c + \sum_{j \in J} v_jd_j^c.
\]

### 3.4.3 Allocation based on volumes or on stand alone costs

A straight forward allocation is to distribute the total cost of the grand coalition, \( c(N) \), among the participants according to a volume or a cost weighted measure. This is expressed by the formula

\[
y_j = w_j c(N),
\]

where \( w_j \) is equal to participant \( j \)’s share of the total transported volume, or, alternatively, equal to

\[
c(\{j\}) / \sum_{i \in N} c(\{i\}).
\]

This is the preferred model among the companies when asked. It is easy to understand, easy to show and it is easy to compute. However, as we will see it may provide allocations that are no seen as fair. One reason why this has not been observed earlier is that in normal wood bartering, the actual costs (or common profits) are not revealed.

### 3.5 Equal Profit Method

In this project we found some disadvantages with previous allocation models when it came to the acceptance of the cost allocation among the companies. It was difficult not to show that all companies had a similar relative profit compared to the individual cost. In a negotiation situation it would be beneficial to have an initial allocation where the relative savings are as similar as possible for all participants. This lead us to propose a new cost allocation principle. We therefore suggest a new method which is motivated by finding a stable allocation, such that the maximum difference in pairwise relative savings is minimized. We call this the *Equal Profit Method* (EPM).

The relative savings of participant \( i \) is expressed as

\[
\frac{c(\{i\}) - y_i}{c(\{i\})} = 1 - \frac{y_i}{c(\{i\})}
\]
By the assumption, that a cost allocation is stable, we have that $c(\{i\}) \geq y_i$.

Thus, the difference in relative savings between two participants, $i$ and $j$, is equal to

$$\frac{y_i}{c(\{i\})} - \frac{y_j}{c(\{j\})}$$

To find this allocation we need to solve the LP problem

$$\begin{align*}
\text{min} & \quad f \\
\text{s.t.} & \quad f \geq \frac{y_i}{c(\{i\})} - \frac{y_j}{c(\{j\})}, & \forall (i, j) \\
& \quad \sum_{j \in S} y_j \leq c(S), & S \subset N \\
& \quad \sum_{j \in N} y_j = c(N)
\end{align*}$$

The first constraint set is to measure the pairwise difference between all participants profits. The variable $f$ is used in the objective to minimize the largest difference. The two other constraint sets define all stable allocations. Since the objective is a combination between participants, it is not a weighted nucleolus. In the literature of this field, we have not been able to find an allocation method with similar objective. Therefore, to our knowledge, this allocation method is new.

In the case the core is empty we propose to use the so called epsilon-core. We should note that during our tests we never got this case. It means that we add a minimum penalized slack in the constraints defining the core. The implication of an empty core is that the grand coalition is not stable, i.e., some companies may break out and start their own coalition. By using an epsilon-core we can keep the grand coalition ”stable” and noting the existence of a coalition that would have an incentive to break out. Alternatively we can seek the maximal number of players present in a game for which the core exists. However, how this subgroup of players should be selected remains to be studied in future research.

### 3.6 An illustrative numerical example

In order to illustrate the difference between a cost allocation, computed according to the Equal Profit Method (EPM), and cost allocations based on other well-known concepts, we consider a small example including three participants. The cost of the coalitions $S$, $c(S)$, are given
by \( c\{1\} = 4, \ c\{2\} = 7, \ c\{1\} = 5, \ c\{1,2\} = 10, \ c\{1,3\} = 8, \ c\{2,3\} = 10, \ c\{1,2,3\} = 12. \)

The mathematical model based on the EPM can now be stated as

\[
\begin{align*}
\text{min} \quad f \\
\text{s.t.} \quad f &\geq y_1/4 - y_2/7 \quad (1) \\
&\geq y_1/4 - y_3/5 \quad (2) \\
&\geq y_2/7 - y_1/4 \quad (3) \\
&\geq y_2/7 - y_3/5 \quad (4) \\
&\geq y_3/5 - y_1/4 \quad (5) \\
&\geq y_3/5 - y_2/7 \quad (6) \\
y_1 &\leq 4 \quad (7) \\
y_2 &\leq 7 \quad (8) \\
y_3 &\leq 5 \quad (9) \\
y_1 + y_2 &\leq 10 \quad (10) \\
y_1 + y_3 &\leq 8 \quad (11) \\
y_2 + y_3 &\leq 10 \quad (12) \\
y_1 + y_2 + y_3 & = 12 \quad (13)
\end{align*}
\]

Here \( y_i \) is the cost allocated to participant \( i \) and \( f \) the maximal pairwise difference of relative savings. A unique optimal solution to this model is \( y_1 = 3, \ y_2 = 5.25, \ y_3 = 3.75, \) resulting in a relative saving equal to 25% for all three participants. Thus, the objective value \( f \) is equal to zero. If we compute the cost allocation based on the nucleolus for this small example, we obtain \( y_1 = 3, \ y_2 = 5.5 \) and \( y_3 = 3.5. \) Further, the cost allocation based on the Shapley value coincides with the one based on the nucleolus.

These cost allocations can be illustrated in a triangle, plotted in barycentric coordinates, in which each point represents a suggested cost allocation \((y_1, y_2, y_3)\), such that \( y_1 + y_2 + y_3 = 12 \) (see figure 3). The vertices of the triangle are defined by the intersections of the lines corresponding to the constraints (7)-(9), resulting in \((0,7,5), (4,3,5)\) and \((4,7,1)\).

In this triangle, for example, the line \( y_1 = 4 \) is the same as the line \( y_2 + y_3 = 8. \) The lines corresponding to the constraints (10)-(12) are then added to the triangle. The shaded area represents the core (the stable allocations). The points representing the EPM, the Shapley value and the nucleolus are shown. In order to briefly illustrate the shape of the objective function value, \( f \), we show this value for a number of points in the core. In the point representing the Shapley value/the nucleolus this value is 0.09, which corresponds to relative savings equal to 25%, 21.4% and 30% for participants 1, 2 and 3, respectively.
Figure 3: Geographical illustration of the solutions from the different allocations.

Finally, in Table 1 we observe the excess vectors of the two points. Recall that, the excess of a coalition is the difference between the total cost of a coalition and the sum of the costs allocated to its members. The two smallest elements of the two excess vectors are equal to 1 for both points. The Shapley value/the nucleolus has a lexicographically greater associated excess vector than the EPM, since the third smallest element is equal to 1.5, which is greater than 1.25. In this example the points representing the Shapley value and the nucleolus coincide. However, in general, these points are different, as can be observed in the computational results in the next section of this paper.

<table>
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<th>{2}</th>
<th>{3}</th>
<th>{1,2}</th>
<th>{1,3}</th>
<th>{2,3}</th>
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<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1</td>
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<tr>
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<td>1.25</td>
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Table 1: The excess vector of the two solutions.
4 Numerical results

4.1 DSS system FlowOpt

Wood flow analysis includes much work preparing data and interpreting results. Those actions are very time-consuming if done manually. It is also hard to control accuracy and find errors in the data. The FlowOpt system makes it easier to handle data and interpret results in a semi-automatic way. The time used for an analysis is considerably decreased when using the system. Basically, the process to make a wood flow analysis using FlowOpt has four separate parts:

1. Collecting data
2. Pre-processing and set up of data for optimization
3. Optimization
4. Processing and interpreting the results, report generation

Each of the process steps are supported in the system, see figure 4. The "Main application” is the central part of the FlowOpt system. The application is connected to a database storing the data about supply, demand, nodes, railway system etc. The interface offers different functionality for viewing geographical data and results, report generation and editing the data.

Information about, for example, supply and demand is company specific and denoted "Raw information”. Road information from the National Road database (NVDB) is used when distances are calculated. All information necessary for the analysis is stored in a separate database, denoted "Case”. The optimization module is located in a separate application. All data generation for the model are done in ”Data generation”. All data is then translated into a mathematical model by use of a set of input/output routines and the AMPL modeling language (see [2]). As a solver we make use of the CPLEX-optimization system (see [7]). Results from the optimization module are then imported back into the main application where different report options are possible. Normally, the results are exported to Excel or further calculations are done in a database in order to analyze specific key figures. The results are also interpreted in the main application where the viewer chooses different wood flows to show in the GIS system. This
makes it easy, for example, to view catchments areas for different industries. The flow is shown either with straight lines as the crow flies or with lines marking the exact route. The viewed wood flows can also be exported as shape files in order to make more advanced presentations in commercial GIS tools.

The collection of data can be done in different ways, but normally data is collected from company databases. The system can either work with historical data or make prediction on future demand, supply and different train and truck capacities. This depends on the purpose of the analysis. Historical transportation data can be used to compare with an optimized solution of the same historical data. Prediction of future demand and supply is needed to optimize the future wood flow and, for example, the integration between truck and train.

4.2 Case study

The data used in this paper has been taken from a case study done by the Forestry Research Institute of Sweden for eight participating forest companies. The data is taken from transports carried out during one month. It involves all transports from the eight companies and includes information on from/to nodes, volume and assortment. There is a relatively large difference in size between the companies. Table 2 shows the volume transported for each of the companies.
<table>
<thead>
<tr>
<th>Company</th>
<th>Volume</th>
<th>Proportion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Company 1</td>
<td>36,786</td>
<td>4.17%</td>
</tr>
<tr>
<td>Company 2</td>
<td>77,361</td>
<td>8.76%</td>
</tr>
<tr>
<td>Company 3</td>
<td>44,509</td>
<td>5.04%</td>
</tr>
<tr>
<td>Company 4</td>
<td>6,446</td>
<td>0.73%</td>
</tr>
<tr>
<td>Company 5</td>
<td>301,660</td>
<td>34.16%</td>
</tr>
<tr>
<td>Company 6</td>
<td>89,318</td>
<td>10.12%</td>
</tr>
<tr>
<td>Company 7</td>
<td>232,103</td>
<td>26.29%</td>
</tr>
<tr>
<td>Company 8</td>
<td>94,769</td>
<td>10.73%</td>
</tr>
<tr>
<td>Total</td>
<td>882,952</td>
<td>100.00%</td>
</tr>
</tbody>
</table>

Table 2: Company and volume expressed in cubic meters.

In order to make a comparison, the same distance table and cost functions are used for both optimized and actual transportation carried out. There are several comparisons that are interesting and we compute the following results.

**Real** Single company with actual cost.
**opt1** Full coordination with direct flows.
**opt2** Full coordination with backhauling flows.
**opt3** Single company with direct flows.
**opt4** Single company with backhauling flows.

With the above results we can compare the potential of coordination by comparing (opt1) and (opt3). We can also estimate the overall savings by comparing (Real) with (opt2). We do not choose to use any backhauling in our comparison. This is because the information about the actual transportation does not include any information about backhauling. Therefore we do not know the extent to which backhauling were used in practice. In Table 3 the results from each of the scenarios is given. We note that the savings from solving for each individual company to a full coordination provides a saving of about 8%. If we also consider the potential savings as compared to the actual transportation we get a saving of about 14%.

4.3 Cost allocation

In Table 4 we show cost allocation results, computed according to the different concepts presented earlier. As a basis for these computations, FlowOpt has been used to solve the transportation problem with direct flows for each possible coalition of companies, that is, for $2^8 - 1 - 8 = 247$ different coalitions. The total transportation cost obtained for each coalition $S$, is then used to initiate the value of $c(S)$. In the column “Individual” the companies’ costs of operating alone
Company Real opt 1 opt 2 opt 3 opt 4
---
Company 1 1,934 1,541 1,514 1,884 1,879
Company 2 3,894 3,894 3,894 3,778 3,676
Company 3 2,103 1,748 1,636 2,067 2,050
Company 4 333 303 294 333 332
Company 5 16,241 12,947 12,728 14,785 14,663
Company 6 5,084 4,363 4,328 4,959 4,846
Company 7 10,704 10,500 10,542 10,340 10,226
Company 8 4,828 4,067 3,856 4,742 4,712
Total 45,121 39,363 38,792 42,888 42,384

Table 3: Costs (in kSEK) for the real and optimization runs.

are shown. In the preceding columns we give the cost allocations obtained by the concepts and methods. For each computed cost allocation, the savings as compared to the individual costs are given.

<table>
<thead>
<tr>
<th>Company</th>
<th>Individual</th>
<th>Volume</th>
<th>Shapley</th>
<th>Shadow</th>
<th>ECM</th>
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<tr>
<td></td>
<td>Cost</td>
<td>Cost %</td>
<td>Cost</td>
<td>Cost %</td>
<td>Cost %</td>
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<tr>
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<td>1,970</td>
<td>4,6</td>
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<tr>
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<td>285</td>
<td>14,4</td>
<td>315</td>
<td>5,4</td>
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<td>3,953</td>
<td>20,3</td>
<td>4,501</td>
<td>9,2</td>
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<td>0,6</td>
<td>9,755</td>
<td>5,6</td>
</tr>
<tr>
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<td>4,742</td>
<td>4,195</td>
<td>11,5</td>
<td>4,070</td>
<td>14,2</td>
</tr>
<tr>
<td>Sum</td>
<td>42,885</td>
<td>39,083</td>
<td>8,9</td>
<td>39,083</td>
<td>8,9</td>
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</table>

<table>
<thead>
<tr>
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<th>Individual</th>
<th>ACAM</th>
<th>Nucleolus</th>
<th>CGM</th>
<th>EPM</th>
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<tbody>
<tr>
<td></td>
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<td>Cost</td>
<td>Cost %</td>
<td>Cost %</td>
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<td>9,8</td>
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<td>8,9</td>
<td>4,547</td>
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<td>9,850</td>
<td>4,7</td>
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<tr>
<td>Company 8</td>
<td>4,742</td>
<td>4065</td>
<td>14,3</td>
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<td>14,2</td>
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<tr>
<td>Sum</td>
<td>42,885</td>
<td>39,083</td>
<td>8,9</td>
<td>39,083</td>
<td>8,9</td>
</tr>
</tbody>
</table>

Table 4: Distribution of costs and their savings as compared to the individual costs.

The cost allocations computed according to the Shapley value, Volume, ACAM, nucleolus, CGM and EPM are stable. We observe that the core is non-empty and, therefore, at least the nucleolus must represent a stable cost allocation. The fact that also the Shapley value, the
ACAM, CGM and the EPM do so is interesting. The cost allocations based on shadow prices and the ECM method produce an unstable cost allocation since at least one participant is worse of in these allocations as compared with the participant working on its own. The cost allocation based on volume weights can be considered as the simplest method that we use to split costs, and no consideration is taken to the geographical distribution of the demand points. Consequently, the potential savings, as indicated in Table 4, of two companies with equal volume but with different geographical distribution of their demand points, may differ.

It is interesting to note, that according to the cost allocation based on the ECM, a number of companies loose as compared to when they operate alone. The reason for this is that the weights used to allocate the non-separable cost do not take into account each company’s individual cost and savings. It can be observed, that the two largest companies are favoured by this allocation. The cost allocation based on shadow prices does not seem to be appropriate, as one of the largest companies does loose. There can be several reasons for this. The problem has multiple dual solutions and we just use one in the tests. Other may have another property. Another practical reason may be that the company’s supply and demand locations are in areas where the marginal savings are smaller than elsewhere.

The most evenly spread cost savings are, as expected, produced by the EPM, where the maximal pairwise difference is minimized. The cost allocations obtained by the Nucleolus, ACAM and the CGM are similar. The extra information about coalitions that is added in CGM does obviously not provide any large effects in how the total cost is divided among the companies. Finally, we note that the different concepts and methods that we have used, provide us with suggested cost allocations that do not differ too much. The three methods with the most differing results, both in savings and in the fact that the cost allocations are not stable, are the volume weight method, the method based on shadow prices and the ECM.

5 Practical aspects

There are large potentials in collaborative planning. However, there are also a number of practical aspects to consider before such a process can be implemented. Below we discuss a number of these aspects.
Negotiations

In a practical situation the results in the previous section relies on the fact that all companies agree in advance to accept a cost allocation computed in any of the suggested approaches. In practice this may be difficult as there is a large difference in size between the companies or their position in the overall wood supply chain. It is very likely that the largest companies are the key drivers and have a stronger position in a negotiation. Moreover, negotiations for wood bartering between companies is normally not generated by case studies such as the one described in this paper. Instead, the initiative is taken by the timber managers representing each company. Personal relations is very important. The results from analyzes can be used in negotiations between several companies or by a freestanding organization for wood bartering. This would show the benefits and can be the decisive fact to initiate a collaboration.

Legality

An aspect of how the collaboration should be carried out concerns the law governing restrictive practices. Co-ordination of the transportation planning must be performed in a way that it cannot be interpreted as formation of cartels. The exact ruling seems to be in a bit of grey zone but the current interpretation by many companies is that collaboration and wood bartering between companies is allowed as long as it does not interfere with the overall wood market. For example, the companies are not allowed to collaborate in the buying of wood from forest land owners e.g. use the same price lists.

Tactical versus operational planning

In this paper, we have focused on collaboration between companies on a tactical level by making a destination planning of the wood. This case study has involved truck transportation but can also used when trucks are integrated with train and ship transportation. There is also additional possibilities on a lower level where the actual routing of the trucks are carried out. This is often done by several hauliers and they get a certain part of the destinations found. The hauliers may be working for a single or a combination of forest companies and they should of course be allocated destinations such that the potential for backhauling is maximized. This in order to
make the individual routing of logging trucks more efficient. This implies that the OR model [P2] described earlier should be used on a shorter planning horizon, say every week. The exchange of information can be done either between transport managers or between individual drivers.

**Shared information and quality**

A key factor for successful co-ordination of transportation planning between companies is accurate and reliable information. In our case study we made use of stored data in a common forest database. We could therefore easily compare with transportation done in practice. However, for future planning it is important that the supply and demand data is accurate. The demand is normally known with high accuracy. The supply is not known one month in advance and needs to be estimated. This must be based on what areas to be harvested and the proportion between assortments. If the planning horizon should be shorter, actual harvested volumes can be used. Companies do however use different information systems and data collection and there is a need to set up a framework where all information is kept in a standardized way.

**Valuation of wood**

In our case study the majority (70%) of the potential savings derive from transportation of sawlogs. This is natural since the saw timber is bought and harvested with respect to a certain saw mill which not always is the closest one. This means that those companies that prioritize to buy sawlogs before pulplogs will have most benefits in this analysis. It also means that it will be difficult to realize the entire potential. It is important to be able to make decisions on special bucking patterns, to produce e.g. certain dimensions for saw mills, once the destination of logs are done (as long as the assortment can be used). In Sweden, all companies use one standard to classify assortments. This contrasts many other countries e.g. Finland where it is done within each company. A common classification is an important prerequisite for collaborative planning.
**Decision support system**

There are questions about what type of organization that should collect sensitive information on supply, demand and costs and make the collaborative planning. We believe that there needs to be an independent organisation to carry out and suggest the wood bartering. This is already used in the operations to measure volume/weight of all delivered logs to mills and keeping track on all transportation carried out by trucks. This information is then used for invoicing between companies.

Companies may have different planning horizons or different storage strategies and this needs to be included in the OR model. In order for the planning to work it is required that each company involved contribute with supply and demand such that it is in balance i.e. it is possible to establish a feasible plan for each participating company. Which data that is provided depends on the agreement but some companies may choose not to include parts of their supply and demand as they want to plan this elsewhere. There may be several reasons for this e.g. some very special assortments. This would of course limit the total potential.

In order for the collaborative planning to work, the planning and wood bartering needs to be analyzed continuously. Here the actual transported volumes can be analyzed to compare against the underlying planning solutions. This could be done either by representatives from the co-operating companies, a company owned by the participating companies or by an external participant like a special logistic company.

**6 Concluding remarks**

It is very unusual that as many as eight forest companies together analyze the potential savings of cooperating within the area of transportation. However, the result of the analysis shows that there are a lot of money to save, up to 14% of the cost. Also, the environmental effects of better cooperation between the companies are very positive with about 20% reduction of emissions from the trucks. The calculated potential only includes savings caused by more efficient transportation planning, i.e. the logs are transported to the nearest mill. However, better utilization of the truck fleet will also reduce the number of needed trucks which will lead to lower costs.
Situations when several companies are cooperating will be more important in order to improve the transportation efficiency. Today, there exist systems and OR methodology that can establish coordinated plans. However, the coordination is limited to few companies that agrees on bartering volumes but not how to distribute savings. As more companies will be a part of the coordination it is not viable to agree only on volumes.

We have studied a number of economic models and proposed a new method in how the savings can be distributed taking various properties into account. These have been applied on a case study with eight companies involved in forest transportation in Sweden. The concepts based on EPM, Shapley, ACAM, nucleous and CGM provide stable cost allocations and are interesting as a basis for splitting costs. The concept based on the proposed EPM where the savings are as equal as possible provides an allocation that the companies express to be easy to accept.

As a future research we want to study how a negotiation process may take place when companies have different negotiation power. We also want to test a platform in several case studies where we can account for different planning periods and test the sensitivity in information quality.

References


