Paradoxes in Networks Supporting Competitive Electricity Markets

Mette Bjørndal
Kurt Jørnsten
Department of Finance and Management Science
Norwegian School of Economics and Business Administration
Helleveien 30
N-5045 BERGEN
Norway

Abstract

Grid investments are normally done in electrical networks in order to achieve a well functioning integrated electricity market and/or making the network more secure, i.e. less sensitive to link failures. In general, there are two aspects to be considered when making a new grid investment, the first is that of detecting beneficial investments, and the second is how to induce them under the chosen market regime. We will show that network "improvements", i.e. strengthening a line or building a new line, may in fact be detrimental to social surplus, and that some agents will have incentives to advocate these changes.

1. Braess' Paradox and Generalizations

In user-optimizing traffic assignment problems, where each individual user chooses the path with the lowest travel cost, it is well known that the equilibrium flow in a network is generally different from the system optimal flow, i.e. the flow minimizing total travel cost. In his original example, Braess [3] showed that adding a new link to a congested network may in fact *increase* travel cost for all, and this phenomenon is referred to as the Braess' paradox. Braess' paradox and variations of it have been the subject of several papers, like Murchland [20], Stewart [26], Frank [14], Dafermos and Nagurney [11], Steinberg and Zangwill [25] and Steinberg and Stone [24], among others.

More recently, Penchina [22] and Pas and Principio [21] have studied the classical Breass' traffic network configuration[1] with a single origin-destination pair and with fixed and variable user cost on the links, representing for instance travel and congestion cost respectively. Given the cost parameters, demand is varied and it is illustrated that the paradox typically occurs for intermediate traffic demand, whereas for low and high demand the additional link is beneficial. This means that when traffic demand increases over time, networks can "grow into" or "grow out from" the paradox region.

In relation to this, Penchina discusses different cures, including tolls and reversible one-way signs, showing that the "best" remedy depends on traffic, and although system optimum is achieved under marginal cost pricing, in some cases there is a trade-off between the optimality and complexity of the suggested cure. Similarly, Pas and Principio show that the paradox-region can be divided into two sub-ranges. In the first (for relatively lower demand) marginal cost pricing results in a flow pattern, in which the additional link is used, and the overall system performance is improved. In the second, marginal cost pricing results in the additional link *not* being used. This means that in this sub-range, not only will the additional link increase travel time in the user equilibrium flow pattern (Braess' paradox), the additional link is not warranted even under marginal cost pricing.

Yang and Bell [30] also study the classical Braess' network adding throughput capacities to the links and showing that at a given service level, a new link may reduce the throughput capacity of the network. Alternatively, at the same level of throughput queues may develop when the new link is introduced. The concept of reserve capacity, in the form of a flow-multiplier, is introduced as a means to detect and avoid capacity expansions that are detrimental to overall throughput capacity.

Hallefjord et al. [15] discuss paradoxes in traffic networks in the case of elastic demand. When travel demand is elastic it is not evident what a paradoxical situation is, and in this case there is a need for characterizations of different paradoxes. An example is given where total flow decreases while travel time increases due to adding a new link to the network. This is a

[1] In some papers, like Cohen and Horowitz [10] and Calvert and Keady [7] [8], it is referred to as the Wheatstone bridge topology.

rather extreme type of paradox. A different paradox is when the network “improvement” leads to a reduction in social surplus.

The reason for the traffic equilibrium paradoxes is the behavioral assumption that a traveler chooses the path that is best for himself, without paying attention to the effect this has on the other users (eventually including himself). In user equilibrium a user cannot decrease travel time by unilaterally changing his travel route, leading us to seeing the equilibrium as a Nash equilibrium of an underlying game. Korilis et al. [18] investigate the non-cooperative structure of certain networks, where the term non-cooperative emphasizes that the networks are “operated according to a decentralized control paradigm, where control decisions are made by each user independently, according to its own individual performance objectives”. Nash equilibria are generally Pareto inefficient as demonstrated by Dubey [13], and Korilis et al., who use the Internet as an example while referring more generally to queuing networks.

Cohen and Horowitz [10] give examples of Braess’ paradox for other non-cooperative networks like mechanical systems (strings) and hydraulic and electrical networks, and point to the need for specifications of conditions under which general networks behave paradoxically. This is partly provided by Calvert and Keady [7] [8] and Korilis et al. [18] propose methods for avoiding degradation of performance when adding resources to non-cooperative networks.

In the following sections we will give examples of paradoxical situations that can occur in electrical networks due to electrons behaving “non-cooperatively”. This behavior is reflected by the power flow equations. When computing the equilibria, we assume competitive deregulated electricity markets. In that respect our analysis follows the same line of research in electricity markets that was performed by Hallefjord et al. [15] for elastic traffic equilibria.

2. Grid Investments in Electricity Networks

We consider real power only and the lossless and linear “DC” approximation of the power flow equations (Wu and Varaiya [28], Wood and Wollenberg [27], McGuire [19]). Assuming all line-reactances equal to 1, the optimal dispatch problem of an *n*-node network with *m* links can be formulated as follows:

(2-1) max $\sum_{i=1}^{n}\left(\int_{0}^{q_i^d} p_i^d(q)dq - \int_{0}^{q_i^s} p_i^s(q)dq\right)$

(2-2) s.t. $q_i^s - q_i^d = \sum_{j\neq i} q_{ij}$ $\quad i = 1,\ldots,n-1$

(2-3) $\sum_{ij\in L_l} q_{ij} = 0$ $\quad l = 1,\ldots,m-n+1$

(2-4) $\sum_{i=1}^{n}(q_i^s - q_i^d) = 0$

(2-5) $q_{ij} \le C_{ij}$ $\quad 1 \le i, j \le n$,

where $p_i^d(q_i^d)$ is the demand function of node i, q_i^d is the quantity of real power consumed in node *i*, $p_i^s(q_i^s)$ is the supply function of node i, and q_i^s is the quantity of real power produced in node *i*. C_{ij} is the capacity of link ij, and q_{ij} is the power flow over the link from i to j.

The objective function (2-1) expresses the difference between consumer benefit (the area under the demand curve) and the cost of production (the area under the supply curve). Equations (2-2) correspond to Kirchhoff's junction rule, and there are $n-1$ independent equations. Equations (2-3) represent Kirchhoff's loop rule, where $L = (L_1,\ldots,L_{m-n+1})$ represents a set of independent loops (Dolan and Aldous [12]), and L_l is the set of directed arcs in a path going through loop *l*. Equation (2-4) stands for conservation of energy, while inequalities (2-5) are the capacity constraints.

Solving (2-1) (or alternatively (2-1)-(2-4) to obtain line flows) gives the unconstrained dispatch and a uniform price of energy (the system price). The convex program (2-1)-(2-5) corresponds to the optimal dispatch problem, from which optimal nodal prices can be found (see for instance Schweppe et al. [23], Hogan [16], Wu et al. [29] or Chao and Peck [9]). In optimal dispatch the locational prices can vary over all nodes, even if there is only one congested link.

In Wu et al. [29] a 3-node example is given, showing that strengthening a line by increasing its admittance may lead to larger minimum cost. The network and initial optimal dispatch is displayed in Figure 2-1. In optimal dispatch the nodal prices will be related by $p_1 < p_2 < p_3$ since line 1-3 is congested (for an argument, see Wu et al.). When the admittance of line 2-3 is increased, the power flow equations change, and flow will increase on path 1-3-2 if injections are maintained. This will result in line 1-3 becoming overloaded and injection in node 1 must be reduced. Hence, by increasing the admittance, the former feasible power flow becomes infeasible. This can be viewed as the physical paradox that results from the underlying physical equilibrium model. If consumption is to be maintained, injection in node 3 must increase, leading to larger minimum cost. Hence, an economic paradox that is the result of the underlying characteristics of the physical equilibrium model occurs.

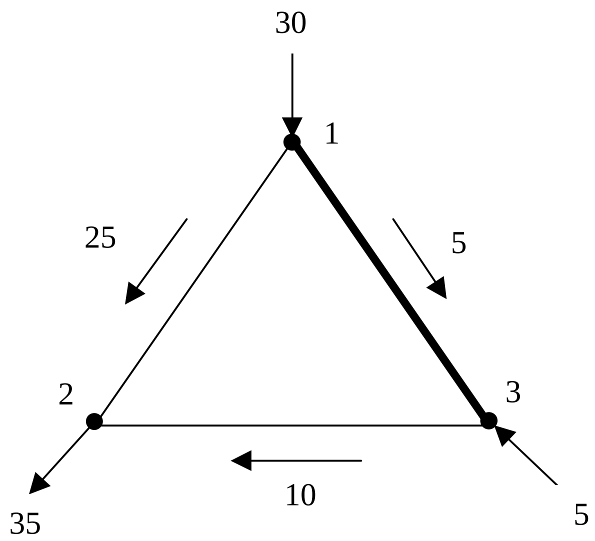


Figure 2-1 Increasing Admittance Increases Cost

In a similar 3-node example exhibited in Figure 2-2, Bushnell and Stoft [4] show that a new line hurts the network but still collects congestion rent.

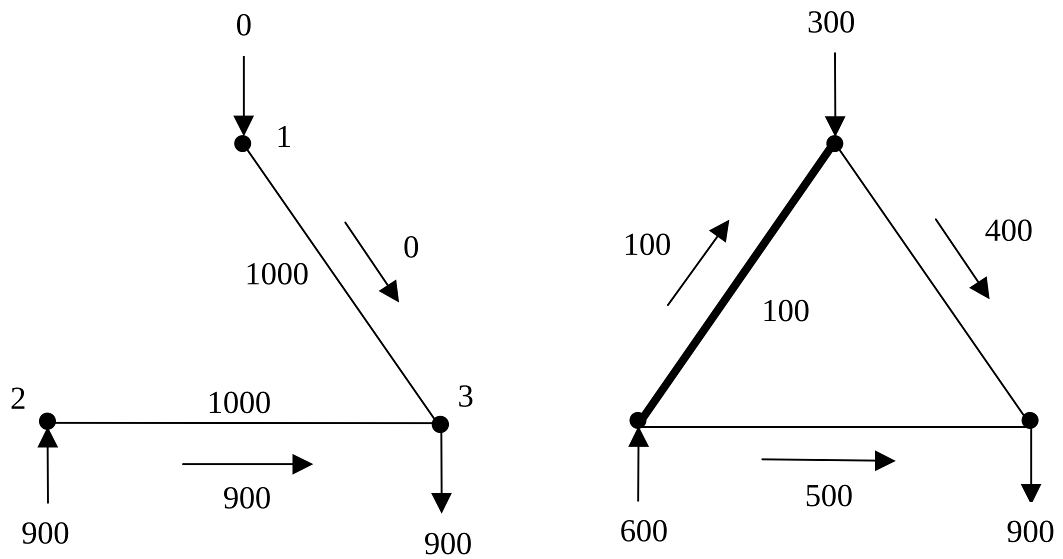


Figure 2-2 New Line Increases Cost

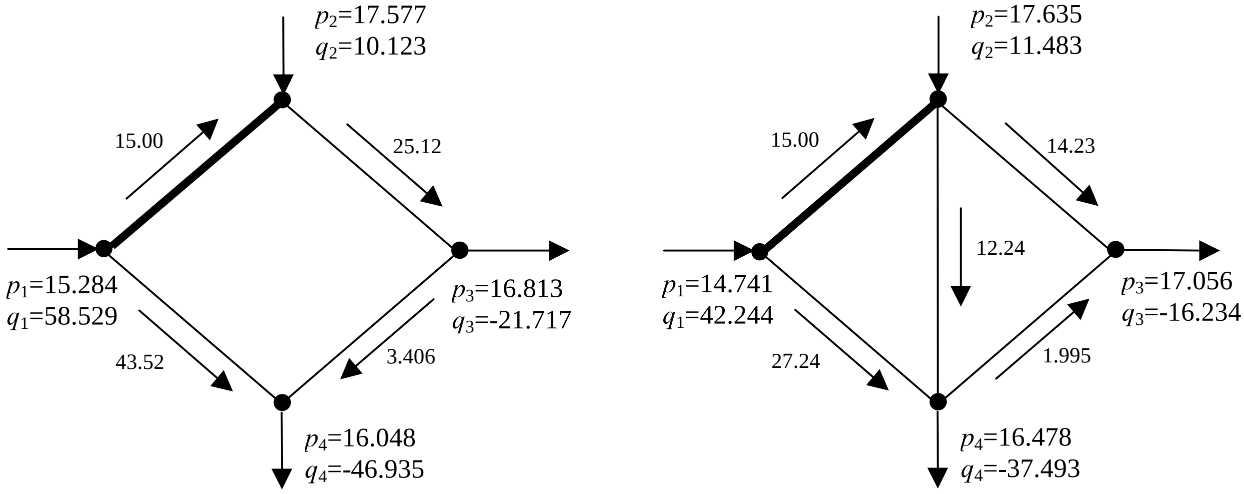
In the example, there is high cost production in node 1 and relatively lower cost production in node 2. Consumption takes place in node 3 where there is a fixed demand equal to 900 MW. Initially, there are only two links, 1-3 and 2-3, each with a capacity of 1000 MW, and demand is supplied entirely by the low cost producers in node 2.

In part B of Figure 2-2, a new line has been built between nodes 1 and 2. This is a weak line with a capacity of only 100 MW, and it introduces loop flow, having as a consequence that the transfer capacity between node 2 and 3 is greatly reduced. Assuming reactances equal to 1 on every link and no production in node 1 to generate counter flow on line 1-2, it is reduced from 1000 to 300 MW. By inducing injections in node 1, the minimum cost of supplying 900 MW to node 3 is obtained by injecting 600 MW in node 2 and 300 MW in node 1, which is obviously a more costly dispatch. The new line is congested in direction 2 to 1, and since $p_1 > p_2$, the new line receives a positive congestion rent, $q_{21} \cdot (p_1 - p_2)$.

In the following we will give examples of paradoxes in a 4-node network with the Wheatstone bridge topology and with elastic demand and production in every node. We assume linear cost and demand functions, represented by $p_i = c_i q_i^s$ and $p_i = a_i - b_i q_i^d$ where p_i is the price in node *i*, q_i^s is the quantity produced in node *i*, q_i^d is the quantity consumed in node *i*, and a_i, b_i and c_i are positive constants. Net injection in node *i* is given by $q_i = q_i^s - q_i^d$. With input data given in Table 2-1 and a thermal capacity of 15 units on line 1-2, optimal dispatch and optimal prices are given in Figure 2-3. Part A shows the situation without line 2-4, while part B includes this line. We use the linear and lossless "DC" approximation of the power flow equations, assuming reactances equal to 1 one every line.

Table 2-1 Cost and Demand Parameters

NODE	CONSUMPTION		PRODUCTION
	a_i	b_i	c_i
1	20	0.05	0.1
2	20	0.05	0.3
3	20	0.05	0.4
4	20	0.05	0.5



Part A: No Line between Nodes 2 and 4
Social Surplus: 2878.526
Grid Revenue: 45.848

Part B: New Line between Nodes 2 and 4
Social Surplus: 2852.660
Grid Revenue: 69.444

Figure 2-3 Optimal Dispatch before and after Line 2-4

By introducing the new line, total production and consumption has been reduced together with social surplus. On the other hand, grid revenue defined as the *merchandizing surplus*, $MS = -\sum_i p_i q_i = \frac{1}{2}\sum_i \sum_j (p_j - p_i) \cdot q_{ij}$, increases. The effect on individual agents varies, i.e. some agents loose while others are better off, as displayed in Table 2-2. If surplus changes for an agent, it means that the nodal price that he faces has been altered. More specifically, if the price of node i increases as a consequence of the new line, producer i gains, while consumer i looses. If the price falls, the opposite is valid.

Table 2-2 Allocational Effect of New Line

	Node 1		Node 2		Node 3		Node 4	
	Before	After	Before	After	Before	After	Before	After
Production	152.843	147.415	58.589	58.783	42.031	42.641	32.097	32.955
Consumption	94.314	105.171	48.466	47.300	63.749	58.874	79.031	70.448
Net Exports	58.529	42.244	10.123	11.483	-21.717	-16.234	-46.935	-37.493
Producer Surplus	1168.048	1086.554	514.901	518.321	353.328	363.646	257.552	271.511
Consumer Surplus	222.379	276.522	58.724	55.933	101.597	86.655	156.149	124.075
Surplus of Region	1390.427	1363.076	573.624	574.254	454.925	450.301	413.701	395.585

Considering the surplus of each region (i.e. the combined producer and consumer surpluses of each node), it is evident that in general, some regions are better off due to the new line while others loose. However, it is easy to construct examples in which every region looses because of the new line. For instance, changing the example above by letting $c_2 = 0.37$, makes every region worse off, while the grid revenue increases when line 2-4 is introduced.

In the discussion so far, we have considered optimal nodal pricing as the means of managing congestion. Zonal pricing constitutes an alternative, which is used in practice, for instance in Nordic market, and the characteristics of zonal pricing are studied in Bjørndal and Jørnsten [2]. When the number of zones K ($K \le n$) and the allocation of nodes to zones $Z_1, \ldots, Z_K$ are determined, the optimal zonal prices can be found by solving problem (2-1)-(2-5) with additional constraints:

(2-6) $$\begin{cases} p_i^s(q_i^s) = p_{Z_k} \\ p_i^d(q_i^d) = p_{Z_k} \end{cases} \qquad i \in Z_k,\ k = 1, \ldots, K,$$

where p_{Z_k} is the price in zone Z_k. Equations (2-6) guarantee that prices are uniform over nodes belonging to the same zone. It is obvious that the social surplus of the zonal solution is less than or equal to the surplus of the optimal dispatch (which again is less than or equal to social surplus in unconstrained dispatch). Moreover, it is obvious that a finer partition of the grid (dividing a zone into two or more "subzones" by allowing more prices) will increase social surplus or leave it unchanged.

In the given example, assuming only two zones, there are four zone allocations that separate nodes 1 and 2[2]. These are displayed in Figure 2-4. Different zone allocations affect social surplus, and for the parameters of our example, Z4 is best without the new line, while Z1 is best when the new line is included. This illustrates that modifications to the grid should lead to a reconsideration of zone allocations.

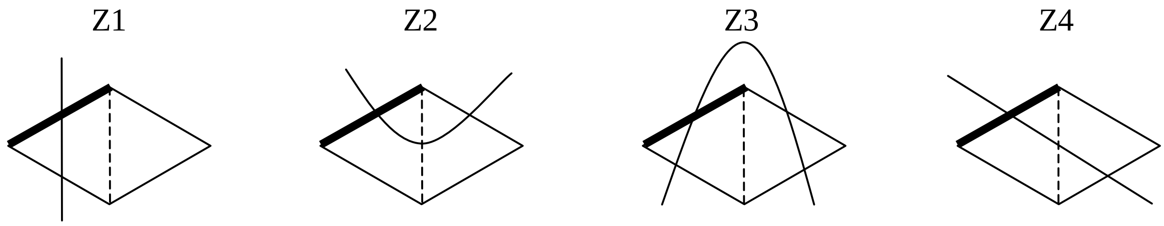
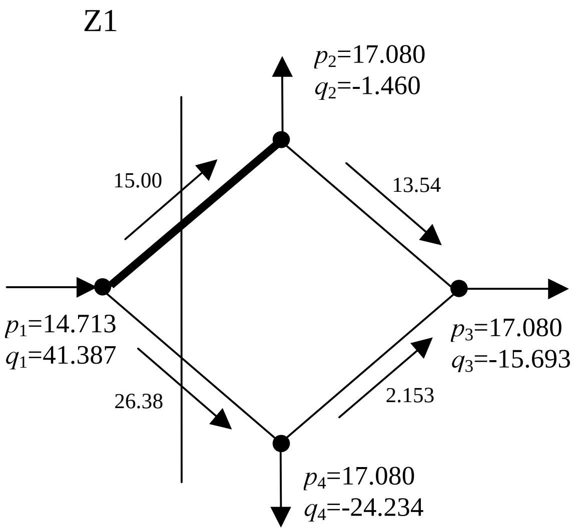
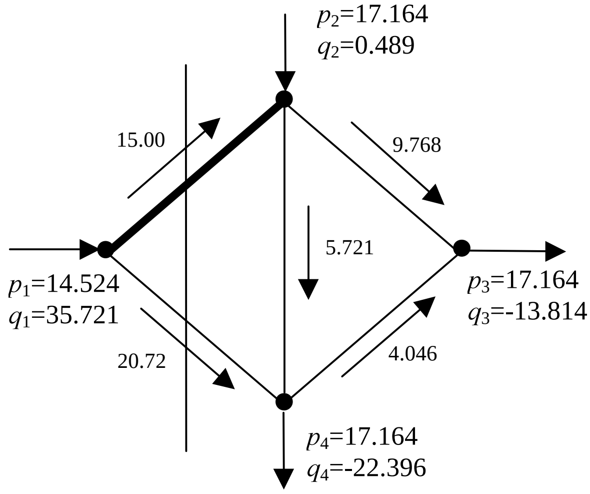


Figure 2-4 Allocations to Two Zones

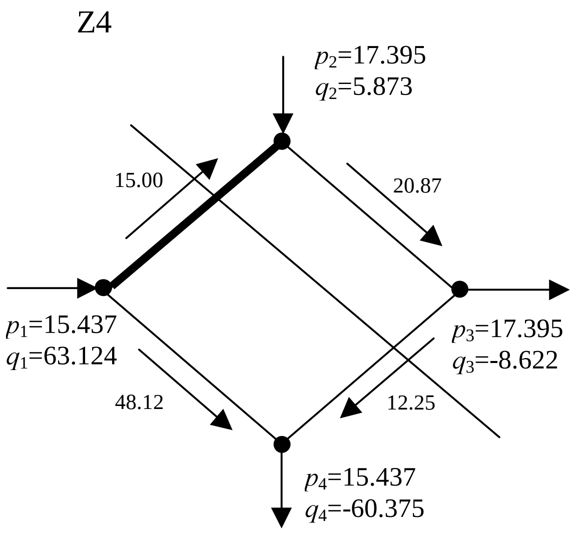
Prices, net injections and power flows for Z1 and Z4 are displayed in Figure 2-5, together with total social surplus and grid revenue. As is evident from the numbers, also under zonal pricing total social surplus is reduced when the new line is built. This is so for fixed zone allocations (i.e. the partition of nodes into zones remains the same after the new line is in place), but it is also valid even if the best zone allocation is chosen at every point. For fixed zone allocations grid revenue is reduced when building the new line. However, if the new line changes the partition of nodes from Z4 to Z1, grid revenue increases considerably, thus providing a strong incentive on the part of the grid owners to build the line.



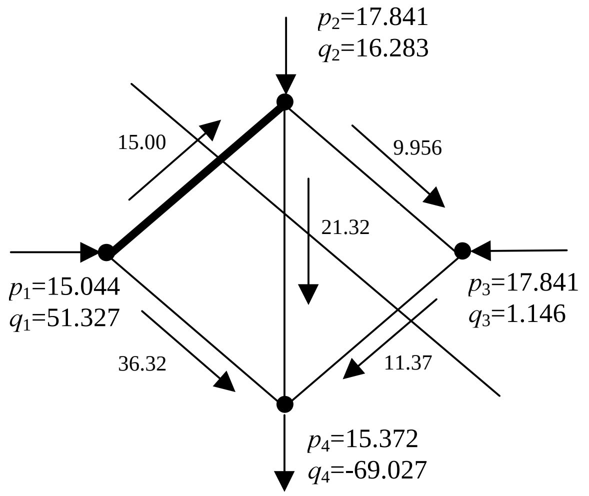
Part A: No Line between Nodes 2 and 4
Social Surplus: 2858.235
Grid Revenue: 97.979



Part B: New Line between Nodes 2 and 4
Social Surplus: 2844.051
Grid Revenue: 94.296



Part C: No Line between Nodes 2 and 4
Social Surplus: 2869.871
Grid Revenue: 5.380



Part D: New Line between Nodes 2 and 4
Social Surplus: 2821.270
Grid Revenue: -49.495

Figure 2-5 Zonal Solutions Z1 and Z4 before and after the New Line

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[2] In practical implementations, it is often required that a zone boundary should cut the link with the capacity problem.

In Table 2-3 we show the surpluses for each region. In general, the change of surplus for individual agents can be positive or negative. In Z1 every region surplus as well as the grid revenue decreases due to the new line. If parameters are changed so that $c_2 = 0.35$ and the thermal capacity of line 1-2 is 5 units, the effect of the new line on every region would be negative when choosing the social beneficial zone allocations (i.e. switching from Z4 to Z1 when building line 2-4). Grid revenue on the other hand would increase.

Table 2-3 Region Surpluses

	Z1		Z4	
	Before	After	Before	After
Node 1	1361.881	1354.600	1399.745	1377.242
Node 2	571.474	571.434	572.168	577.110
Node 3	449.917	448.685	446.096	444.489
Node 4	376.983	375.036	446.482	471.924

In the examples cited so far, the reductions in social surplus are relatively minor. In the original example in Table 2-1 the reduction in total social surplus is equal to 25.866, or 0.9%. This is partly due to the assumption of identical demand functions in every node. By allowing more unequal distributions of consumption, the reductions can be of considerable size. For instance, increasing $b_i, i = 1,2$ to 0.25, i.e. the size of the markets in nodes 1 and 2 are assumed to be only 20% of the markets in nodes 3 and 4, social surplus in optimal dispatch is reduced from 2541.968 to 2394.397, i.e. by 5.8%, when the new line is built. This is more than 2.5 times the cost of the thermal limit itself, as social surplus in unconstrained dispatch is equal to 2600.506. If there is no consumption in nodes 1 and 2, social surplus is reduced from 2395.869 to 2129.125, i.e. by 11.1%. Also when increasing demand by shifting the demand curves positively (for instance by raising the a_i's), the paradox becomes more severe.

The persistence of the paradox depends also on the cost parameters. Consider for instance varying c_2. When $c_2 \in [0, 0.080)$ the new line improves social surplus. When $c_2 \in [0.080, 0.102)$ the new line has no effect on social surplus because the thermal limit is not binding in optimal dispatch (neither with nor without line 2-4). Finally, when $c_2 \geq 0.102$ the new line reduces social surplus, implying that the paradox also occurs when production in node 2 is so costly that it is not being used. The reduction reaches a maximal value at $c_2 = 0.350$. Varying c_4 in the same manner, the thermal capacity is binding for all values of c_4. When $c_4 < 0.179$ line 2-4 improves social surplus, whereas the paradox arises for $c_4 > 0.179$.

From the treatment of the “DC”-approximation in Wu et al. [29], we know that

$$q_{ij} = \frac{1}{x_{ij}} \sin(\delta_i - \delta_j) = Y_{ij} \sin(\delta_i - \delta_j),$$

where δ_i is the phase angle at node *i*, q_{ij} is the power flow over line *ij*, x_{ij} is the reactance of line *ij*, and the admittance Y_{ij} of line *ij* is equal to the reciprocal of the reactance of the line. Since the sine function has a maximal value of 1, we must have that $q_{ij} \leq Y_{ij}$. Considering also the thermal limit C_{ij} of line *ij*, q_{ij} is bounded by $\min\{C_{ij}, Y_{ij}\}$. This means that “strengthening” a line has two interpretations: increasing the admittance or increasing the thermal limit.

From the optimal dispatch problem, we know that the shadow price of the thermal limit is non-negative, i.e. $\mu_{ij} \geq 0$, which means that social surplus cannot be reduced by improving the thermal limit of any line. What we have shown by the previous examples is that whenever there is at least one binding thermal limit, say on line *ij*,

$$\frac{\partial \text{Social Surplus}}{\partial Y_{kl}}$$

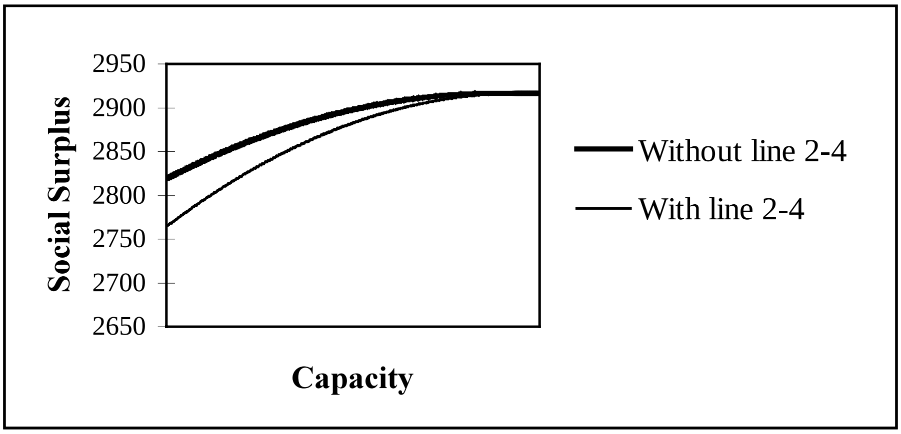
may be negative for some link *kl*. I.e. by either increasing the admittance of an existing line or by building a new line[3], we may reduce social surplus.

Consider now varying the thermal capacity of line 1-2. In Diagram 2-1 social surpluses are shown as functions of C_{12}. The functions are concave and increasing and the difference

[3] I.e. increasing the admittance from the 0 level.

between the curves is the greatest for $C_{12} = \varepsilon$ and vanishes when C_{12} is so large that the thermal limit is no longer binding in any of the network configurations considered. This occurs at $C_{12} = 42.587$, which is the flow over line 1-2 in unconstrained dispatch assuming line 2-4 is included in the network. From this point, social surplus is constant and equal to 2916.525, and increasing the thermal capacity is not beneficial in either network configuration.

Diagram 2-1 Social Surplus and Thermal Capacity of Line 1-2



As is shown by Wu et al. [29], in optimal dispatch, the merchandizing surplus given by $\frac{1}{2}\sum_i \sum_j (p_j - p_i) q_{ij}$ is equal to the *congestion rent* defined by

$$CR = \sum_i \sum_j \mu_{ij} C_{ij} .$$

Since line 1-2 is the only congested line in our example[4], grid revenue is equal to $\mu_{12} C_{12}$, i.e. for a given thermal capacity C_{12}, the size of the grid revenue is determined by the value of

$$\mu_{12} = \frac{\partial \text{Social Surplus}}{\partial \text{C}_{12}} .$$

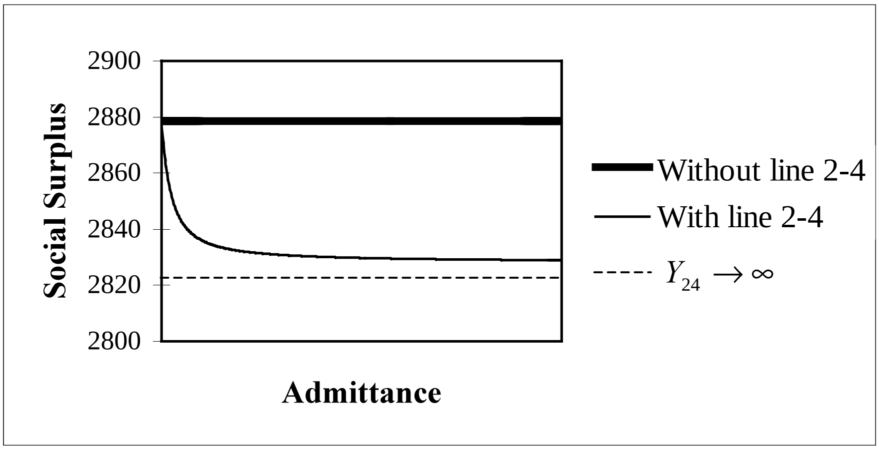
[4] Assuming $\mu_{12} > 0$, while $\mu_{ij} = 0$ for $ij \neq 12$.

As is indicated by the curves of Diagram 2-1, building line 2-4 will increase grid revenue since at every $C_{12} < 42.587$ the social surplus function *with* line 2-4 is steeper than the function depicting social surplus *without* line 2-4.

Note however that whether the grid revenue increases due to the new line is not indicative of whether the paradox occurs. Grid revenue may increase also when the new line is beneficial. For instance, letting $c_4 = 0.15$, total social surplus increases from 3448.992 to 3457.022 when the new line is built. Grid revenue increases from 58.969 to 64.530 i.e. total social surplus increases more than the grid revenue, leaving a net increase for the market participants due to the new line.

In Diagram 2-2 social surplus is shown as a function of the admittance of line 2-4. For reference, social surplus without line 2-4 is also exhibited. We note that the difference between social surplus with and without line 2-4 increases with the admittance Y_{24}. When $Y_{24} \to \infty$, social surplus approaches the value 2828.161 asymptotically, signifying that the paradox becomes more severe the stronger is the new line, but there is a maximal degradation of social surplus equal to 2878.526-2828.161 = 50.365.

Diagram 2-2 Social Surplus and Admittance of Line 2-4



Load factor β_{ij}^{kl} is the fraction of a trade from node *k* to node *l* that flows over the directed arc *ij*, and load factors are such that $\beta_{ij}^{kl} = -\beta_{ij}^{lk}$, and $\beta_{ij}^{kl} = -\beta_{ji}^{kl}$. In general, load factors vary with the level of the power flows in the system. However, given the admittances of the lines, load factors are constants in the "DC" approximation. The load factors of line 1-2 for different trades can be expressed as functions of Y_{24}. When the new line is introduced with an admittance of Y_{24}, the power flow equations become the following:

Kirchhoff's junction rule: $q_1 = q_{12} + q_{14}$

$q_2 = -q_{12} + q_{23} + q_{24}$

$q_3 = -q_{23} + q_{34}$

Kirchhoff's loop rule: $q_{24} = -Y_{24} q_{12} + Y_{24} q_{14}$

$q_{24} = Y_{24} q_{23} + Y_{24} q_{34}$

Conservation of energy: $q_1 + q_2 + q_3 + q_4 = 0$.

By solving the power flow equations for different trades, we find the load factor matrix

$$\mathbf{B}_{12}^{Y_{24}} = \begin{pmatrix} 0 & \frac{3+2Y_{24}}{4(1+Y_{24})} & \frac{1}{2} & \frac{1+2Y_{24}}{4(1+Y_{24})} \\ -\frac{3+2Y_{24}}{4(1+Y_{24})} & 0 & -\frac{1}{4(1+Y_{24})} & -\frac{1}{2(1+Y_{24})} \\ -\frac{1}{2} & \frac{1}{4(1+Y_{24})} & 0 & -\frac{1}{4(1+Y_{24})} \\ -\frac{1+2Y_{24}}{4(1+Y_{24})} & \frac{1}{2(1+Y_{24})} & \frac{1}{4(1+Y_{24})} & 0 \end{pmatrix}$$

where the entry of row k and column l is β_{12}^{kl}. The negative numbers indicate that the corresponding trades generate counter flows on line 1-2.

When $Y_{24} \to \infty$, trades between nodes 2, 3 and 4 have no influence on line 1-2, which can be seen from

$$\mathbf{B}_{12}^{\infty} = \lim_{Y_{24} \to \infty} \mathbf{B}_{12}^{Y_{24}} = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 \end{pmatrix}.$$

Nodes 2, 3 and 4 thus become one market with identical nodal prices. Net injection in node 1 on the other hand, distributes equally on lines 1-2 and 1-4 (load factors are equal to $\frac{1}{2}$), implying that the maximal export from region 1 is equal to $30 = 2 \cdot C_{12}$. An interpretation of this situation is that nodes 2 and 4 are electrically "the same", which is similar to a cost of zero on line 2-4 in a traffic equilibrium network. In the case of our electrical network, this makes the paradox maximal.

The paradox of the example of Table 2-1 and Figure 2-3 can be interpreted in terms of load factors. The load factor matrix without line 2-4 is equal to

$$\mathbf{B}_{12}^{0} = \begin{pmatrix} 0 & \frac{3}{4} & \frac{1}{2} & \frac{1}{4} \\ -\frac{3}{4} & 0 & -\frac{1}{4} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{4} & 0 & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 \end{pmatrix},$$

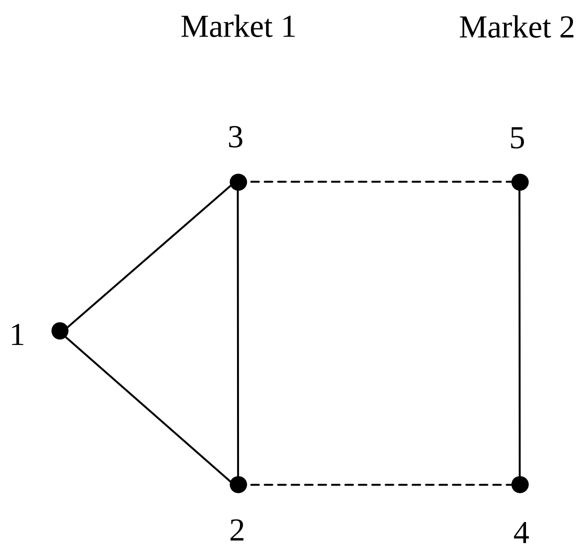
whereas the load factor matrix *with* line 2-4 (with admittance equal to 1) is equal to

$$\mathbf{B}_{12}^{1} = \begin{pmatrix} 0 & \frac{5}{8} & \frac{1}{2} & \frac{3}{8} \\ -\frac{5}{8} & 0 & -\frac{1}{8} & -\frac{1}{4} \\ -\frac{1}{2} & \frac{1}{8} & 0 & -\frac{1}{8} \\ -\frac{3}{8} & \frac{1}{4} & \frac{1}{8} & 0 \end{pmatrix}.$$

Considering optimal dispatch without line 2-4, $q_1 = 58.529$, $q_2 = 10.123$, $q_3 = -21.717$ and $q_4 = -46.935$. As is evident from matrices $\mathbf{B}_{12}^{0}$ and $\mathbf{B}_{12}^{1}$, the load factors of trades between net injection and net consumption nodes have developed unfavorably when introducing line 2-4. The positive load factors β_{12}^{13} and β_{12}^{14} stays the same or increases, meaning that the corresponding trades use as much or more of the capacity of line 1-2 under the new network configuration. The negative load factors β_{12}^{23} and β_{12}^{24} have increased, indicating that the trades that they represent, produce smaller counter flows on line 1-2, thus relieving the capacity constraint to a lesser extent. Under the new network configuration, the injection vector (58.529, 10.123, -21.717, -46.935) is no longer feasible. According to the characterization used by Bushnell and Stoft [6], the old dispatch belongs to the “newly infeasible region”, and the “newly feasible” region that follows from the new line, provides no better dispatch, thus the paradox. It could be interesting to investigate if traffic equilibrium paradoxes can be interpreted using something similar to the load factors in electricity networks.

3. Market Integration

A consequence of the paradoxical characteristics of certain electricity networks is that in the presence of congestion constraints, social surplus can be reduced when markets are integrated. In Figure 3-1 market 1 consists of nodes 1, 2 and 3, while market 2 consists of nodes 4 and 5. We assume linear cost and demand functions, with parameters given in Table 3-1. We want to consider integrating the markets by building lines 2-4 and 3-5. Disregarding any thermal constraints we find that social surplus would increase from 3126.177 to 3157.895. The system price settles on 16.842, which is higher than the price of market 1 and lower than the price of market 2.



Separate Markets
Unconstrained Dispatch:
Social Surplus: 3126.177
Price Market 1: 16.271
Price Market 2: 17.778

Integrated Market
Unconstrained Dispatch:
Social Surplus: 3157.895
Market Price: 16.842

Figure 3-1 Market Integration - Unconstrained Dispatch

Table 3-1 Cost and Demand Parameters

NODE	CONSUMPTION		PRODUCTION
	a_i	b_i	c_i
1	20	0.05	0.1
2	20	0.05	0.8
3	20	0.05	0.4
4	20	0.05	0.6
5	20	0.05	0.3

Assume now there is a capacity limit of 10 units on line 1-2. In Figure 3-2 we show optimal dispatch without the connecting lines. Social surplus is equal to 3000.433. In Figure 3-3 the new lines have been built, and social surplus is reduced to 2988.241, implying that the thermal limit on line 1-2, which is internal to market 1, prevents the realization of potential benefits from market integration.

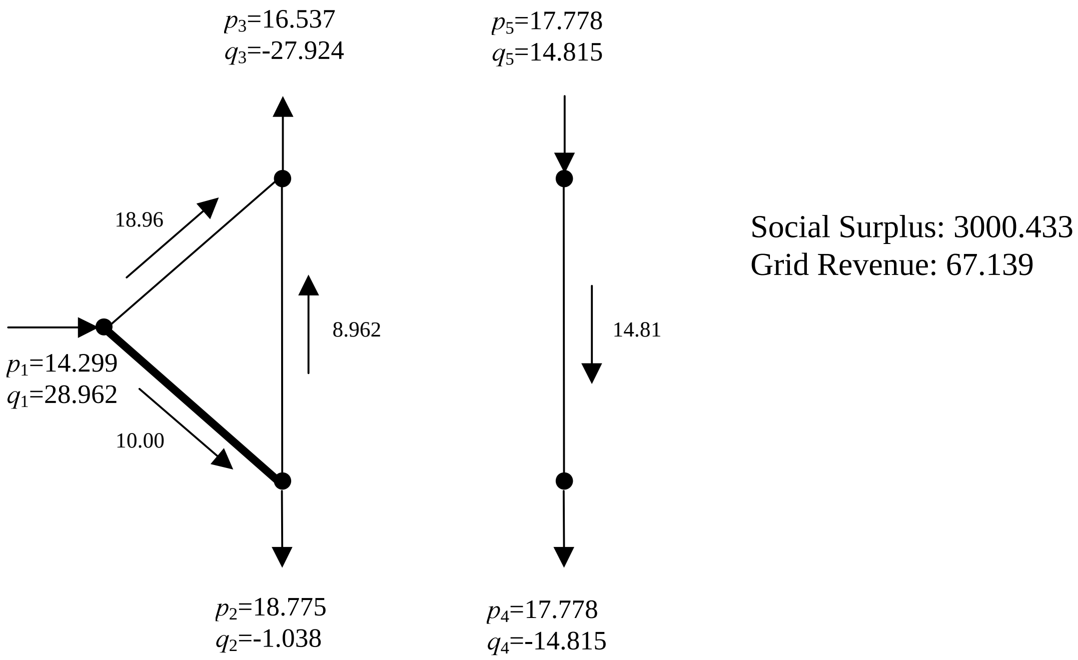


Figure 3-2 Optimal Dispatch - Before Integration

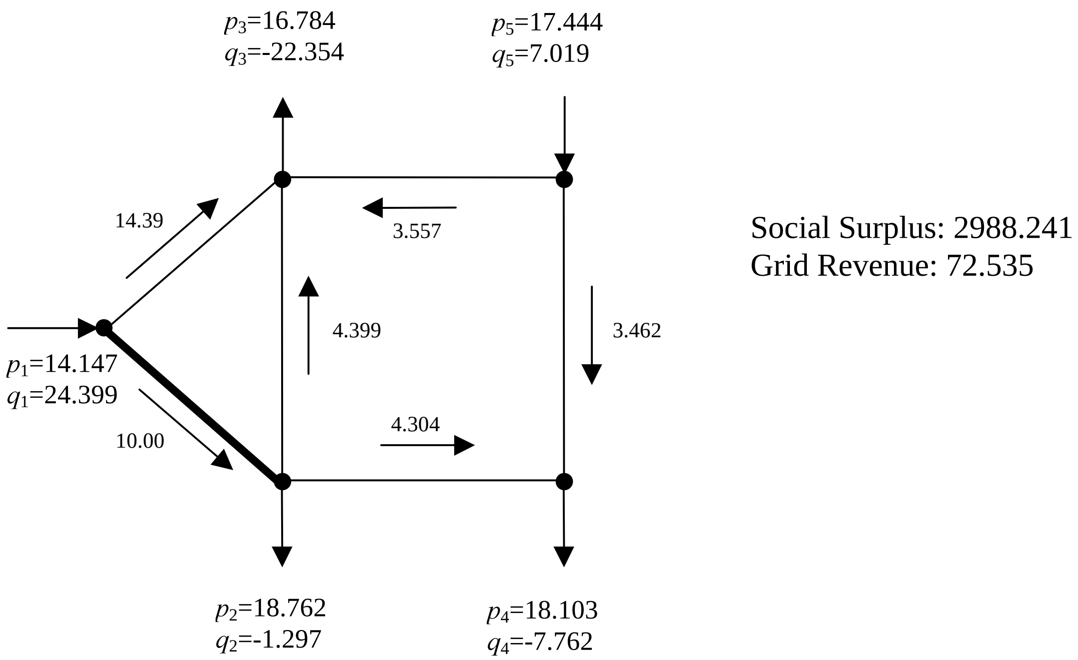


Figure 3-3 Optimal Dispatch - After Integration

4. Suggested Cures

Given that an investment has already been carried out, in traffic equilibrium networks marginal cost pricing can lead to improved overall system performance from the grid modification even when Braess' paradox occurs in user equilibrium (Pas and Principio [21]). In electricity networks there is no equivalent methodology, since electrons do not respond to marginal cost pricing. To alter flows for a given set of injections, we would have to alter line impedances.

Considering the investment decision itself, the obvious way to avoid the paradox in our Wheatstone bridge example is to build line 1-3 instead of line 2-4. This would resolve the capacity problem of line 1-2, but may be unacceptable for other reasons, for instance investment cost. Generally, the issue of how to encouraging beneficial investments and discouraging detrimental investments has been treated in the literature, for instance by Baldick and Kahn [1], Bushnell and Stoft [4] [5] [6] and Hogan [17]. As is shown by Bushnell and

Stoft [4] [5], transmission congestion contracts (TCCs), where new contracts are allocated according to a *feasibility rule*, which helps internalizing the external effects of detrimental grid investments, can provide at least a partial solution.

However, as is demonstrated by some of the examples in this chapter, and also pointed to by Bushnell and Stoft [6], the performance of a network depends on expected dispatch, which is influenced by future supply and demand conditions, which are constantly changing and subject to uncertainty. Thus, as market conditions change, so can the performance of the different network configurations considered. This is further complicated by typically long asset lifetimes and the lumpiness of the investment decisions, which sometimes makes it desirable to expand the network in a manner that is not immediately beneficial but will be in the long run. Ideally, we should compare different expansion *paths* rather than various fixed networks, as the investment problem is dynamic in nature.

5. Conclusions

Depending on the parameters of the problem considered (cost, demand, thermal capacity and admittance) a new line may be detrimental to social surplus. In general, some agents are better off while others loose. In this article we provide examples where, in optimal dispatch, every region looses while the grid revenue increases. For fixed zone allocations there is also the possibility that every region-surplus *and* the grid revenue is reduced as a consequence of a new line. In this article it is also demonstrated that a thermal limit, which is internal to a market, may result in market integration being disadvantageous. The possibility of such paradoxical effects and the incentives that they provide to different agents must clearly be taken into consideration both in the process of grid development and market development.

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