Career Concerns, Monetary Incentives and Job Design

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Abstract

We study optimal incentive contracts when commitments are limited, and agents have multiple tasks and career concerns. The agent career concerns are determined by the outside market. We show that the principal might want to give strongest explicit incentives for agents far from retirement to account for the fact that career concerns might induce behavior in conflict with the principal’s preferences. Furthermore, we show that maximized welfare might be decreasing in the strength of the career concerns, that optimal incentives can be positively correlated with various measures of uncertainty, and that career incentives have strong implications for optimal job design.

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1. Introduction

The purpose of this paper is to study optimal incentive contracts in government agencies. These organizations are characterized by limited commitments between principals and agents, by agents that have multiple tasks and career concerns, and by principals pursuing goals that, unlike financial objectives, are too complex to be summarized in one aggregate performance measure which can be rewarded directly. According to Dewatripont, Jewitt, and Tirole (1999a) these observations, and especially the last one, will imply that governmental agencies may operate more or less on a fixed budget, and that career concerns are paramount in prodding officials to pursue the agencies’ goals (p. 201). Furthermore, and as emphasized by Wilson (1989), government agencies invariably employ professionals whose career concerns are at least partly determined by their professional environment.

In recent years there has however been a trend towards more extensive use of monetary incentives in governmental agencies. Specialized health care in many OECD-countries is a prominent example where monetary incentives are introduced, e.g. through prospective payment systems. Individual performance pay is also adopted within hospitals as a means to improve performance. For example, the Detroit-based Sullivan, Cotter and Associates Inc., which tracks not-for-profit health care organizations found that 69% of institutions, most of them hospitals and medical centers, in 2000 offered some type of incentive plan. In addition, 74% of the responding institutions that collect physician performance-data, base salary or salary increases on individual performance (Sullivan, Cotter and Associates, 2001). Hence, explicit economic incentives have come to play an important role in the design of incentive schemes.

A question that naturally arises is then how the interplay between monetary incentives and professional career concerns affects individual behavior, and thereby affects the possibility of an organization to achieve its goals. More specifically, how can the management of a governmental agency, by offering agents monetary incentive contracts, induce behavior consistent with its preference? What does the optimal incentive scheme look like in the presence of professional career concerns, what are the implications for job design, and what are the implications for welfare? These are the questions we address in this paper. To do so we put forward a simple dynamic multitask career concern model with monetary incentives. To emphasize the observations mentioned above we assume that the principal’s gross benefit cannot be rewarded directly, and that the agent’s career concerns are determined by the professional environment. We also assume that commitment to long-term contracts is limited.

The following example illustrates the type of situations we have in mind. Consider a physician’s choice between treating more patients or spending more time on fewer patients within a fixed time-budget. Both types of actions will typically improve patients’ health status, and thus contribute to the hospital management’s (the principal’s) gross benefit. An aggregate performance signal on the improvement in patients’ health status
will however, typically not be available. As a result management must base incentive contracts on alternative performance measures, e.g. the number of patients treated. But there often exist additional observable, although not verifiable, signals that reveal information about the agent’s effort, such as measures of the quality of the treatment given. Such signals are typically not verifiable since it is too costly to specify ex ante the quality aspects of treatment in terms that can be verified ex post by a third party. On the other hand, the quality of the given treatment provides some information about the physician’s ability to both the inside principal and outside hospitals (or the outside market) through professional networks, and hence, there are career incentives related to such signals.3

The general conclusion we obtain is that the optimal incentive scheme must balance professional career concerns in two ways. Firstly, monetary incentives must balance career incentives on the task which can be economically rewarded. Secondly, monetary incentives must balance how the agent should divide his/her effort among the tasks. This general conclusion is rather intuitive, but the optimal incentive scheme we derive has many implications that we believe give contributions both to our understanding of the public sector, and to the theory of incentives. We now describe these implications, and the relevant literature, in more detail.

The first observation we make is that the optimal incentive scheme may be strongest earliest in agents’ careers. This result resembles the fact often observed in government agencies where subordinates get paid overtime, while more senior officers are paid a fixed salary. In the theory of incentives it is however often argued that optimal incentives are increasing over time if agents have career concerns.4 This result was put forward by Gibbons and Murphy (1992) who showed that an optimal compensation contract optimizes the combination of monetary and career incentives. And as career incentives decrease over time, it is necessary to boost monetary incentives for agents close to retirement to induce a certain effort level. The key to understand why their result is at variance with ours is to note that Gibbons and Murphy (1992) modelled incentive contracts in situations where there exists an aggregate contractible measure of the principal’s gross benefits.5 When such an aggregate measure exists, the division of effort between different tasks can be delegated to the agent. Technically this is equivalent to modelling agents that only exert effort on one task, as Gibbons and Murphy do. This implies that monetary incentives and career concern incentives become substitutes in their framework; higher career concerns reduce the required monetary incentives needed to induce a certain effort level. Since career concerns are strongest earliest in agents’ careers, the

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3See e.g. Le Grand (1999), Grout, Jenkin, and Propper (2000), Croxson, Propper, and Perkins (2001) and Gaynor, Rebitzer, and Taylor (2001) for evidence that physicians’ behavior are driven by both career concerns and monetary incentives.

4The fact that career concerns is a means to provide incentives for exerting effort was first discussed by Fama (1980) and Holmstrom (1982). Fama (1980) argued that incentive contracts are not necessary since agents are disciplined by career concerns, while Holmstrom (1982) showed that career concern incentives are not sufficient to induce efficient effort. Building on this fact, Gibbons and Murphy (1992) added explicit contracts to the Fama-Holmström model.

5Specifically they examined the relationship between chief executive compensations and stock market performance.
required monetary incentives needed to induce a certain effort level are lower for agents far from retirement.

We do however believe that such aggregate performance measures are not available in many governmental organizations, e.g. in health care. Hence, it is more problematic for (health care) principals to let agents determine for themselves how to split effort between tasks. In these organizations, monetary incentives must be set not only in response to career incentives on a single task, but also to serve the function of balancing the agent’s effort between tasks. As a result monetary incentives and career concern incentives are \textit{complements} between the tasks. That is, higher career concerns (on one task) imply higher monetary incentives on other tasks to induce the same split of efforts between the tasks.

It has been pointed out that a complementarity effect between monetary and career incentives may arise for another reason, namely when there is technological complementarity between effort and talent in the way they affect performance. Dewatripont, Jewitt, and Tirole (1999b) show that a complementarity effect may arise in the single-task case if the effort structure is multiplicative in this way. In this case these authors show that multiplicity of equilibria can arise: market expectations about high or low effort can be self-fulfilling. In addition Dewatripont, Jewitt, and Tirole (1999b) show that complementarity effects between these two types of incentives may arise such that raising monetary incentives may increase career incentives either locally around a certain equilibrium or globally to affect the set of equilibria. Note, however that these results do not hold when they consider an additive effort structure (as in the model presented here). Furthermore, the main focus in Dewatripont, Jewitt, and Tirole (1999b) is on career incentives, and not on the interplay between monetary incentives and career concerns.

Second, we show that the presence of career effects produce incentives that can be highly non-monotone in observable measures of uncertainty. Consequently, we offer a possible explanation for the fact that empirical studies observe both a positive and negative correlation between risk and incentives. Specifically, and in contrast with the theoretical prediction of the traditional principal-agent model, we show that optimal monetary incentives are increasing in the noise of the verifiable signal. The reason is simple; more noise on this signal shifts the attention the market gives to performance from this signal to other signals when calculating the agent’s talent. This shift in attention reduces the agent’s career incentives on the verifiable task, making it necessary for the principal to offer stronger monetary incentives to restore the balance between total incentives on the two tasks.

Third, we find that career concern incentives might be harmful for welfare. The intuition behind this result is that career effects may be so strong that the agent’s cost of providing more effort outweighs the associated increase in production value.

Fourth, we provide new insight into the question of whether implicit incentives take

\footnotetext[6]{See also Dewatripont, Jewitt, and Tirole (1999a).}
\footnotetext[7]{Prendergast (2000) gives an overview of the empirical literature on the tradeoff of risk and incentives. See also Prendergast (1999, 2002).}
\footnotetext[8]{Holmstrom (1982) contains a similar result.
the form of career incentives or ratchet effects. Prendergast (1999) considers this issue. He considers a model where monetary contracts are based on a (single) subjective assessment of performance, and the agent can exert productive effort as well as unproductive ‘bias activity’ to influence the assessment. There are then career incentives when the agent is equally productive in all firms and they compete for his services. Monetary and implicit incentives are substitutes, and it is pointed out that monetary incentives are increasing in the noisiness of the subjective performance measure. It is also pointed out that the implicit incentives take the form of ratchet effects when the agent’s talent has productive value only for the inside firm.

Contrary to Prendergast’s focus on subjective performance assessments and unproductive influence activity, ours is on ‘productive multitasking’ with verifiable performance measures being available for some, but not all tasks. In this setting implicit incentives take the form of either career concerns or ratchet effects depending on whether the market values the agent’s talent more than the inside firm values the agent’s effort-productivity. That is, in our model ratchet effects come into being when one unit talent is less productive in the market than one unit effort for the inside firm.

Finally we consider the case where the principal hires several agents. The main issue under consideration is how the principal should organize tasks among agents in cases where it is possible to separate tasks (e.g., medical research and treatment of patients). That is, should each agent have sole responsibility for one task, or should the principal offer the agents jobs in which they both bear joint responsibility for both tasks? We find that joint responsibility leads to weaker individual career incentives compared to sole responsibility. In some situations such weak career incentives are detrimental, and to such an extent that sole responsibility is the better organizational design. In other situations career incentives are too strong when jobs are separated, and then joint responsibility is the better design. These results indicate that career concerns have strong implications for optimal job design.

The paper is organized as follows. In section 2 we outline the model and the first-best solution. Section 3 characterizes optimal contracts when the principal is hiring one agent, and the multi-agent problem is analyzed in section 4. Finally, section 5 presents some concluding remarks.

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2. The model

There is one agent, two tasks (with associated signals $z$ and $q$), and two periods. For concreteness we can think of the tasks as provision of quantity and quality, respectively, for some product. The tasks compete for the agent’s attention, and efforts are thus substitutes in the agent’s cost function. The agent’s choice of efforts determines the agent’s total contribution to the principal, denoted by $y_t$. That is, $y_t$ reflects everything the principal cares about, except for wages, in period $t$. We assume that no contract on $y$ can be enforced in court because it is prohibitively costly to specify this outcome ex ante in such a way that it can be verified by a third party ex post. We do however assume that all parties—insiders as well as outsiders—observe the $y$—signal ex post.10

Contrary to the signal on the agent’s total contribution, the performance signal associated with one task ($z$) is verifiable, so monetary incentives for that task can be provided through this (production) signal. Hence incentives on the production signal serves as a means to increase the agent’s total contribution for the principal. Since this signal is verifiable, all parties observe it.

The performance signal associated with the other task (the quality signal, $q$) is non-verifiable. Yet some incentives are provided for this task through career concerns. We consider the case where these career concerns are determined by the outside market (or outside principals or the professional environment). All parties—insiders as well as outsiders—observe the $q$—signal, and favorable realizations of this signal improve the agent’s standing on the job market. To sum up, the principals offer the agent (linear) payments

$$w^i_t = A^i_t + \alpha^i_t z^i_t,$$ 

where $i = I, O$ denotes the inside and outside principals, respectively.11

The agent which is risk-neutral privately chooses $(a_t, b_t)$, where $a_t$ ($b_t$) is effort supplied into the production of $z_t$ ($q_t$). The private cost (in monetary units) is $C(a_t, b_t)$, where $C(\ldots)$ is strictly convex, and efforts are substitutes for the agent: $C_{ab} := \frac{\partial^2 C}{\partial a \partial b} > 0$.

To simplify the algebra we assume a quadratic cost function

$$C(a, b) = \frac{1}{2}a^2 + \frac{1}{2}b^2 + \gamma ab, \quad 0 \leq \gamma < 1$$

When the agent works for principal $i$ ($i = I, O$) the relevant signals are

$$y^i_t = h^i \eta + f^i a_t + g^i b_t + \varepsilon^i_t, \quad i = I, O$$
$$z^i_t = \eta + a_t + \nu^i_t,$$
$$q^i_t = \eta + b_t + \tau^i_t,$$

where $\eta$ is the agent’s unknown ability. The ability $\eta$ is drawn at the beginning of the first period from a normal distribution with mean $m_0$ and variance $\sigma^2_\eta$, i.e. $\eta \sim N(m_0, \sigma^2_\eta)$. We also assume that $\varepsilon^i_t \sim N(0, \sigma^2_{\varepsilon^i_t})$, $\nu^i_t \sim N(0, \sigma^2_{\nu^i_t})$, $\tau^i_t \sim N(0, \sigma^2_{\tau^i_t})$, and the productivity

10 Kaarbøe and Olsen (2001) study a similar model where insiders have more information than outsiders.

11 The focus on linear contracts can be justified by appeal to a richer dynamic model in which linear payments are optimal (Holmström and Milgrom, 1987).
parameters \((h^i, f^i, \text{ and } g^i)\) are positive. All noise terms are independent of each other and of ability \(\eta\). All parties observe \(x^i_t = (y^i_t, z^i_t, q^i_t)\). The principals competing in every period for the agent’s services can observe neither the actions taken by the agent nor his ability. They only observe the output \(x^i_t\), and use it in every period to update their beliefs about his ability. The net benefit for the (risk-neutral) principal who employs the agent is given by \(y^i_t - w^i_t\), and total surplus for the principal and the agent is given by \(S^i_t(a^i_t, b^i_t) := y^i_t - C(a^i_t, b^i_t), \ i = I, O\).

We further assume that, after an agent has worked for a principal, a special relationship is formed between the two, e.g. due to the agent learning specific ways to perform the tasks, resulting in an increased fixed benefit for this principal from keeping the agent in his service. The additional benefit is sufficiently large that the inside principal will always want to retain the agent, even if unfavorable signals are observed in the first period. This kind of assumption is in line with assumptions made in the existing literature (e.g. Gibbons and Murphy 1992; Meyer and Vickers 1997). To simplify notation we will drop superscript \(I\) when referring to variables generated inside this relationship. Thus \(x_t = (y_t, z_t, q_t)\) and \((h, f, g)\) refer to information signals and productivity parameters, respectively, when the agent works for the inside principal. Finally let only one-period contracts be feasible.

### 2.1. First-best efforts

As a reference case consider the first-best solution (when efforts are contractible). Principal \(i\) will then choose efforts each period to maximize \(f^i a^i_t + g^i b^i_t - C(a^i_t, b^i_t)\). Focusing on the 'inside' principal, we find that the optimum is as follows.

1. **When marginal productivities on the two tasks are sufficiently close, \(f - \gamma g > 0\) and \(g - \gamma f > 0\), it is optimal to induce efforts on both tasks.** First-best efforts and value (each period) are then \(a^{FB} = \frac{f - \gamma g}{1 - \gamma^2}, b^{FB} = \frac{g - \gamma f}{1 - \gamma^2}\) and \(S^{FB} = h m_0 + \frac{1}{2} f^2 + \frac{g^2 - 2 \gamma f g}{1 - \gamma^2}\), respectively, where \(m_0\) is expected ability.

2. **Otherwise, when marginal productivities are not close, it is optimal to concentrate effort only on the most productive task.** For \(g - \gamma f < 0\) we have \(b^{FB} = 0\), \(a^{FB} = f\) and \(S^{FB} = h m_0 + \frac{1}{2} f^2\). For \(f - \gamma g < 0\) we have \(a^{FB} = 0\), \(b^{FB} = g\) and \(S^{FB} = h m_0 + \frac{1}{2} g^2\).

To see the intuition for case II note that, starting from \(a = f, b = 0\) the marginal cost of exerting effort on \(b\) is \(C_b(a, 0) = \gamma a = \gamma f\). If this cost exceeds the marginal value \(g\), it is not worthwhile to exert effort on the \(b\)—task.

We are here primarily interested in the case where it is first-best efficient to have the agent working on both tasks, so we will in the following assume that marginal productivities are close, so that case I applies. Note that for equal productivities \((g = f)\), the first-best value is \(S^{FB} = h m_0 + \frac{f^2}{1 - \gamma^2}\). This value is clearly higher than what can be obtained by concentrating effort on only one task.
3. Optimal contracts for one agent

Consider now the case where contracts can only be written on signal $z_t$. Assume further that only short-term contracts can be written. The agent starts working for the inside principal in period 1. In the second period the agent may leave and seek outside employment. We assume that there is a (small) positive probability $p > 0$ that the agent must leave for exogenous reasons, such as a move triggered by a job change for the agent’s spouse etc., and that an outside principal cannot observe whether the agent leaves voluntarily or due to such exogenous events. Competition among the outside principals will then ensure that the agent is offered a contract, $w^O_2(x_1)$, that earns zero expected profits for such a principal.\footnote{We assume that outside principals offer relatively simple contracts and hence do not offer screening contracts.} This will be an equilibrium because (a) the inside principal will in any case match this offer, hence (b) there is no reason for the agent to leave voluntarily (no self-selection), and (c) an outside principal cannot therefore deduce anything helpful about the agent’s type from her behavior on the job market.

To characterize the optimal contract note that the agent’s problem in an arbitrary period is given by

$$\max_{a,b} \{ A + \alpha a + \beta b - C(a,b) \},$$

where $\beta$ is the career incentive on the $q$-task and $\alpha$ is the effective incentive on the $z$-task.\footnote{There may be career incentives, say $\beta_a$, also on the latter task, and then $\alpha = \alpha^x + \beta_a$, where $\alpha^x$ is the explicit incentive on that task.} The first-order conditions (for an interior solution) are $C_a - \alpha = 0$, and $C_b - \beta = 0$, where $C_i := \frac{\partial C_i}{\partial a}$, $i = a, b$. These conditions define efforts as functions of effective incentives; $a = a(\alpha, \beta)$ and $b = b(\alpha, \beta)$. For later reference we differentiate the first-order conditions w.r.t $\alpha$ and obtain $b_a := \frac{\partial b}{\partial a} = -\gamma a$, and $a_a := \frac{\partial a}{\partial a} = \frac{1}{1 - \gamma^2}$, where $\gamma := C_{ab}$.

We now characterize the optimal contract in the second, and last, period. In this period there is no career incentives. Hence $b_2 = 0$ and total expected surplus when the agent is working for principal $i$ is given by

$$E S^i_2 = h^i E (\eta \mid x_1) + f^i a(\alpha_2) - C(\alpha_2, 0),$$

where $E$ is the expectation operator. By differentiating this expression we obtain

$$\frac{\partial}{\partial \alpha_2} E S^i_2 = (f^i - C_a) a_{\alpha_2}.$$  

It is obviously optimal to set $\alpha^*_2 = f^i$. Hence, the agent is offered incentives that are efficient for the $z$-task in isolation in the second period.

Competition among outside principals will ensure that the agent is offered a contract that earns zero expected profits for such a principal, i.e.: $E (y^O_2 \mid x_1) = E (w^O_2 \mid x_1)$. In order to retain the agent the inside principal must match this offer, hence the wage contract $(w_2 = A_2 + \alpha_2 z_2)$ must satisfy

$$A_2 + \alpha_2^* E (z_2 \mid x_1) - C^* \geq E (y^O_2 \mid x_1) - C^{*O},$$

where $C^*$ and $C^{*O}$ are the effort costs if the agent works for the inside or the outside principal, respectively. It follows that the salary component $A_2$ satisfies $A_2 = $
\((h^O - \alpha_2^*) E(\eta \mid x_1) + \text{const}\), where the constant is independent of \(x_1\). The optimal second-period wage contract thus takes the form:

\[
w_2(x_1) = (h^O - \alpha_2^*) E(\eta \mid x_1) + \alpha_2^* z_2 + \text{const}, \quad \text{where}
\]

\[
E(\eta \mid x_1) = E\eta + R_z (z_1 - E z_1) + R_q (q_1 - E q_1) + R_y (y_1 - E y_1).
\]

The exact expressions for the regression coefficients \(R_i = \frac{\partial}{\partial x_i} E(\eta \mid x_1), i = y, q, z\) are contained in Appendix A.2. Here we simply note that \(R_i \in [0, 1]\) and depends on the noise terms \(\sigma_i^2, i = \eta, y, z, q\), as well as on the productivity parameter of ability \(h\). Furthermore we note that the if the \(z\)-signal is more noisy than the \(q\)-signal (i.e., \(\sigma_z^2 > \sigma_q^2\)), more weight is put on \(z\) relative to \(z\) in estimating the agent’s ability.

After characterizing the second-period wage contract we turn to period one. First of all we notice that since the second period compensation depends on the first period signals, \(x_1 = (y_1, z_1, q_1)\), the agent has incentives to exert effort in the first period to affect his market value. Working for the inside principal the agent thus chooses effort according to

\[
\max_{a_1, b_1} \{ \alpha_1 a_1 - C(a_1, b_1) + (h^O - \alpha_2^*) E(\eta \mid x_1) + \text{const} \}.
\]

where \(E(\eta \mid x_1)\) is calculated on the basis of expected (equilibrium) efforts, so that the marginal effect of an effort deviation on this expectation is given by the relevant regression coefficients \(R_i\). The first-order conditions for the agent are then:

\[
\begin{align*}
a_1 & \geq 0, \\
C_a & \geq \alpha_1 + (R_z + f R_y) (h^O - \alpha_2^*):= \alpha_1 + \beta_a, \\
b_1 & \geq 0, \\
C_b & \geq (R_q + g R_y) (h^O - \alpha_2^*):= \beta_b.
\end{align*}
\]

(3.1a) \hspace{1cm} (3.1b)

where \(\beta_i\) is the implicit (career incentive) on task \(i = a, b\), and the inequalities hold with complementary slackness.

To characterize optimal first-period incentives we differentiate the expression for total expected surplus in period one, \(E y_1 - C(a_1, b_1)\), and obtain \(\frac{\partial}{\partial a_1} E S = (f - C_a) a_a + (g - C_b) b_a\), where \(b_a = -\gamma a_a\) and \(a_a = \frac{1}{1 - \gamma}\) for interior solutions (as before). Interior solutions (efforts on both tasks) are optimal for the principal when the implicit incentive on the \(b\)-task exceeds some critical value \((\beta_b > \beta_{\text{crit}})\), see below. In that case we can substitute from the agent’s first-order conditions into the expression for \(\frac{\partial}{\partial a_1} E S\) to see that the optimal first-period monetary incentive is given by

\[
\alpha_1^* = \alpha_2^* - \beta_a + \gamma (\beta_b - g), \quad (\alpha_2^* = f).
\]

We can now analyze how optimal monetary incentives vary over time.

First we consider the case where one unit talent is less productive than one unit effort on the \(z\)-task, i.e. the case where \(h^O \leq f\). In this case both \(\beta_a\) and \(\beta_b\) are non-positive. Hence, the agent will choose zero effort on the \(q\)-task \((b_1 = 0)\), and the principal will consequently ensure that total incentives on the \(z\)-task equal the productivity parameter on that task, i.e. \(\alpha_1^* + \beta_a = f\). When \(\beta_a < 0\) there is a ratchet effect associated with
the $z$-task, and optimal monetary incentives in period one have to be larger than the optimal incentives in the second period, thus $\alpha_1^* > f = \alpha_2^*$.

If $hO > f$, things are different. In this case there are career incentives on both tasks, and the agent will optimally choose to provide effort on both tasks if incentives on the two tasks do not deviate too much. Now optimal monetary incentives must balance not only the career incentives on the $z$-task but also how the agent should divide his effort between the two tasks. Note that this latter effect depends on how close the career incentives on the $q$-task are to the first-best effort on this task, i.e., on $\beta_b - g$, and on how close substitutes the tasks are in the agent’s cost function, i.e., on $\gamma$. When the size of these two effects are small, that is when either $\beta_b - g$ or $\gamma$ are close to zero, optimal first-period monetary incentives are set mainly in response to the career incentives on the $z$-task. Since career incentives are positive, first-period monetary incentives will be lower than second-period incentives.

On the other hand, when either $\beta_b - g$ or $\gamma$ are large, the principal puts less emphasis on the $q$-signal and thus raises first-period monetary incentives to induce more effort on the $z$-task. If in addition the career incentives on the $z$-task are low, e.g., because principals put a relatively small weight on this signal in estimating the agent’s ability ($\sigma_z^2$ large), first period monetary incentives will typically be larger than second-period incentives.

The following proposition sums up this discussion and provides a formal characterization of optimal incentives. See Appendix A.1 for a proof.

**Proposition 1.** i) Suppose $hO \leq f$, i.e. that talent is less productive on the outside than ‘quantity effort’ is on the inside. Then the agent will not to provide any effort on the $q$-task, and there is a ratchet effect associated with the $z$-signal. Furthermore, optimal monetary incentives are strongest early in the agent’s career.

ii) Suppose $hO > f$. Then there are career effects on both tasks. There is a critical value $\beta^{crit} \in (0, g)$ for the implicit incentive on the $q$-task such that the following holds:

(a) For $\beta_b \leq \beta^{crit}$ it is optimal to induce effort only on the $z$-task, and the optimal monetary incentive on that task satisfies $\alpha_1^* + \beta_a = f$. Monetary incentives are then lowest early in the agent’s career.

(b) For $\beta_b > \beta^{crit}$ it is optimal to induce efforts on both tasks, and the optimal monetary incentive (on the $z$-task) is given by (3.2). Monetary incentives are then strongest (weakest) early in the agent’s career if and only if $\gamma (\beta_b - g) - \beta_a > (<) 0$. The optimal first-period value is in this case

$$S^* = hm_0 + (g\beta_b - \frac{\beta_b^2}{2}) + \frac{1}{2} \frac{(f - \gamma g)^2}{1 - \gamma^2}$$

We now relate Proposition 1 to the existing literature of monetary incentives and career concerns.

The fact that optimal monetary incentives can be strongest early in the agent’s career in the presence of career effects is at variance with the predictions from the theoretical model in Gibbons and Murphy (1992), and is due to the fact that monetary incentives
here also serve the task of allocating the agent’s effort between the two tasks. In this sense monetary and implicit incentives effectively become complementary if there is strong substitutability between the tasks in the agent’s cost function (large $\gamma$), or if career incentives are too high on the $q-$task, $\beta_b > g$ such that higher career incentives on that task imply a shift in focus implying higher monetary incentives on the other task.\(^{14}\)

Secondly, we comment on the budget-run agencies-result from Dewatripont, Jewitt, and Tirole (1999b). This result is stated in their Proposition 3.3, and says that if $i$) the principal only cares about the sum of the agent’s effort (and not its distribution among tasks), $ii$) that only one task is contractible, $iii$) that efforts are perfect substitutes in the agent’s cost function, and $iv$) that there are equal and positive career incentives on all non-verifiable tasks, then in the additive case, the agency is run as a fixed-budget agency. In our framework this situation is captured in equation 3.2 when both tasks are equally productive for the inside principal, i.e. $f = g$, the noisiness of the $q-$ and $z-$signals are the same so that $\beta_a = \beta_b$, and when $\gamma \lesssim 1$, so that efforts are almost perfect substitutes in the agent’s cost function. With these restrictions, the principal cannot give the agent monetary incentives, and the equilibrium effort levels are implemented by giving the agent a fixed budget. In our view, this shows that the budget-run agencies-result from Dewatripont, Jewitt, and Tirole (1999b) builds on strong assumptions, and that monetary incentives typically can be provided without abandoning the other tasks.

Thirdly, we note that implicit incentives take the form of either career concerns or ratchet effects depending on whether the market values the agent’s talent more than the inside firm values the agent’s effort-productivity. Specifically ratchet effects come into being when one unit talent is less productive in the market than one unit effort for the inside firm. This result is to be contrasted to the result in Prendergast (1999) who shows that implicit incentives take the form of ratchet effects when the agent’s talent has productive value only for the inside firm.

Finally, we note that this model produces comparative statics results in line with those of Holmstrom (1982) regarding uncertainty about the agent’s ability: career incentives are monotonically increasing in the ability variance, $\sigma^2_n$. Note however that optimal first-period monetary incentives are increasing (decreasing) in the ability variance only when $\gamma$ is high (low). This result follows from the fact that optimal monetary incentives are increasing in career concerns associated with the $q-$task and decreasing in career incentives associated with the $z-$task. Hence the relative magnitude of these two career effects will determine if first-period monetary incentives increase or decrease with the ability variance. This magnitude again depends on the degree of substitutability between the two tasks in the agent’s cost function. In the same vein, optimal incentives are decreasing in the market noise $\sigma^2_q$.\(^{15}\)

\(^{14}\)Dewatripont, Jewitt, and Tirole (1999b) show that a similar result may arise in the case where the effort structure is multiplicative. Note, however, that this result does not hold when they consider an additive effort structure (as in the model presented here). Kaarboe and Olsen (2001) show a similar result in the case where the tasks are perfect substitutes for the agent. Then explicit incentives on one of the tasks must equal the career incentives on the other task if the principal prefers effort on both tasks.

\(^{15}\)This conclusion is however not so straight forward as it may seem, since the career incentives...
More interestingly, optimal monetary incentives are here increasing in the noise of the verifiable signal, $\sigma_z^2$. The reason is simple; more noise in this signal reduces the implicit career incentive on this task and increases the career incentive on the other task. This shift in career incentives is induced by the market now putting relatively less weight on the more noisy signal when updating its beliefs regarding the agent’s ability. Since implicit incentives as a result shift towards the non-verifiable task, the principal must offer more monetary incentives to restore the balance between total incentives on the two tasks. The following proposition sums up the discussion. See Appendix A.4 for a proof.

**Proposition 2.** Suppose $h^O > f$, and define $\tau := \frac{\sigma_q^2(\sigma_q^2 + h\sigma_z^2)}{\sigma_z^2(\sigma_q^2 + h\sigma_y^2)}$. Then optimal monetary incentives are

i) increasing in the ability noise if $\gamma > \tau$, and decreasing for $\gamma < \tau$.

ii) decreasing in the market noise ($\sigma_q^2$), and

iii) increasing in the noise of the verifiable signal.

We now turn to welfare analysis. More specifically we want to analyze how implicit incentives on the two tasks affect the total expected surplus for the principal and the agent. A generalization of Proposition 1 (from period one to any period $t$) shows that the optimal value for the principal and the agent is given by\footnote{See also Appendix A.5.} $S^* = h\beta_b + (g\beta_b - \frac{\beta_y^2}{2}) + \frac{1}{2}(f^2 - 2g\gamma)^2$, for $\beta_b > \beta_{\text{crit}}$. From this expression we immediately have the following proposition, which is parallel to one of the results obtained in Meyer and Vickers (1997) and Holmstrom (1982), (part i) and part ii) respectively.

**Proposition 3.** Suppose it is optimal to induce effort on both tasks. Then

i) Welfare is independent of implicit incentives on the verifiable task (can be neutralized by monetary incentives).

ii) First-period welfare varies non-monotonously with the implicit career incentives on the non-verifiable task.

4. Joint vs. sole responsibility

We now consider the case where the principal wants to hire two agents. The main issue under consideration is how the principal should organize the tasks among the agents. That is, in situations where it is possible to split the tasks, should each agent have sole responsibility for one task, or should the principal offer the agents jobs in which they both bear joint responsibility for both tasks? One such situation arises e.g. if the tasks are treatment of patients and medical research or teaching.\footnote{See also Holmstrom and Milgrom (1991) and Meyer, Olsen, and Torsvik (1996) for analyses of optimal job-design.}Associated with the $q$-task may in fact increase with the market noise if the principal values the $q$-task so highly that she chooses to implement no effort on the $z$-task. Since our focus is on multitasking we abstract from this situation.

\[^{16}\text{See also Appendix A.5.}\]

\[^{17}\text{See also Holmstrom and Milgrom (1991) and Meyer, Olsen, and Torsvik (1996) for analyses of optimal job-design.}\]
We assume that the agents are identical and that agents’ abilities are uncorrelated. Hence, the signals generated by one of the agents are uninformative about the other agent’s ability (and efforts). Finally, let $S_i^t$ $i = S, J$ denote total expected surplus for the principal and the agents in period $t = 1, 2$ for the case of sole respectively joint responsibility.

As in the case of one agent, we first characterize the first-best solution.

4.1. First-best

Again we assume an interior solution, i.e.: $f - \gamma g > 0$ and $g - \gamma f > 0$, or $\gamma < \min \{ \frac{f}{g},\frac{g}{f} \}$.

From section 2.1 we know that effort on both tasks is optimal, and that the optimal value is

$$S_{FB} = \frac{g^2}{2} + \frac{1}{2} \frac{(f - \gamma g)^2}{1 - \gamma^2} = \frac{f^2}{2} + \frac{1}{2} \frac{(g - \gamma f)^2}{1 - \gamma^2}$$

This value is higher than concentration on any single task ($S_{FB} > \max \{ \frac{1}{2} f^2, \frac{1}{2} g^2 \}$) since

$$S_{FB} - \frac{g^2}{2} = \frac{1}{2} \frac{(f - \gamma g)^2}{1 - \gamma^2} > 0, \quad S_{FB} - \frac{f^2}{2} = \frac{1}{2} \frac{(g - \gamma f)^2}{1 - \gamma^2} > 0$$

The last term is the contribution from effort being ‘spread’ to the second task.\(^{19}\)

If two agents are working for the principal and have sole responsibility for one task it follows that total surplus is $S^S = \frac{1}{2} f^2 + \frac{1}{2} g^2$. On the other hand, if both have joint responsibility we get

$$S^J = 2S_{FB} = \frac{1}{2} f^2 + \frac{1}{2} g^2 + \frac{1}{2} \frac{(f - \gamma g)^2}{1 - \gamma^2} + \frac{1}{2} \frac{(g - \gamma f)^2}{1 - \gamma^2}$$

From this it follows that $S^J > S^S$, and thus the first-best optimal job design is one where the agents have joint responsibility.

4.2. Job Design and Agency

Sole responsibility. In this case one agent is working on task $a$, and one on task $b$. Four information signals are generated in each period, $y^1_t, y^2_t, z_t$ and $q_t$, where $y^1_t$ denotes the total contribution of agent $i = 1, 2$. We first consider agent one who is working on task $a$. His choice problem in period $t$ is $\max_{a_t} \left\{ A_t + \alpha_t^S a_t^1 - C(a_t, 0) \right\}$, where $\alpha_t^S$ is the total incentive on task $a$, i.e. the sum of explicit ($\alpha_t^S$) and implicit ($\beta_t^S$)incentives: $\alpha_t^S = \alpha_t^S + \beta_t^S$. Note that the implicit incentives are determined by the information signals generated by agent one, that is by $y^1_{t-1}$, $z_{t-1}$. Solving the agent’s maximization problem gives us the first-order condition: $\alpha_t^S = C_{a_t} = a_t$. Since total expected surplus

\(^{18}\)To simplify notation we drop the the term $hm_0$ in this section.

\(^{19}\)The intuition for this is that, starting from $a = f, b = 0$ the marginal cost of exerting effort on $b$ is $C_b(a, 0) = \gamma a = \gamma f$. When this cost is less than the marginal value $g$, it is advantageous to exert some effort on the $b$--task.
in period $t$ is given by $y^1_t - C(a_t)$, optimal monetary incentives are adjusted such that $f - C_{a_t} = 0$. Hence, optimal monetary incentives will adjust the implicit incentives such that the agent’s effort is efficient for the $z$-task in isolation, i.e., $a_t = f$.

Similar considerations for agent two gives us $C_{b_t} = b_t = \beta_{b_2}^S$, where $\beta_{b_2}^S$ is the implicit (career) incentive on the $b$-task in this setting. Total maximal expected welfare for both agents and the principal (seen from period one) is thus $S^{**} = \frac{3}{2}f^2 + g - \frac{1}{2}\beta_{b_1}^S \beta_{b_2}^S$. Note that $\beta_{b_2} = 0$, and that the principal realizes this such that both agents are working on task $a$ in the second period.

**Joint responsibility.** Suppose now we assign jobs such that both agents are working on both tasks. Hence six information signals are generated in each period ($y_i^t, z_i^t, q_i^t$, $i = 1, 2$). We know that this job design is optimal in a first-best world where monetary incentives can be provided on both tasks. The question is here whether the agency problems associated with this design may be worse than those associated with the design where agents have sole responsibility. We see that the information structures (e.g. the number of signals) are different for the two designs, and we will show that this may in fact make sole responsibility the better alternative.

By solving the agents’ maximization problem for the current case (joint responsibility), and by assuming an interior solution, we know from Proposition 1 that maximal welfare in period $t$ is given by $S_t^J = (g - \frac{1}{2}\beta_{b_1}^J)\beta_{b_1}^J + \frac{1}{2}(\frac{1}{\gamma_2} - \beta_{b_1}^S)^2$, where $\beta_{b_1}^J$ is the implicit (career) incentive on the $b$-task in this case. Note that $\beta_{b_2}^J = 0$, such that equilibrium efforts in period 2 are $b_2 = 0, a_2 = f$, and hence $S_2^J = S_2^S = 2(\frac{1}{2}f^2)$ . From this it follows that the principal’s decision about job design is determined by comparing total expected welfare for sole, respectively joint, responsibility in period one. The principal’s decision on job design is thus determined by

$$S_1^J - S_1^S = 2(g - \frac{1}{2}\beta_{b_1}^J)\beta_{b_1}^J - \left(g - \frac{1}{2}\beta_{b_1}^S\right) \beta_{b_1}^S + \frac{(f - \gamma g)^2}{1 - \gamma^2} - \frac{1}{2}f^2 < 0. \quad (4.1)$$

The first two parts in this expression reflect the contribution from the $b$-task; the two latter parts thus reflect the contribution to total welfare from the $a$-task. We first consider the $b$-task.

From the agent’s first order condition (equation (3.1b)) it follows that $\beta_{b_1}^i = (R_{i}^q + gR_{i}^q) \left(h^O - \alpha_z^i\right)$, $i = S, J$, where $R_{i}^S = \frac{\partial}{\partial \eta} E (\eta \mid y_i^t, q_i)$, and $R_{j}^J = \frac{\partial}{\partial \eta} E (\eta \mid y_i^t, z_i^t, q_i^t)$, $i = 1, 2$ and $j = q, y$. The exact expressions for these regression coefficients are contained in Appendix A.6. Here we simply note that $R_{i}^S = \lim_{\sigma_i^2 \to \infty} R_{i}^J$, and that $\frac{\partial R_{i}^J}{\partial \sigma_i^2} > 0$, $j = y, q$. Note that these facts imply that $\beta_{b_1}^J < \beta_{b_1}^S$. That is, joint responsibility leads to weaker individual career incentives on the non-verifiable task compared to sole responsibility. When each agent works on both tasks (joint responsibility) the market can base its assessment of each agent’s ability on three agent-specific signals ($y_i^t, z_i^t, q_i^t$). The weight put on the non-verifiable $q$-signal is then smaller than if the market can base its assessment only on two agent-specific signals ($y_i^t, q_i^t$), and this leads weaker incentives on the non-verifiable task under joint compared to sole responsibility. Such weak career incentives may be detrimental, and to such an extent that sole responsibility becomes a better organizational design.
Suppose for instance that parameters are such that we have $\beta_{b_1}^S$ close to the marginal productivity $g$, while $\beta_{b_1}^J$ is small and close to the critical value $\beta_{crit}$. By definition of $\beta_{crit}$ we see that for $\beta_{b_1}^J = \beta_{crit}$ we have $S^{*J} - S^{*S} = \frac{1}{2} f^2 - (g - \frac{1}{2} \beta_{b_1}^S) \beta_{b_1}^S$. When $f < g$ we further see that there is a range of values for $\beta_{b_1}^S$ (including $\beta_{b_1}^S = g$) where this difference is negative, and hence where sole responsibility will be the optimal design. Further considerations of this difference show the following (see the appendix):

**Proposition 4.** Sole responsibility leads to stronger career incentives than joint responsibility, and may for this reason be a better way to assign jobs. In particular, sole responsibility is better than joint responsibility when $\beta_{b_1}^S$ is 'large' and $\beta_{b_1}^J$ and $\beta_{b_1}^S$ are 'close' ($\beta_{b_1}^J \leq \beta_{b_1}^S < \beta_{b_1}^S$ for some critical $\beta_{b_1}^S$). For $f < g$ sole responsibility is better than joint responsibility also when $\beta_{b_1}^S$ is close to $g$ while $\beta_{b_1}^J$ is close to $\beta_{crit}$.

5. Conclusion

Incentives contracts in governmental agencies must typically be based on performance measures that do not exactly match the principal’s gross benefits. In addition, agents working in these organizations often perform multiple tasks and have career concerns. The main focus is this paper has been to analyze how these facts affect the optimal incentive schemes between principals and agents when only one-period contracts are feasible. To do so we have put forward a simple dynamic multitask career concern model with monetary incentives where the principal’s gross benefit cannot be rewarded directly, and where the agent’s career concern are determined by the professional environment.

The general conclusion we have obtained is that the optimal incentives scheme must balance the professional career concerns in two ways. Firstly, monetary incentives must balance the career incentives on the task which can be economically rewarded. Secondly, monetary incentives must balance how the agent should divide his/her effort among the tasks. Even though this general conclusion is quite in line with simple intuition, we will stress that the optimal incentive schemes we derive overturn some of the guidelines that emerge from a single task analysis. For example we have shown that optimal monetary incentives can be non-monotone or strongest earliest in agents’ careers, and that career concerns have strong implications for optimal job design.
References


Appendices

A. Technicalities

In this appendix we provide more details regarding some of the calculations in this paper.

A.1. Proof of Proposition 1

It remains to prove part (ii) of the proposition. Recall the maintained assumptions $f - \gamma g > 0, g - \gamma f > 0$. For the purpose of this proof we let $\alpha$ denote the effective incentive on the a-task, while $\beta$ denotes the implicit incentive on the b-task. We have $\beta > 0$. The principal chooses $\alpha$ to solve

$$
\max_\alpha S = fa + gb - \left(\frac{1}{2}a^2 + \frac{1}{2}b^2 + \gamma ab\right)
$$

s.t.

$$
\alpha - (a + \gamma b) \leq 0, \quad a \geq 0
$$

$$
\beta - (b + \gamma a) \leq 0, \quad b \geq 0
$$

where the inequalities in the IC constraints (for the agent’s choice of efforts) hold with complementary slackness. For given $\alpha, \beta$ there are three subcases:

(i) $\alpha \leq \gamma \beta$: Then $a = 0, b = \beta$ and $S = g\beta - \frac{1}{2}\beta^2$.

(ii) $\gamma \beta < \alpha < \beta$: Interior solution with

$$
a = \frac{\alpha - \gamma \beta}{1 - \gamma^2}, \quad b = \frac{\beta - \gamma \alpha}{1 - \gamma^2}
$$

$$
S = \frac{1}{2} \frac{2\alpha \gamma \beta + 2f \alpha + 2g \beta - 2f \gamma \beta - 2g \gamma \alpha - \beta^2 - \alpha^2}{1 - \gamma^2}
$$

(iii) $\alpha \geq \frac{\beta}{\gamma}$: Then $b = 0, a = \alpha$ and $S = f \alpha - \frac{1}{2} \alpha^2$.

Note that $S$ as a function of $\alpha (S(\alpha))$ is non-concave and has kinks at $\alpha = \gamma \beta$ and at $\alpha = \frac{\beta}{\gamma}$. The right-hand derivative at the former point is seen to be $\frac{\partial S}{\partial \alpha} (\gamma \beta^+) = \frac{f - g \gamma}{1 - \gamma^2} > 0$, hence $\alpha \leq \gamma \beta$ (and thus no effort on the a-task) is never optimal. We further find

$$
\frac{\partial S}{\partial \alpha} (\frac{\beta^-}{\gamma}) = \frac{1}{1 - \gamma^2} (f - g \gamma - (\frac{1}{\gamma} - \gamma) \beta)
$$

$$
\frac{\partial S}{\partial \alpha} (\frac{\beta^+}{\gamma}) = f - \frac{\beta}{\gamma}
$$

Consider now various cases for $\beta$.

(A) $\beta \geq \gamma f$.

Note that the assumption $g > \gamma f$ implies $\frac{f - g \gamma}{1 - \gamma^2} < f$, and hence that $\beta \geq \gamma f$ implies $\frac{\partial S}{\partial \alpha} (\frac{\beta^-}{\gamma}) < 0$ and $\frac{\partial S}{\partial \alpha} (\frac{\beta^+}{\gamma}) \leq 0$. This means that optimal $\alpha$ satisfies $\gamma \beta < \alpha < \frac{\beta}{\gamma}$ (case ii above). Straightforward calculations show that the optimum is

$$
\alpha^* = f - g \gamma + \gamma \beta
$$

$$
S^* = (g \beta - \frac{\beta^2}{2}) + \frac{1}{2} \frac{(f - g \gamma)^2}{1 - \gamma^2}
$$
(B) $\gamma \frac{\alpha^*}{1-\gamma} < \beta < \gamma f$.

Now we have $\frac{\partial S}{\partial \alpha}(\beta) < 0$ and $\frac{\partial S}{\partial \gamma}(\beta) > 0$. Hence there are two local optima: $(\alpha^*, S^*)$ given above and the local optimum where only the $a$-task is pursued, i.e. $\alpha = f$ and $S = S^a = \frac{1}{2} f^2$. Comparing the local maxima we find $S^* > S^a$ iff $\beta > \beta^{crit}$ given by

$$\beta^{crit} = g - (g - \gamma f)(1 - \gamma^2)^{-1/2}$$

Note that $0 < \gamma \frac{\alpha^*}{1-\gamma} < \beta^{crit} < \gamma f < g$ when $g > \gamma f$ and $f > \gamma g$.

(C) $\beta \leq \gamma \frac{\alpha^*}{1-\gamma}$. In this case we have $\frac{\partial S}{\partial \alpha}(\beta) \geq 0$ and $\frac{\partial S}{\partial \gamma}(\beta) > 0$, hence it is optimal to choose $\alpha > \frac{\beta}{\gamma}$, i.e. $\alpha = f$ is optimal.

Overall we can conclude that

$$S_{\text{max}} = \frac{1}{2} f^2 \quad \text{(with } a = f, b = 0\text{)} \quad \text{for } \beta < \beta^{crit}$$

$$S_{\text{max}} = S^* \quad \text{(with } a > 0, b > 0\text{)} \quad \text{for } \beta > \beta^{crit}$$

\[\blacksquare\]

A.2. Regression coefficients

We consider the case of one agent and two periods ($t = 1, 2$). The information signals are

$$y_t = h\eta + fa_t + gb_t + \epsilon^y_t$$
$$z_t = \eta + a_t + \epsilon^z_t$$
$$q_t = \eta + b_t + \epsilon^q_t.$$

We seek $E(\eta \mid y_1, z_1, q_1)$. The covariance matrix is

$$\begin{bmatrix}
\sigma^2_{\eta} & \alpha \sigma^2_{\eta} & \sigma^2_{\eta} \\
\alpha \sigma^2_{\eta} & h^2 \sigma^2_{\eta} + \sigma^2_{y} & \alpha \sigma^2_{\eta} + \sigma^2_{z} \\
\sigma^2_{\eta} & \alpha \sigma^2_{\eta} + \sigma^2_{z} & \sigma^2_{\eta} + \sigma^2_{q}
\end{bmatrix}$$

where

$$\sigma^2_{\eta} = \text{var}(\eta)$$
$$\sigma^2_{y} = \text{var}(\epsilon^y_t)$$
$$\sigma^2_{z} = \text{var}(\epsilon^z_t)$$
$$\sigma^2_{q} = \text{var}(\epsilon^q_t)$$

By inverting and applying well-known formulas (e.g., DeGroot (1970)) we get

$$R_y = \frac{\partial}{\partial y} E(\eta \mid x_1) = \frac{h\sigma^2_{\eta} \sigma^2_{\eta} \sigma^2_{q}}{h^2 \sigma^2_{\eta} \sigma^2_{\eta} \sigma^2_{q} + \sigma^2_{y} \sigma^2_{\eta} \sigma^2_{q} + \sigma^2_{z} \sigma^2_{\eta} \sigma^2_{q} + \sigma^2_{y} \sigma^2_{z} \sigma^2_{q}}$$

$$R_z = \frac{\partial}{\partial z} E(\eta \mid x_1) = \frac{\sigma^2_{\eta} \sigma^2_{y} \sigma^2_{q}}{h^2 \sigma^2_{\eta} \sigma^2_{\eta} \sigma^2_{q} + \sigma^2_{y} \sigma^2_{\eta} \sigma^2_{q} + \sigma^2_{z} \sigma^2_{\eta} \sigma^2_{q} + \sigma^2_{y} \sigma^2_{z} \sigma^2_{q}}$$

$$R_q = \frac{\partial}{\partial q} E(\eta \mid x_1) = \frac{\sigma^2_{\eta} \sigma^2_{y} \sigma^2_{z}}{h^2 \sigma^2_{\eta} \sigma^2_{\eta} \sigma^2_{q} + \sigma^2_{y} \sigma^2_{\eta} \sigma^2_{q} + \sigma^2_{z} \sigma^2_{\eta} \sigma^2_{q} + \sigma^2_{y} \sigma^2_{z} \sigma^2_{q}}$$
where $h = h'$. 

### A.3. The derivatives

Simple calculations give the following derivatives

$$
\frac{\partial R_z}{\partial \sigma_z^2} = -\sigma_y^2 \sigma_q^2 \eta \left( h^2 \sigma_y^2 \sigma_q^2 + \sigma_y^2 \sigma_q^2 + \sigma_y^2 \sigma_q^2 \right) < 0,
$$

$$
\frac{\partial R_y}{\partial \sigma_z^2} = h (\sigma_y^2)^2 \left( \sigma_q^2 \right)^2 \frac{\sigma_y^2}{D^2} > 0,
$$

$$
\frac{\partial R_q}{\partial \sigma_z^2} = (\sigma_y^2)^2 (\sigma_q^2)^2 \frac{\sigma_q^2}{D^2} > 0,
$$

$$
\frac{\partial R_y}{\partial \sigma_q^2} = \sigma_y^2 \sigma_q^2 \eta \left( h \sigma_y^2 \sigma_q^2 + \sigma_y^2 \sigma_q^2 + \sigma_y^2 \sigma_q^2 \right) > 0,
$$

$$
\frac{\partial R_q}{\partial \sigma_q^2} = \sigma_y^2 \sigma_z^2 \frac{\sigma_q^2}{D^2} > 0,
$$

$$
\frac{\partial R_y}{\partial \sigma_q^2} = h \sigma_y^2 \left( \sigma_z^2 \right)^2 \frac{\sigma_y^2}{D^2} > 0,
$$

$$
\frac{\partial R_q}{\partial \sigma_q^2} = \sigma_y^2 \sigma_z^2 \frac{\sigma_q^2}{D^2} > 0,
$$

$$
\frac{\partial R_y}{\partial \sigma_q^2} = -\sigma_y^2 \sigma_z^2 \eta \left( h^2 \sigma_y^2 \sigma_q^2 + \sigma_y^2 \sigma_q^2 + \sigma_y^2 \sigma_q^2 \right) < 0,
$$

where $D := (h^2 \sigma_y^2 \sigma_z^2 \sigma_q^2 + \sigma_y^2 \sigma_z^2 \sigma_q^2 + \sigma_y^2 \sigma_z^2 \sigma_q^2 + \sigma_y^2 \sigma_z^2 \sigma_q^2)$.

### A.4. Proof of Proposition

Proposition 2 claims that

$$
i) \quad \frac{\partial \alpha_z^*}{\partial \sigma_z^2} = -\frac{\partial \beta_a}{\partial \sigma_z^2} + \gamma \frac{\partial \beta_b}{\partial \sigma_z^2} > 0 \text{ if } \frac{\partial \alpha^*_i}{\partial \sigma_z^2} > 0 \text{ and } \frac{\partial \alpha^*_i}{\partial \sigma_z^2} = -\frac{\partial \beta_a}{\partial \sigma_z^2} + \gamma \frac{\partial \beta_b}{\partial \sigma_z^2} < 0
$$

iii) $\frac{\partial \alpha_z^*}{\partial \sigma_z^2} = -\frac{\partial \beta_a}{\partial \sigma_z^2} + \gamma \frac{\partial \beta_b}{\partial \sigma_z^2} > 0$.

**AD** i) In Appendix A.3 we show that $\frac{\partial R_i}{\partial \sigma_z^2} > 0$ for $i = y, z, q$. Hence an increase in the ability noise increases the career incentives on both tasks, i.e. $\frac{\partial \beta_i}{\partial \sigma_z^2} > 0$, $i = a, b$. Since, certeris paribus, increased career incentives on the $z$-task and on the $q$-task
have opposite effect on the monetary incentives will the total effect be determined by their relative strength. This latter effect depends on $\gamma$. More specifically, the career effects from the $q$-signal dominates when $\gamma > b_q \frac{b_y}{\sigma_q^2} (b_y + hb_qg)$. This follows since

$$\frac{\partial \alpha_1^*}{\partial \sigma_q^2} = \left( -\frac{\partial R_z}{\partial \sigma_q^2} + \gamma \frac{\partial R_y}{\partial \sigma_q^2} - (f - \gamma g) \frac{\partial R_y}{\partial \sigma_q^2} \right) (h - f)$$

$$= \left( \sigma_y^2 \sigma_q^2 \left( -\sigma_y^2 \sigma_q^2 + \gamma \sigma_y^2 \sigma_q^2 - \gamma \sigma_y^2 \sigma_q^2 f + h \sigma_q^2 \sigma_q^2 \gamma g \right) \right) \frac{h - f}{D^2} > 0 \text{ iff } \left( -\sigma_y^2 \sigma_q^2 + \gamma \sigma_y^2 \sigma_q^2 - \gamma \sigma_y^2 \sigma_q^2 f + h \sigma_q^2 \sigma_q^2 \gamma g \right) > 0$$

Hence $\frac{\partial \alpha_1^*}{\partial \sigma_q^2} > 0$ iff $\gamma > \sigma^2_q \frac{\sigma^2_y + \gamma \sigma^2_q}{\sigma^2_q + \gamma \sigma^2_q}$

**AD ii)** We have

$$\frac{\partial \alpha_1^*}{\partial \sigma_q^2} = -\frac{\partial \beta_a}{\partial \sigma_q^2} + \gamma \frac{\partial \beta_b}{\partial \sigma_q^2}$$

$$= \left( \gamma \frac{\partial R_y}{\partial \sigma_q^2} - \frac{\partial R_z}{\partial \sigma_q^2} - (f - \gamma g) \frac{\partial R_y}{\partial \sigma_q^2} \right) \frac{(h^0 - f)}{D^2}$$

In Appendix A.3 we show that $\frac{\partial R_y}{\partial \sigma_q^2} > 0$, $\frac{\partial R_y}{\partial \sigma_q^2} > 0$, and $\frac{\partial R_z}{\partial \sigma_q^2} < 0$. Hence $\frac{\partial \beta_b}{\partial \sigma_q^2} > 0$. Furthermore $\frac{\partial \beta_b}{\partial \sigma_q^2} > 0$ only for ‘large’ values of $g$. This follows since

$$\frac{\partial R_y}{\partial \sigma_q^2} - g \frac{\partial R_y}{\partial \sigma_q^2} = \frac{\sigma^2_y \sigma^2_q - \gamma \sigma^2_q}{D^2} \left( \frac{h^0 - f}{D^2} \right)$$

Hence,

$$\frac{\partial \beta_b}{\partial \sigma_q^2} \frac{1}{\gamma (h^0 - f)} = \frac{\partial R_y}{\partial \sigma_q^2} + g \frac{\partial R_y}{\partial \sigma_q^2}$$

$$= \frac{\sigma^2_y \sigma^2_q - \gamma \sigma^2_q}{D^2} \left( \frac{h^0 - f}{D^2} \right) \frac{1}{\gamma (h^0 - f)}$$

To ensure an interior solution we also need the restriction $f - \gamma g > 0$. Hence the $\frac{\partial \beta_b}{\partial \sigma_q^2} > 0$—effect will never dominate, and the conclusion follows.

**AD iii)** We have

$$\frac{\partial \alpha_1^*}{\partial \sigma_q^2} = -\frac{\partial \beta_a}{\partial \sigma_q^2} + \gamma \frac{\partial \beta_b}{\partial \sigma_q^2}$$

$$= \left( \frac{\partial R_z}{\partial \sigma_q^2} + \gamma \frac{\partial R_y}{\partial \sigma_q^2} - (f - \gamma g) \frac{\partial R_y}{\partial \sigma_q^2} \right) \frac{(h - f)}{D^2}$$

$$= \frac{\sigma^2_y \sigma^2_q - \gamma \sigma^2_q - \gamma \sigma^2_q f - \gamma \sigma^2_q \gamma g}{D^2} > 0$$

since $h - f > 0$. Hence $\frac{\partial \alpha_1^*}{\partial \sigma_q^2} > 0$ for $\gamma > 0$.  

\[ \blacksquare\]
A.5. The Welfare analysis

By solving the optimization problem,
\[ S^*_t = \max_{a_t} S_t(a_t, b_t) \]
\[ \text{s.t.} \quad a_t = a_t(\alpha_t, \beta_a, \beta_b, \gamma), \quad b_t = b_t(\alpha_t, \beta_a, \beta_b, \gamma) \]
we obtain \( \alpha^*_t(\beta_a, \beta_b, \gamma, f, g) \) and \( a_t = a_t(\alpha^*_t, \beta_a, \beta_b, \gamma), b_t = b_t(\alpha^*_t, \beta_a, \beta_b, \gamma) \).

Then by the envelope property \( \left( \frac{\partial S_t}{\partial a_t} \frac{\partial a_t}{\partial \mu} + \frac{\partial S_t}{\partial b_t} \frac{\partial b_t}{\partial \mu} = 0 \right) \) we have, for any parameter \( \mu = \beta_a, \beta_b, \gamma \)
\[ \frac{\partial S^*_t}{\partial \mu} = \frac{\partial S_t}{\partial a_t} \frac{\partial a_t}{\partial \mu} + \frac{\partial S_t}{\partial b_t} \frac{\partial b_t}{\partial \mu} \]
\[ = (f - \frac{\partial C}{\partial a_t}) \frac{\partial a_t}{\partial \mu} + (g - \frac{\partial C}{\partial b_t}) \frac{\partial b_t}{\partial \mu} \]
\[ = (f - [\alpha_t + \beta_a]) \frac{\partial a_t}{\partial \mu} + (g - \beta_b) \frac{\partial b_t}{\partial \mu} \quad \text{(by agent’s Fob)} \]
\[ = (-\gamma [\beta_b - g]) \frac{\partial a_t}{\partial \mu} + (g - \beta_b) \frac{\partial b_t}{\partial \mu} \quad \text{(by optimal } \alpha_t \text{)} \]
\[ = [g - \beta_b] \left( \frac{\partial a_t}{\partial \mu} + \frac{\partial b_t}{\partial \mu} \right) \]
From equation (3.1b) the agent’s second first-order condition we have
\[ 0 = C_{ba} \frac{\partial a_t}{\partial \beta_a} + C_{bb} \frac{\partial b_t}{\partial \gamma} = C_{bb} \left( \gamma \frac{\partial a_t}{\partial \beta_a} + \frac{\partial b_t}{\partial \gamma} \right) \]
\[ 1 = C_{ba} \frac{\partial a_t}{\partial \beta_b} + C_{bb} \frac{\partial b_t}{\partial \gamma} = C_{bb} \left( \gamma \frac{\partial a_t}{\partial \beta_b} + \frac{\partial b_t}{\partial \gamma} \right). \]
Hence we can conclude
\[ \frac{\partial S^*_t}{\partial \beta_a} = 0 \quad \text{and} \]
\[ \frac{\partial S^*_t}{\partial \beta_b} = [g - \beta_b] \frac{1}{C_{bb}} = [g - (R_q + gR_y)(h^o - f)] \frac{1}{C_{bb}}. \]

A.6. Joint vs. sole responsibility

Note that
\[ R^S_y = \frac{\partial}{\partial y} \mathbb{E} \left( \eta \mid y^2, q_1 \right) = \lim_{\sigma^2 \to \infty} R^J_y = \frac{h\sigma_y^2 \sigma_q^2}{h^2 \sigma_y^2 \sigma_q^2 + \sigma_y^2 \sigma_q^2 + \sigma^2 \sigma_q^2} \]
\[ R^S_q = \frac{\partial}{\partial q} \mathbb{E} \left( \eta \mid y^2, q_1 \right) = \lim_{\sigma^2 \to \infty} R^J_q = \frac{\sigma^2 y^2}{h^2 \sigma_y^2 \sigma_q^2 + \sigma_y^2 \sigma_q^2 + \sigma^2 \sigma_q^2} \]
\[ R^J_y = \frac{\partial}{\partial y} \mathbb{E} \left( \eta \mid x_1 \right) = \frac{h\sigma_y^2 \sigma_x^2}{h^2 \sigma_y^2 \sigma_x^2 + \sigma_y^2 \sigma_x^2 + \sigma^2 \sigma_x^2} \]
\[ R^J_q = \frac{\partial}{\partial q} \mathbb{E} \left( \eta \mid x_1 \right) = \frac{\sigma^2 y^2 \sigma_x^2}{h^2 \sigma_y^2 \sigma_x^2 + \sigma_y^2 \sigma_x^2 + \sigma^2 \sigma_x^2}. \]
These expressions verify the claims leading to the relation $\beta_J < \beta_S$.

To further analyze the difference in (4.1), consider the expression

$$S^*J - S^*S = 2(g - \frac{1}{2}\beta^J)\beta^J - (g - \frac{1}{2}\beta^S)\beta^S + \frac{(f - \gamma g)^2}{1 - \gamma^2} - \frac{1}{2}f^2$$

We see that the contour for $S^*J - S^*S = 0$ is a hyperbola in $\beta_J , \beta_S$-space, see the figure for an illustration. The figure has $x = \beta_J$ on the horizontal axis and $y = \beta_S$ on the vertical axis. The relevant part of this space to consider is $\beta_{crit} < \beta_J < \beta_S$. For $\beta_J = \beta_S$ (along the diagonal) we have $S^*J - S^*S < 0$ only if $\beta_J$ is sufficiently large. (The right-hand branch of the contour cuts the diagonal to the left of $\beta_J = 2g$ if $\gamma$ is not too small. $(S^*J - S^*S = \frac{(f-\gamma g)^2}{1-\gamma^2} - \frac{1}{2}f^2$ for $\beta_J = \beta_S = 2g$.) The left-hand branch of the contour cuts the line $\beta_J = \beta_{crit}$ if and only if $g > f$. (For $\beta_J = \beta_{crit}$ we have by definition of $\beta_{crit}$ that $S^*J - S^*S = \frac{1}{2}f^2 - (g - \frac{1}{2}\beta^S)\beta^S \geq \frac{1}{2}(f^2 - g^2)$.) So for $g > f$ there is an area between $\beta_J = \beta_{crit}$ and the left-hand part of the contour where $S^*J - S^*S < 0$.

These considerations verify the statements made in the proposition.

$g = 2$, $\gamma = .6$