Endogenous Verifiability in Relational Contracting

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Abstract

We analyze a repeated principal-agent trust game where the principal makes a specific investment by paying the agent up-front, expecting an agreed upon quality level in return. The verifiability of the agent’s action is endogenously determined by the principal’s investment in writing an explicit contract. Since verification is not certain, explicit contracting is insufficient, and the parties must engage in relational (implicit) contracting. First, we analyze how variations in trust (the discount factor) affect the contract equilibrium. Interestingly, we find that more trust may lead to lower levels of specific investments. This occurs when the surplus from trust is realized mainly through lower explicit contract costs. Second, we extend the literature on the interaction between explicit and relational governance by analyzing how variations in verification technology affect contract equilibrium. Since verification technology determines the cost necessary to achieve a given probability of verification, this analysis can also explain interesting aspects of legal systems.

Keywords: Trust, Relationship Specific Investments, Relational Contracts, Endogenous Verifiability

1 Introduction

For contracting parties it is always to some extent uncertain whether a legal court is able to enforce the real intents of their contract. The strict requirements of verifiability makes it costly, if not impossible, for the parties to arrange their transactions and design

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their contract in a way that makes it completely protected by the court. If property
rights allocation do not provide the parties with proper incentives to transact, they must
therefore rely on self-enforcement. Through repeated transactions they can make it costly
for each other to breach the contract, by letting breach ruin future trade. But the self-
enforcing range of contracts is limited, so the court’s ability to enforce the contract is
not without consequence for the contracting parties. If possible, the parties may thus
have incentives to take costly actions that affect the ability of the court. Both ex ante
preparations in terms of detailed contracting, and ex post revelation of information can
increase the probability of fair court enforcement. In this paper, we focus on the former,
assuming that the parties are able to improve the verifiability of the contracted upon
actions by careful ex ante contract specifications. We assume that careful contracting can
improve the court’s ability to verify whether an action is equal to the one described in
the contract. Then we ask: what happens to the self-enforcing contract equilibrium if a
party takes actions ex ante that affect the verifiability of actions ex post? In order to
give some answers to this question, we analyze a simple repeated principal-agent game
where the verifiability of the agent’s actions is endogenously determined by the principal’s
investments in drafting an explicit contract pertaining to the quality of the agent’s output.

The existing principal-agent literature assumes at the outset that some variables are
verifiable, and thus enforceable by courts, and some are not. Models with Costly State
Verification (CSV) have focused on contract design problems where enforcement and thus
verifiability is a decision variable, but the CSV approach considers verification of exogenous
state variables, not endogenous variables as here (see Krasa and Villamil (2000) for a
generalization of the CSV models). Ishiguro (2002) analyzes a model with endogenous
verifiability of endogenous effort variables. In his model, verifiability is determined by the
parties’ choices when disputes arise, while we analyze a model where verifiability depends
on choices taken ex ante. Ishiguro does not consider the effect of endogenous verifiability
in repeated relationships, as we do.

In repeated game models of relational (also called implicit) contracts, verifiability is al-
ways exogenously given. By definition, a relational contract relies only on self-enforcement;
effort variables are non-verifiable, and the parties honor the contract as long as the present
value of honoring exceeds the present value of reneging. MacLeod and Malcomson (1989)
made a definite treatment of relational contracts with symmetric information. Schmidt
and Schnitzer (1995) analyze a repeated relationship where some actions can be verified
and some cannot, while Baker, Gibbons and Murphy (1994) analyze a model with one action generating one verifiable, but imperfect signal and one perfect, but non-verifiable signal. But the verifiability of a given action or signal is exogenously given in these models. The present paper is, to our knowledge, the first to analyze endogenous verifiability of actions in a repeated principal-agent model.\(^1\)

1.1 Summary of Results

In our model the principal makes a specific investment by paying the agent up-front and expecting an agreed upon quality level in return. If they transact repeatedly, the principal can safeguard her investment by threatening to end the relationship if the agent abuse her trust. In addition she can write a detailed contract specifying quality in order to increase the probability of court verification. Since both quality and verifiability is costly, the principal faces trade-offs. These trade-offs reveal several interesting relationships:

Trust

First, it complicates the relationship between trust and specific investments. Let the famous merger between General Motors (GM) and Fisher Body in 1926 be a backdrop. The classic interpretation is that GM acquired Fisher Body because of relationship specific underinvestment and contractual hold-up (see Klein, Crawford and Alchian, 1978; Williamson, 1985; Hart, 1995; Klein 2000 ). Others oppose to this view, arguing that the relationship between GM and Fisher Body prior to 1926, exhibited trust rather than opportunism (see Casadeus-Masanell and Spulber, 2000; Coase, 2000; Freeland, 2000). Are these views necessarily incompatible? Cannot underinvestment go hand in hand with trust?

We know that the problem of relationship specific underinvestment is due to incomplete contracting. If contracts are complete, first-best investments can easily be achieved. Moreover, we know that trust can be a source of contractual incompleteness: Why spend effort in writing a detailed contract with somebody you trust? It is somewhat surprising, then, that in the incomplete contract literature, or more precisely in the literature on self-enforcing relational contracts, the degree of relationship specific investments is always a

\(^1\) Enforcement mechanisms are also an issue in the emerging economics literature on courts (e.g. Djankov, La Porta, Lopez-De-Silanes, Schleifer, 2003; Johnson, McMillan, Woodruff, 2002; Glaeser and Shleifer, 2002). This literature is concerned with factors that decide the court’s ability to enforce contracts, but do not model endogenous verifiability of effort variables.
positive function of trust: the higher trust-level, the closer one gets first-best investments. If trust generates contractual incompleteness, could it not also generate underinvestment? We show that once we allow for different levels of contractual incompleteness, modeled as endogenously determined probabilities of legal enforcement, the relationship between trust and relationship specific appropriable investments is not crystal clear. In fact, under certain conditions the level of specific investments can be a negative function of trust. Assume P invests s in A and expects q in return, where s is appropriable by A if q cannot be verified. For P to do this, A must be trustworthy. The more trustworthy A is, i.e. the greater A’s incentives are to honor trust, the more P will invest in A. Moreover assume that P spends money and effort in contract specifications in order to safeguard s. The more she spends in contract specifications, the higher is the probability, v, that a court can verify q, and thus legally enforce the contract. Now, what happens if the trust-level increases, i.e. A’s incentives to honor trust increases? First, there is an income effect: P will realize the surplus from higher trust by ordering a larger q, and/or reducing contract costs by lowering v. But there may also be a substitution effect: If higher trust makes q more costly in terms of v, i.e. if the level of v that is necessary to achieve a given level of q increases, P will substitute lower v for q. If this substitution effect dominates the income effect of higher trust, the investment level becomes negatively related to trust. Hence, the surplus from trust is not necessarily realized through higher investment-levels, it might as well be realized through lower contract costs, which in turn lowers specific investments.

**Verification technology**

Following Chakravorty and MacLeod (2004), the present paper views explicit contracts 'as part of the technology of exchange'. A proper level of explicit contracting is part of the efficient solution. The verification technology, i.e. the court’s ability to verify whether an action is equal to the one described in the contract, will decide the efficient level of contracting. Verification technology can be defined as the effort necessary to achieve a given level of verifiability (i.e. probability of verification) for a given transaction. By studying the effect of exogenous changes in verification technology we can (i) explain certain interesting aspects of legal systems and (ii) gain insight in the interactions between

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2Chakravorty and MacLeod models a situation where increased explicit contracting in form of project design and planning results in more certainty regarding the desired ex post design. Also Bajari and Tadelis (2001) endogenize contract incompleteness by choosing the proper degree of planning. In their model increased planning reduces ex post transaction costs due to costly renegotiation.
explicit and relational governance.

With respect to (i) we find an ambiguous relationship between verification technology and the equilibrium verifiability level. In particular we find that the surplus from better verification technology can be realized through lower contract costs and thus lower verifiability. This suggests that the verifiability level, and thus the predictability of the court, is not a good measure for the ability of the court. The model also predicts lower equilibrium verifiability level in civil law than in common law systems.

With respect to (ii), the model complements the literature on explicit versus relational contracting. It is well established in the literature that formal, verifiable contracts can both limit and expand the self-enforcing range of relational contracts (most notable Baker, Gibbons and Murphy, 1994; and Schmidt and Schnitzer, 1995). The logic behind this ambiguity is that better formal contracts enhances the long-term fallback position of the contracting parties, but at the same time reduces the short term gain from deviation. We complement this literature with a new aspect: If something with the formal contract is changed, not only the efficiency of the contract is changed, but also the probability of verification. This has some implications for a theory of the interaction between explicit and relational governance. In particular we find that explicit and relational governance can be complements even if there is a positive surplus from spot contracting.

Legal breach remedies

The model may also contribute to our understanding of how legal breach remedies affect the scope of contracting. In the main part we assume that the court applies ‘specific performance’ as a breach remedy, i.e. the breacher is required to perform what was contracted upon. Since there is no uncertainty over the value of the transaction in our model, specific performance is equivalent to ‘expectation damages’, in which the breacher has to pay the amount that makes the victim equally well off as under contract performance. Specific performance and expectation damages are standard legal breach remedies, but in the last part we analyze the effect of a less common remedy, namely ‘reliance dam-

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3 Other models within the ‘self-enforcing contract literature’ that addresses the relationship between explicit and relational governance are Pearce and Stacchetti (1998), Klein (1996), and Dixit (2004). Earlier, but less formal contributions, are MacNeil (1980) and North (1990). Also the ‘reciprocity literature’ has made important contributions, viewing explicit contracting as a move away from a reciprocal relationship to a strict ‘economic’ interaction (see e.g. Frey, 1997; Fehr and Gachter, 2000; Lubell and Scholz, 2001).

4 Expectation damages is the most typical remedy, but specific performance is increasingly used in commercial contexts (see Edlin and Reichelstein, 1996)
ages\textsuperscript{5}, where the breacher compensates the victim such that the latter is equally well off as before the contract was signed. We show that this remedy increases the scope of relational contracting. The intuition is simple: Since the contract is valuable for both parties, 'reliance damages' reduce the expected short-term gain from breach and hence create a greater scope for relational contracting.

The remainder of the paper is organized as follows: Section 2 presents the basic model. In Sections 3 and 4 we analyze how the relational contract equilibrium varies with trust and verification technology, respectively. In Section 5 we briefly discuss how variations in breach remedies affects the scope of relational contracting. Section 6 concludes.

2 The Model

Consider a relationship between a risk neutral principal and a risk neutral agent where the principal pays the agent $s$ to make the agent deliver a good with a quality that the principal values $q$. Payoff to the principal is $q - s$. Payoff to the agent is $s - C(q)$ where $C(q)$ is the cost of producing quality $q$, and $C'(q) > 0$, $C''(q) > 0$, $C(0) = 0$. Total surplus from the transaction is then $\Pi(q) = q - C(q)$. Reservation payoffs are normalized to zero.

We will consider the relationship as a one-sided trust game where the principal pays the agent $s$ up-front, trusting that the agent will deliver on the agreed upon quality $q$. The agent can honor that trust by delivering $q$, or he can abuse trust by not delivering a value adding quality, that is $q' = 0$.\textsuperscript{6} Note that the principal’s choice of $q$ reflects her level of specific investments, since $s$ is appropriable by the agent and expected $q$ is a strictly positive function of $s$.

Assume first that the there is no probability that the court can verify $q$. Assume also that the principal is the owner of the good produced, so that the agent cannot hold-up values ex post. Played only once, the Nash equilibrium is $[s = 0, q = 0]$, hence a one-shot transaction (spot contract) has low value - normalized to zero.\textsuperscript{7}

If the game is played repeatedly, and the discount factor is sufficiently high, then trigger strategies constitute a subgame perfect Nash equilibrium where some $[s, q]$ is played in

\textsuperscript{5}Although less common in private contracting, it is the default remedy in government contracting (Che and Chung, 1999).

\textsuperscript{6}Trust-abuse is equivalent to any $q' \neq q$, but if the agent is to abuse trust, his optimal deviation is to play $q' = 0$

\textsuperscript{7}This corresponds to the 'spot employment' contract in Baker, Gibbons and Murphy (2002).
every stage game. What trigger strategies? We assume that the principal plays as follows: If the agent honored the contract in all previous periods $\tau < t$, then principal honors the contract in period $t$. If not, the principal offers a spot contract $s = 0$ forever after, where the agent responds by playing $q = 0$. The agent will thus honor trust as long as 

$$\frac{1}{1-\delta}(s - C(q)) \geq s,$$

where $\delta$ is the discount factor. The principal does not have incentives to deviate ex post; it is the ex ante condition $q \geq s$ that binds. It is possible to find such $s$ iff $q \geq \frac{C(q)}{\delta}$. An efficient contract maximizes surplus $q - C(q)$, subject to $q \geq \frac{C(q)}{\delta}$. This yields first-best quality level $q = q^F$ for $\delta$ so large that $q^F > \frac{C(q^F)}{\delta}$. For lower $\delta$, optimal quality satisfies $q = \frac{C(q)}{\delta}$.

Assume now that the parties to a certain extent can rely on a third party (the court) to verify $q$. Let $v$ be the probability that the true value of the good will be verified in a court of law in case of contract breach. Verification implies legal enforcement. If the parties can write contracts specifying large penalties in case of contract breach, first-best is easily implemented when there is a positive probability of verification. In line with actual court behavior, it is reasonable to assume, however, that contracted penalties are denied if they are sufficiently large relative to the actual loss from contract breach.

We assume that if a contract breach is verified, the court applies 'specific performance' as a breach remedy, i.e. the breacher is required to perform what was contracted upon, which in our model means that the agent is legally forced to deliver $q$ to the cost of $C(q)$. As noted, since there is no uncertainty over the value of the transaction in our model, specific performance is equivalent to 'expectation damages', in which the breacher has to pay the amount that makes the victim equally well off as under contract performance.

With expectation damages, Ishiguro (2002) shows that first-best is achievable in one-shot transactions (under some strict assumptions) if a party spends effort in verifying actions ex-post. Here we assume that it is not possible to take actions ex post production that affects the probability of legal enforcement. Efforts to improve verifiability must be

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8 The common law tradition prevents courts from enforcing terms stipulating damages that exceed the actual harm. Civil law countries show more willingness to enforce contract penalties, but penalties are reduced if they are unreasonable large.

9 Specific performance and expectation damages typically differs if there is uncertainty over costs, for instance if the true cost function is revealed ex post contract signing, but ex ante complete performance.

10 If this assumptions seems strong, recall that it is implicitly made in all principal-agent models (except Ishiguro’s). It can, for instance, be assumed that ex post verification effort is too costly (expensive lawyers etc.).
taken ex ante. Let $K(v)$ be the cost to achieve verifiability level $v$. We can interpret $K$ as the costs associated with writing explicit contracts specifying qualities of the good.

If full verification, i.e. $v = 1$, is possible, first-best quality is in principle achievable in one-shot transactions. A contract is then fully verifiable at cost $K(1)$, and if this cost is not too high, the parties can engage in a one-shot explicit contract with surplus $q^F - C(q^F) - K(1)$. For the most part, we will, however, assume that $0 \leq v < 1$, i.e. the principal cannot achieve $v = 1$. This is the standard assumption in the relational contract literature (that is: $v = 0$ is the standard assumption). The assumption says that even if the contract is extremely detailed and seemingly covers all contingencies, there will always be some uncertainty as to whether the court in fact is able to verify the true intent of the contract. In Section 4.1 we will discuss implications of allowing $v = 1$.

Let us then consider the relational contract constraints. If the agent now abuses trust, he will with probability $v$ nevertheless be forced to deliver $q$ at the cost $C(q)$. The agent will therefore honor trust if

$$\frac{1}{1 - \delta}(s - C(q)) \geq s - vC(q) \quad (1)$$

Note that $q > 0$ is impossible in one shot transactions (i.e. when $\delta = 0$) since the expected value from reneging exceeds the expected value from honoring when $v < 1$.

As noted, with up-front payments, the principal has nothing to renege from ex post.\(^{11}\) It is the ex ante condition that binds. The ex ante condition simply says that the per period expected surplus must be positive, that is

$$q - D(\delta)K(v) \geq s \quad (2)$$

where $D(\delta)K(v)$ is the ‘dynamic cost’ of contracting. We will mostly discuss two cases: i) $D(\delta) = 1 - \delta$ which means that the principal only carries contract costs $K$ in the first period, and ii) $D(\delta) = 1$ where $K$ is carried every period.

It is possible to find an $s$ satisfying (1) and (2) iff

$$G(v, q) = q - \frac{1}{\delta}(1 - (1 - \delta)v)C(q) - D(\delta)K(v) \geq 0 \quad (3)$$

\(^{11}\)If salary $s$ is to be paid ex post, it is the principal, not the agent, that has incentives to deviate ex post. As in the case with up front payments, $q > 0$ is not achievable in one shot transactions since $q - s < q - vs$ when $v < 1$. 

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An efficient contract should maximize per period surplus

\[ q - C(q) - D(\delta)K(v) \]

subject to (3). The problem is illustrated in Figure 1.12

The heavy curve represents the constraint, with admissible \((q, v)\) values in the north-west region (and bounded by \(v = 1\)). The thin curves are iso-surplus (indifference) curves, with the lower curve representing a higher surplus. With no verification investment \((v = 0)\) the maximal surplus that can be achieved is \(q^0 - C(q^0)\), where \(q^0 = q^0(\delta) < q^F\) is defined by \(q^0 = C(q^0)/\delta\). In the figure the upper indifference curve represents this surplus. The figure illustrates that one can achieve a higher surplus by making some ‘verification investments’ and thus move along the curve representing the constraint.

Given some regularity conditions the constraint can be represented as \(v = v(q, \delta)\), and the objective then takes the form

\[ F(q, \delta) = q - C(q) - D(\delta)K(v(q, \delta)) \tag{4} \]

The first order condition for an interior maximum is

\[ F_q(q, \delta) = 1 - C'(q) - D(\delta)K'(v(q, \delta))v_q(q, \delta) = 0, \tag{5} \]

or \(v_q = \frac{1 - C'(q)}{D(\delta)K'(v)}\); i.e. tangency between the constraint curve and the relevant indifference curve. We will here consider cases where the constraint curve is increasing everywhere \((v_q > 0)\). The optimum will then entail a quality level \(q < q^F\). In appendix we prove the following:

\[ \text{The figure is generated with } D(\delta) = 1, C(q) = \frac{1}{2}q^2, K(v) = av, \delta = \frac{1}{4} \text{ and } a = \frac{1}{8}. \]
Proposition 1 Assume $q_0(\delta) > 0$. The problem has an interior solution with $v \in (0, 1)$ and $q \in (q_0, q^F)$ if $K(0)$ and $K'(0)$ are sufficiently small.

We are now interested in how the optimum varies with the level of trust, represented by $\delta$, and the level of verification technology, represented by the cost $K(v)$ required to obtain a verification probability $v$.

3 Trust

The repeated game approach formalizes an economic concept of trust and trustworthiness (see James Jr. 2002 for a nice survey on the economic concept of trust). A party honors trust if the present value of honoring exceeds the present value of abusing trust. In such, the discount factor is a proxy for trust. In a repeated relationship between P and A, if P knows that A has a high discount factor, P knows that A values future trade with P. Hence P trusts A and A is trustworthy.

A common feature of the self-enforcing relational contracts studied in the literature is that specific appropriable investments is positively related to the parties discount factors. The higher the discount factor, the higher is the present value of the ongoing relationship relatively to the present value of reneging on the contract. When this 'punishment’ from reneging increases, the parties are able to generate higher quasi-rents without running the risk of opportunism. Hence, the higher discount factor, the closer the parties come first-best investments. We will here show that this relationship between discount factors and specific investments is not monotonically positive when there is an endogenous determined probability of contract verification.

From (3) we see that if the trust-level ($\delta$) increases, the constraint is relaxed, so the social surplus must increase. Geometrically the constraint curve shifts downwards. If the 'verification investment’ level $v$ is held fixed, the higher trust level then allows for a higher implementable quality $q$. But both $v$ and $q$ will optimally change with $\delta$. Note that the higher trust level $\delta$ will affect the slope of the constraint curve. If the slope increases ($v q^\delta > 0$), the rate of substitution between $v$ and $q$ increases, so it becomes more costly to substitute quality for lower verification investment. This price change will induce a substitution effect that in isolation will lead to reduced quality. But there may also be an 'income effect’ working in the opposite direction on quality. These considerations
indicate that the equilibrium response to a higher level of trust may be either a higher or a lower level of quality. We will demonstrate that the equilibrium quality level may in fact decrease.

Given an interior solution we obtain from the first order condition (5);

\[ q'(\delta) = \frac{F_{q\delta}(q, \delta)}{-F_{qq}(q, \delta)} \]

Assuming the second order condition \((F_{qq} < 0)\) is fulfilled, we see that \(q'(\delta)\) has the same sign as \(F_{q\delta}\). In the appendix we prove the following:

**Proposition 2** For an interior solution we have

\[ \text{sign } q'(\delta) = \text{sign } \left\{ -D'(\delta)K'(v)(1 - v) + \left[ K'(v) - K''(v)(1 - v) \right] (1 - \delta)v_\delta \right\} \quad (6) \]

where \(v_\delta < 0\)

Consider first the case where \(K\) is paid every period i.e. \(D(\delta) = 1\). This is the relevant case when the principal must modify the contract every period. Note that even if the contract is modified, this does not necessarily mean that equilibrium \(v\) is changed. In fact, we consider stationary contracts where the equilibrium \(v\) and \(q\) is assumed to apply every (remaining) period. This case arises when new technological developments imply that the content of \(q\) changes, but the costs required to produce the object of value \(q\) and the verification level \(v\) do not change. Then contract modifications are required even if costs \(C(q)\) and \(K(v)\) are unaffected.

With \(D(\delta) = 1\) we see from (6) that quality decreases with more trust \((q'(\delta) < 0)\) when \(K'(v) > K''(v)(1 - v)\). This is not a strong requirement. For instance, it holds if verification costs are linear \((K'' = 0)\) in a neighborhood of the equilibrium point.

To better understand this condition, consider the equilibrium relation (5). Computing \(v_q = -G_q/G_v\) from the constraint (3), this relation can be written as

\[ \frac{1 - C'(q)}{D(\delta)K'(v)} = v_q = \frac{\frac{1}{\delta}(1 - (1 - \delta)v)C'(q) - 1}{\frac{1}{\delta}(1 - \delta)C'(q) - D(\delta)K'(v)} \]

Collecting terms (and accounting for \(D(\delta) = 1\) or \(D(\delta) = 1 - \delta\) we can write the relation as follows

\[ D(\delta)K'(v)(1 - v) = (1 - C'(q)) \left( \frac{C(q)}{C'(q)} \right) \quad (7) \]

In the case \(D(\delta) = 1\) we see that this condition defines a relation between \(v\) and \(q\) that is independent of \(\delta\). It thus defines the locus of equilibrium \((q, v)\) points as \(\delta\) varies. The
right hand side is decreasing in $q$ at an interior optimum, and the locus will therefore be upward (downward) sloping if $K'(v)(1 - v)$ is decreasing (increasing). This is precisely what is captured in (6). Figure 2 provides an illustration.13

In Figure 2 the equilibrium locus is downward sloping for small $v$ and upwards sloping for larger $v$. It corresponds to a quadratic cost function $K(v) = kv^2$, where thus $K'(v)(1 - v)$ is increasing for $v < \frac{1}{2}$ and decreasing for $v > \frac{1}{2}$. For low $\delta$ the equilibrium verification level $v$ is high, and increased trust will then lead to a reduction of quality $q$. For higher $\delta$ the equilibrium verification level $v$ is low, and increased trust will then increase equilibrium quality $q$.

Consider next the case where $K$ is carried only in the first period i.e., $D(\delta) = (1 - \delta)$. Here the principal invests in developing a contract in the first period, and then uses this as a standard contract in the remaining periods. Then $D'(\delta) = -1$, so $q'(\delta) < 0$ iff

$$K'(v)(1 - v) < - \left[ K'(v) - K''(v)(1 - v) \right] (1 - \delta)\nu_{\delta}$$

We observe that if $K'(v) < K''(v)(1 - v)$, then $q'(\delta) > 0$. But if $K'(v) > K''(v)(1 - v)$, then $q'(\delta)$ is not necessarily negative, hence, the requirements for $q'(\delta) < 0$ is stricter when $K$ is carried only once. For instance, with linear verification costs we see that $q'(\delta) < 0$ if $(1 - v) < -(1 - \delta)v_{\delta}$ which is only true for low $\delta$.

In this section we have shown that higher trust can reduce the quality level, i.e. the level of specific investments. We have also seen that this relationship is especially relevant in cases where the principal must invest in contract specifications every period. We can say

13The figure is generated with $C(q) = \frac{1}{8}q^2$, so $f' = \frac{1}{4}$, and $K(v) = \frac{1}{2}v^2$. 
that the more contract modifications that are needed during the relationship (that is the higher $D'(\delta)$, where max $D'(\delta)$ is zero), the more likely it is that the ‘surplus from trust’ is realized through lower explicit contract costs instead of higher specific investments. This corollary is quite intuitive. In a changing environment it is costly to constantly make new contract specifications. A high trust level can save on contract outlays, but at the cost of lower specific investments. This also suggests that we will see lower levels of specific investments in changing environments. In a sense this is predicted, and supported empirically, by transactions cost economists. Williamson (1985), among others, predicts lower levels of specific investments in transactions with much uncertainty. We do not model uncertainty, but the difference $D(\delta)K(v) - (1-\delta)K(v)$ reflects an additional cost of dealing with the uncertainty over future contract modifications.

4 Verification Technology

The necessary cost to achieve a given level of legal enforcement may differ from country to country, from industry to industry, and from transaction to transaction. The complexity of the transactions, the strength of enforcement institutions and the practice of legal courts are factors that determine the verification technology. We will here analyze how improvements in this technology affect equilibrium $q$ and equilibrium $v$.

The analysis extend the literature on the interaction between explicit and relational governance. It is well established that explicit and relational governance can both be substitutes and complements, i.e. better formal contracts can both increase and reduce the scope of relational contracting. By far we develop the same result as Baker, Gibbons and Murphy (1994) and Schmidt and Schnitzer (1995), but with some modifications. In their models, explicit contracting and relational contracting are complements only when the value of spot contracting is non-positive, if else the modes of governance are substitutes. The reason for this is as follows: If the explicit part of the contract (i.e. the aspects that can be completely contracted upon) is improved, the short-term gain from reneging on the implicit non-contractible elements of the contract is reduced. But the long term loss from reneging is also reduced. With discount factors that are sufficiently large to implement the relational contract, this reduction in long term loss exceeds the reduction in short term gain. So the total reneging temptation is increased. But if the value from spot contracting is non-positive, then the long term loss from reneging is not affected by improved formal
contracts, while the short term gain is still reduced. Hence, an improvement in the explicit part of the contract reduces the gain from deviation if the value from spot contracting is non-positive. If else it increases the gain from deviation.

The same fundamental mechanism appears in our model, but with a difference. We do not separate between a formal \((v = 1)\) and an informal part \((v = 0)\) of the contract. Instead the degree of formalism is decided by the contract’s \(v\)-level. In Baker et.al. and Schmidt and Schnitzer, an exogenous change of the formal part of the contract is equivalent to a direct change in the efficiency of the contract.\(^{14}\) In our model, an exogenous change of the contract (exogenous change in verification technology) implies a change in the probability of verification. This gives a slightly stronger result for complementarity. As Baker et. al and Schmidt and Schnitzer, we find that explicit and relational governance are complements when spot contracting yields a non-positive surplus. But, as demonstrated in the next subsection (4.1), explicit and relational governance can also be complements if spot contracting yields positive surplus.

First, we continue to consider the case where \(v = 1\) is impossible, implying that spot contracting alone cannot yield positive surplus. In such a setting, a better verification technology relaxes the incentive constraint and thus increases the self-enforcing range of the relational contract. Hence explicit and relational governance are complements. However, improved verification technology does not necessarily imply that verifiability \(v\) and quality \(q\) increase in equilibrium.

For a verification cost function \(K(v, \kappa)\) with \(K_\kappa \geq 0\) we will now find \(\text{sign}(q'(\kappa))\). We obtain the following result (see appendix for proof):

**Proposition 3** For \(K(v, \kappa)\) with \(K_\kappa \geq 0\) we have

\[
\text{sign}(q'(\kappa)) = \text{sign}\left\{-(1 - v)K'_\kappa + \left[K' - (1 - v)K''\right] v_\kappa\right\},
\]

where \(v_\kappa = gK_\kappa\), \(g > 0\), and \(K'_\kappa = \frac{\partial^2 K}{\partial \kappa \partial v}\).

Consider first a fixed cost increment, where \(K'_\kappa = 0\). Then we get

\[
\text{sign} \left[q'(\kappa)\right] = \text{sign} \left[K' - (1 - v)K''\right]
\]

\(^{14}\)In Baker et. al. a better formal contract means that the objective performance measure becomes less distorted. In Schmidt and Schnitzler it means that the cost of writing the verifiable part of the contract is reduced.
Thus quality increases with higher fixed costs iff \( K' - (1 - v)K'' > 0 \). A higher fixed cost will shift the constraint vertically upwards, but not affect (the shape of) indifference curves. The equilibrium relation (7) between \( q \) and \( v \), and hence the equilibrium locus, will then not be affected. When this locus slopes upwards \( (K' - (1 - v)K'' > 0) \) both \( q \) and \( v \) will increase in response to the cost increase. When the locus slopes downwards \( (K' - (1 - v)K'' < 0) \), \( q \) will decrease, but \( v \) will increase. The geometrical picture illustrating these effects is similar to that in Figure 2.15

The previous discussion shows that when \( K'_\kappa = 0 \), improved verification technology (a reduction of fixed costs) leads to lower verification probability in equilibrium. This may seem counterintuitive: The principal in fact buys less of \( v \) if it becomes cheaper. But it is the same logic as in the previous section where the principal realized the surplus from higher trust by reducing contract costs instead of increasing quality. \( (q'(\delta) < 0) \). Here, the principal chooses to realize the surplus from improved verification technology by reducing contract costs instead of increasing quality. And note that the condition for \( q'(\kappa) > 0 \) is the same as the necessary condition for \( q'(\delta) < 0 \) (sufficient condition when \( D(\delta) = 1 \)).

The result that \( \frac{dv}{d\kappa} \) can be positive suggests that the ability of a country’s court system cannot be measured from the predictability of the court. We can interpret verification technology as the court’s ability to verify the true intent of a contract. And we can interpret \( v \) as a proxy for the court’s predictability. Since higher court ability can give lower verification in equilibrium, higher court ability can also imply less predictability in equilibrium. We can say that the gain from a ‘high ability’ court system may be realized through a higher degree of relational contracting rather than a higher degree of explicit contracting.

Consider next a cost change that affects marginal but not fixed costs. Consider some equilibrium \((q^*, v^*)\). If the marginal verification costs increase locally in the sense that \( K'_\kappa(v^*, \kappa) > 0 \) and \( K_\kappa(v^*, \kappa) = 0 \) (i.e. total verification costs is unchanged), where the latter implies \( v_\kappa = 0 \), we have

\[
sign(q'(\kappa)) = sign\left\{-(1 - v)K'_\kappa\right\}
\]

hence \( q'(\kappa) < 0 \). We can interpret a marginal change \( K'_\kappa(v^*, \kappa) > 0 \) and \( K_\kappa(v^*, \kappa) = 0 \) as a marginal move from common to civil law practice. The common law system is assumed

\[\text{Figure 2 shows constraint curves corresponding to different } \delta \text{'s and the equilibrium locus for } D(\delta) = 1.\]

In the present case the constraint curves will exhibit vertical shifts corresponding to different levels of fixed costs. The position, but not the slope of the equilibrium locus, may be affected by \( \delta \) via \( D(\delta) \).
to be more willing to enforce specific contract terms than civil law, which to a larger extent set party-designed contract terms aside if it conflicts with the civil codes. The marginal effect on \( v \) of investing in detailed contracts is thus assumed to be higher in common law, but the civil codes assures that a minimum level of verifiability can be achieved at low costs. We see that \( K_\kappa(v, \kappa) = 0 \) implies \( v_\kappa = 0 \) and thus \( \frac{dv}{d\kappa} = q_\kappa \frac{dK}{d\kappa} + v_\kappa < 0 \), since \( q_\kappa > 0 \) and \( \frac{dK}{d\kappa} < 0 \). Hence a move from common to civil law practice implies lower equilibrium \( v \), which fits with the observation that contracts are typically more detailed in common law countries than in civil law countries.

4.1 Fully Verifiable Contracts

As noted in the introduction, if full verification, i.e. \( v = 1 \), is possible, first-best quality is in principle achievable in one-shot transactions. A contract is then fully verifiable at a finite cost \( K(1) \), and if this cost is not too high, a higher surplus can be achieved in a spot contract with \( v = 1 \) than with \( v = 0 \). This possibility will also affect the relational contract equilibrium, since the parties’ fallback positions are improved. If an explicit spot contract yields a positive per period surplus \( u = qF - C(qF) - D(\delta)K(1) > 0 \), the relational contract constraint becomes

\[
q - \frac{1}{\delta}(1 - (1 - \delta)v)C(q) - D(\delta)K(v) \geq u
\]

It is here worth noting that, although full verifiability \( (v = 1) \) is feasible and yields a positive surplus also in the relational contract, it is typically (given the conditions in Proposition 1) not an efficient solution. When there is some level of trust and hence some scope for relational contracting between the parties, they can trade between product quality and verification investment, and this trade-off then typically leads to less than full verifiability. They will thus not invest resources in contract specification to the extent that the contract becomes perfectly verifiable, even if this is feasible and possibly quite inexpensive. It is in this sense efficient to leave the contract to some extent incomplete.

Another noteworthy aspect is that, in contrast to the case where the value from spot contracting is non-positive, improved verification technology can now reduce total surplus. Explicit and relational governance can thus be substitutes in the present case. This occurs when the better technology improves the spot contract to such an extent that the set of viable relational contracts becomes smaller.

Figure 3 illustrates this kind of situation. The heavy contours in the figure delineate
the set of feasible relational contracts for two verification technologies (verification cost functions). The dashed contour corresponds to the less favorable technology. That is, the dashed contour corresponds to a cost function $K(v, \kappa')$ that is above the cost function $K(v, \kappa)$ associated with the solid contour.\footnote{More precisely, the figure is generated for the case $D(\delta) = 1$ and with cost functions of the form $K(v, \kappa) = K(v, \kappa') = av$ for $v < v_1$, $K(v, \kappa) < K(v, \kappa')$ for $v_1 < v \leq 1$. The upper parts of the contours reflect that to each $q$ there is a maximal $v \leq 1$ that can be implemented. In the previous figures the maximal $v < 1$ was exogenous and not depicted.} We see that the set of feasible relational contracts is smaller for the better technology, and that the attainable surplus is lower for this case.

To see this effect analytically, note that the impact of a marginal cost increase on the maximal surplus is given by

$$\frac{\partial F}{\partial \kappa} = -D(\delta) \left( K_\kappa(v, \kappa) + K'(v, \kappa)v_\kappa \right)$$

where $v_\kappa$ from the constraint (written as $G(q, v, \delta, \kappa) \geq 0$) is given by

$$v_\kappa = \frac{-G_\kappa}{G_v} = \frac{1}{G_v} \left[D(\delta)K_\kappa(v, \kappa) + u_\kappa\right] = \frac{D(\delta)}{G_v} \left[K_\kappa(v, \kappa) - K_\kappa(1, \kappa)\right] \tag{9}$$

The two terms in the expression for $\frac{\partial F}{\partial \kappa}$ account for the direct effect of increased costs on the surplus, and the indirect effect induced by a shift in the constraint curve, respectively. We see that if $v_\kappa > 0$, i.e. if the constraint curve shifts upwards with increased costs, the sign of $\frac{\partial F}{\partial \kappa}$ is unambiguously negative. The surplus will then decrease with higher verification costs. But as illustrated by the figure above, we may have a downward shift of the constraint curve ($v_\kappa < 0$) in the relevant region.

Figure 3: Constraint contours and indifference curves.
At an optimum where the constraint curve is upward sloping \((v_q > 0)\) we have \(G_v > 0\), and the sign of \(v_\kappa\) is then given by the sign of \([K_\kappa(v, \kappa) - K_\kappa(1, \kappa)]\). The latter will obviously be negative if costs increase more at \(v = 1\) than at the relational equilibrium point \(v\), that is if costs increase more in the spot contract than in the relational contract. The last term in the expression for \(\frac{\partial F}{\partial \kappa}\) will then negative, and it may dominate the first term. We thus have the following result.

**Proposition 4** The maximal surplus may increase or decrease with higher verification costs. In particular, at an interior optimum \((v^*, q^*)\) where \(v_q > 0\), we have: (i) if \(K_\kappa(v^*, \kappa) = 0\) and \(K_\kappa(1, \kappa) > 0\), then \(\frac{\partial F}{\partial \kappa} > 0\), while (ii) if \(K_\kappa(v^*, \kappa) > 0\) and \(K_\kappa(1, \kappa) = 0\), then \(\frac{\partial F}{\partial \kappa} < 0\).

Result (ii) from this proposition is similar to Baker et. al. and Schmidt and Schnitzer: Explicit and Relational Governance are substitutes when spot contracting yields positive surplus. But (i) shows that they also can be complements in this situation. We thus have a stronger result for complementarity than Baker et. al. and Schmidt and Schnitzer. The reason for this is that a better verification technology not only improves the efficiency of the contracts. It also increases the probability of verifying a given quality level. This directly supports the implementation of relational contracts. Interestingly, this accords well with recent empirical and experimental studies (Poppo and Zenger, 2002; and Lazzarini, Miller and Zenger, 2004), which find a stronger support for the complementarity hypothesis.

Turning to the effects of increased verification costs on equilibrium quality we obtain the following result.

**Proposition 5** At an interior optimum where \(v_q > 0\), we have

\[
\text{sign}(q'(\kappa)) = \text{sign} \left[-K'_\kappa(1 - v) + \left[K' - K''(1 - v)\right] v_\kappa\right],
\]

where \(v_\kappa\) is given by (9)

The formal expression that determines the sign of \(q'(\kappa)\) is the same as in the case of limited verifiability considered in the previous section. But there is a difference, since the sign of \(v_\kappa\) is ambiguous in the present case. Here this sign depends, as discussed above, on whether verification costs increase more or less in the spot contract than in the relational contract.
Figure 3 illustrates this point. It depicts a situation where $K' = 0$ and $K - K''(1 - v) > 0$ at the equilibrium point $v$ for the relational contract, and where $v_k < 0$ due to $K_k(v, \kappa) - K_k(1, \kappa) < 0$. In this situation the equilibrium quality will decline. Under the same conditions, but a cost function that exhibited $K' - K''(1 - v) < 0$ at the equilibrium point, the equilibrium quality would have improved. A more costly verification process ($K_k \geq 0$) may thus lead to either higher or lower equilibrium quality.

5 Legal Breach Remedies

The previous analysis applied a standard legal breach remedy, namely 'specific performance', which, as noted, equals 'expectation damages' when there is no uncertainty over the value of the transaction. It was assumed that the court would, in the event that quality is verifiable, hold the agent responsible for the provision of the contracted quality (and enforce any ex post payments by the principal). Given that the agent must be paid at least the cost of providing that quality ex ante (e.g. because he has no wealth and cannot borrow from others to finance the specific investment), we saw that this rule leads to the specific IC constraint (3) for the relational contracts.

We will here briefly consider a variation of the court rule. Specifically we will consider another reasonable rule, namely 'reliance damages' in which the agent is held responsible for the funds ($s$) allocated to him up front. Thus, in the event of any deviation $q'$ from the contracted quality $q$, the agent is forced to repay $s - C(q')$ ex post if quality turns out to be verifiable. It is straightforward to see that this rule enlarges the set of allocations that can be implemented in a relational contract. If the agent now abuses trust, he will with probability $v$ be forced to repay unused funds. The agent will therefore honor trust if

\[
\frac{1}{1 - \delta} (s - C(q)) \geq s - vs \quad \text{[recall $q' = 0$ and $C(0) = 0$].}
\]

The principal must still have $q - D(\delta)K(v) \geq s$. It is possible to find such an $s$ iff

\[
q - \frac{C(q)}{(1 - \delta)v + \delta} - D(\delta)K(v) \geq 0 \quad (10)
\]

We see that (10) is a weaker constraint than (3) if

\[
\frac{1}{(1 - \delta)v + \delta} < \frac{1}{\delta} (1 - (1 - \delta)v)
\]

This holds if $0 \leq v < 1$ and $\delta < 1$ (to see this note that $((1 - \delta)v + \delta)\frac{1}{\delta} (1 - (1 - \delta)v) = \frac{1}{\delta} (1 - v)(\delta - 1)^2 v + 1 > 1$).
Since reliance damages yields a weaker relational contract constraint, the parties can implement higher quality with this court rule. The intuition is simple: Since social surplus is higher when the contract is fulfilled than when the contract is cancelled, the parties’ gain from breach is lower under reliance damages than under expectation damages. As a result, the self-enforcing range of the relational contract is increased if the parties use reliance damages instead of expectation damages.

An important insight from the literature on breach remedies, is that when contracts are enforceable by legal courts, legal breach remedies can lead to overreliance (overinvestment) since the promisee is always compensated in case of breach. It many settings reliance damages leads to greater overreliance than expectation damages (Shavell, 1980; Rogerson, 1984), and the latter is therefore considered to be superior.\textsuperscript{17} With reliance damages, the parties are made worse off in case of breach, since they must accept foregone opportunities. Hence, they want to reduce the probability of breach. If the contract is enforceable, the promisee (the principal in our setting) can reduce the probability of breach by increasing her reliance, since increased reliance makes it more costly for the promisor (the agent) to breach.

Overreliance is not an issue in our model since the only reliance is $s(q)$ itself, and since $s'(q) > 0$, overinvestment does not occur. But our model may be a starting point for analyzing optimal breach remedies in relational contracting. Models of optimal breach remedies always assume that contracts are verifiable, except for Edlin and Reichelstein (1996) who integrates the literature on legal remedies with the hold-up literature. We show how repeated interaction and endogenous verifiability may alter conclusions on optimal breach remedies. But for our model to be really fruitful in this respect, richer reliance, and uncertainty over the value of the transaction, should be introduced.

6 Concluding remarks

By discussing both specific investments and explicit contract costs in the same model, we get hold of a relationship between classic references in the literature on transaction costs and hold-up. One interpretation of the model is that the explicit contract costs, $K$, correspond to Coase’s (1937) concept of transactions costs, while the efficiency-loss of not

\textsuperscript{17} An exception: Che and Chung (1999) find that reliance damages give stronger incentives for cooperative investments than expectation damages.
being able to implement first-best allocations corresponds to the type of transaction costs introduced by Klein, Crawford and Alchian (1978) and Williamson (1985), and formalized in the property rights literature (Grossman and Hart, 1986; Hart and Moore, 1990). While Coase focuses on the costs of “negotiating and concluding a separate contract for each exchange transaction...,” Williamson et. al. focus on problems of opportunism and under-investment. By introducing an endogenous probability of legal contract enforcement, we get hold of the substitutability between these types of transactions costs. But perhaps more importantly, the model demonstrates that explicit contract costs are not transaction costs in the meaning of waste. Contracting is an investment, and contract costs must be considered as an endogenous variable determined in equilibrium.

As noted by Tirole (1999), scant attention has been paid to contract enforcement in the incomplete contract literature. In models of relational contracts, enforcement is the central issue, but probabilities of legal enforcement is excluded. By introducing endogenous verifiability in a relational contract set-up, we show how legal institutions can play a role in trust environments. Along these lines the model may serve as a tool for studying the effects of institutional differences in modes and possibilities of legal enforcement.

The perhaps most interesting result from the analysis is how the probability of legal enforcement complicates the relationship between trust and specific investments. If it is true, what the model predicts, that the parties can realize surplus from trust by lowering the effort in writing explicit contracts, we may better understand business relationships that are blessed with trust, but troubled with relationship specific underinvestments.

Appendix

Proof of Proposition 1.
For \( q \geq q_0 \), let \( v(q) \) be the smallest \( v \geq 0 \) that satisfies the constraint
\[
G(v, q) = q - \frac{1}{\delta}(1 - (1 - \delta)v)C(q) - D(\delta)K(v) \geq 0,
\]
provided such \( v \) exists. Note that for \( D(\delta)K(0) < q_0 - C(q_0) \) we have
\[
G(0, q_0) = q_0 - \frac{1}{\delta}C(q_0) - D(\delta)K(0) = -K(0) \leq 0
\]
\[
G(1, q_0) = q_0 - C(q_0) - D(\delta)K(0) > 0
\]
For \( K'(0) \) satisfying
\[
D(\delta)K'(0) < \frac{1}{\delta}(1 - \delta)C(q_0) = q_0 - C(q_0)
\]
we further have
\[
G_v(0, q_0) = \frac{1}{\delta}(1 - \delta)C(q_0) - D(\delta)K'(0) > 0,
\]
Hence $G(v, q_0)$ is concave and has a smallest root $v(q_0) \in [0, 1)$, and moreover $G_v > 0$ at this root:

$$G_v(v(q_0), q_0) = \frac{1}{\delta}(1 - \delta)C(q_0) - D(\delta)K'(v(q_0)) > 0$$

We further have

$$G_q(0, q_0) = 1 - \frac{1}{\delta}C'(q_0) < 1 - \frac{1}{\delta} \frac{C(q_0)}{q_0} = 0$$

Hence $v(q) \in [0, 1)$ is well defined in a right neighborhood of $q_0$ and we have

$$v_q(q_0) = -G_q(0, q_0)/G_v(v(q_0), q_0) > 0$$

Substituting from the constraint in the objective we now have

$$F'(q_0) = 1 - C'(q_0) - D(\delta)K'(v(q_0))v_q(q_0)$$

For $K(0) = 0$ we have $v(q_0) = 0$ and thus $F'(q_0) > 0$ if $K'(0)$ is sufficiently small. Hence the optimal solution must entail $q > q_0$ and $v > 0$. By continuity this will hold also for $K(0)$ positive and small.

For $q > q_0$ we have

$$G_q(0, q) = 1 - \frac{1}{\delta}C'(q) < G_q(0, q_0) < 0$$
$$G_v(0, q) > \frac{1}{\delta}(1 - \delta)C(q) - D(\delta)K'(0) > G_v(0, q_0) > 0$$

So $G(0, q) < G(0, q_0) \leq 0$ and $G(v, q)$ is an initially increasing concave function of $v \geq 0$.

If $G(v, q) = 0$ has a root, the smallest root $v(q)$ thus satisfies

$$G_v(v(q), q) = \frac{1}{\delta}(1 - \delta)C(q) - D(\delta)K'(v(q)) > 0.$$

(For $K(v)$ nonlinear there may in non-generic cases be isolated points $q$ where $G_v(v(q), q) = 0$.)

The optimal solution must entail $q < q^F$. Suppose otherwise. For $q \geq q^F$ and $v < 1$ we have

$$-G_q(v, q) = \frac{1}{\delta}(1 - (1 - \delta)v)C'(q) - 1 > -G_q(1, q^F) = 0$$

We thus have $v_q > 0$ and hence $F'(q) < 0$; a contradiction. This completes the proof.

**Proof of Proposition 2**

To prove the proposition consider

$$F_{\theta \theta} = -D'(\delta)K'(v)v_q(q, \delta) - D(\delta) [K''(v)v_qv_q + K'(v)v_{\theta \theta}]$$

The partials of $v(\cdot)$ can be obtained from the constraint. For later reference we consider a slightly more general constraint, including an additive term $u(\delta)$ as follows:

$$G(q, v; \delta) = q - \frac{1}{\delta}(1 - (1 - \delta)v)C(q) - D(\delta)K(v) - u(\delta) = 0$$

The partials of $G(\cdot)$ are:
\[
G_\delta = \frac{1}{\delta} C(q) - D'(\delta)K(v) - u'(\delta)
\]
\[
G_v = \frac{1}{\delta}(1 - \delta)C(q) - D(\delta)K'(v)
\]
\[
G_q = 1 - \frac{1}{\delta}(1 - (1 - \delta)v)C'(q)
\]
\[
G_{\delta q} = \frac{1}{\delta^2} C'(q)
\]
\[
G_{vq} = \frac{1}{\delta}(1 - \delta)C'(q)
\]
\[
G_{v\delta} = \frac{1}{\delta^2} C(q) - D'(\delta)K'(v)
\]

The partials of \(v()\) are given by
\[
G_q(q, v; \delta) + G_v(q, v; \delta)v_q = 0
\]
\[
G_\delta(q, v; \delta) + G_v(q, v; \delta)v_\delta = 0
\]
and
\[
G_{q\delta} + G_{qv}v_\delta + [G_{u\delta} + G_{uv}v_\delta] v_q + G_v v_{q\delta} = 0
\]

Substituting from these relations we have
\[
G_v v_{q\delta} = -G_q v_\delta - G_v v_q - [G_{v\delta} + G_{vu} v_\delta] v_q
\]
\[
= -\frac{1}{\delta^2} C'(q) - \frac{1}{\delta}(1 - \delta)C'(q)v_\delta - \left[ \frac{1}{\delta^2} C(q) - D'(\delta)K'(v) - D(\delta)K''(v)v_\delta \right] v_q
\]

Consider now
\[
G_v F_{q\delta} = -D'(\delta)K'(v)G_v v_q - D(\delta) \left[ K''(v)G_v v_\delta v_q + K'(v)G_v v_{q\delta} \right]
\]

Substituting from the expression for \(G_v\) in the first two terms, and from the last expression for \(G_v v_{q\delta}\) in the last term, we obtain after some algebra (the algebra is included at the end of this proof):
\[
\frac{\delta}{D(\delta)} G_v F_{q\delta} = K'(v) \frac{E(\delta)}{\delta} C(q) v_q - K''(v)(1 - \delta)C(q)v_\delta v_q + K'(v) \left( \frac{1 - v}{\delta} + (1 - \delta)v_\delta \right) C'(q)
\]

where \(E(\delta) = (-D'(\delta))\delta(1 - \delta)/D(\delta) - 1\). Accounting for \(D(\delta) = 1\) and \(D(\delta) = 1 - \delta\) we may write
\[
E(\delta) = (-D'(\delta))\delta(1 - \delta)/D(\delta) - 1 = -D(\delta).
\]

Moreover, the equilibrium condition (7) implies
\[
C(q)v_q = C(q) \frac{1 - C'(q)}{D(\delta)K'(v)} = C'(q)(1 - v).
\]

Substituting this into (12) we get
\[
\frac{\delta}{D(\delta)} G_v F_{q\delta} = (-D(\delta) + 1)K'(v) \frac{1 - v}{\delta} C'(q) + \left[ K'(v) - K''(v)(1 - v) \right] (1 - \delta)C'(q)v_\delta
\]

It follows from the proof of Proposition 1 that \(G_v > 0\) at the equilibrium point. So \(F_{q\delta}\) has the same sign as the expression on the right-hand side of the last equation. Accounting
again for \( D(\delta) = 1 \) and \( D(\delta) = 1 - \delta \) we may write \( \frac{1 - D(\delta)}{\delta} = -D'(\delta) \). Substituting this into the expression we see that formula (6) holds.

Then consider \( v_\delta \), which is given by

\[
G_v v_\delta = -G_\delta = -\frac{1 - v}{\delta^2} C(q) + D'(\delta) K(v) + u'(\delta)
\]

In the present case we have \( u(\delta) \equiv 0 \) and \( D'(\delta) \leq 0 \). Since \( G_v > 0 \), it follows that \( v_\delta < 0 \).

We finally include the algebra leading from (11) to (12). Making the substitutions as indicated in the paragraph following (11) we have

\[
G_v F_q \delta = -D'(\delta) K'(v) G_v v_q - D(\delta) [K''(v) G_v v_\delta v_q + K'(v) G_v v_q]
\]

\[
= -D'(\delta) K'(v) \left[ \frac{1}{\delta} (1 - \delta) C(q) - D(\delta) K'(v) \right] v_q
- D(\delta) K''(v) \left[ \frac{1}{\delta} (1 - \delta) C(q) - D(\delta) K'(v) \right] v_\delta v_q
- D(\delta) K'(v) \left( -\frac{1}{\delta^2} C'(q) - \frac{1}{\delta} (1 - \delta) C'(q) v_\delta \right) v_q - D(\delta) K''(v) v_\delta v_q
\]

\[
= K'(v) \left( -D'(\delta) \left[ \frac{1}{\delta} (1 - \delta) C(q) - D(\delta) K'(v) \right] + D(\delta) \left( \frac{1}{\delta^2} C(q) - D'(\delta) K'(v) \right) \right) v_q
- D(\delta) K''(v) \left[ \frac{1}{\delta} (1 - \delta) C(q) - D(\delta) K'(v) \right] v_\delta v_q
+ D(\delta) K'(v) \left( \frac{1}{\delta^2} C'(q) + \frac{1}{\delta} (1 - \delta) C'(q) v_\delta \right)
\]

\[
= K'(v) \left( \left( -D'(\delta) \frac{1}{\delta} (1 - \delta) C(q) - (\frac{1}{\delta^2} D(\delta) K'(v)) + D(\delta) \frac{1}{\delta^2} C(q) - D'(\delta) K'(v) \right) v_q
- D(\delta) K''(v) \frac{1}{\delta} (1 - \delta) C(q) - K''(v) D(\delta) K'(v) + K'(v) D(\delta) K''(v) \right) v_\delta v_q
+ D(\delta) K'(v) \left( \frac{1}{\delta^2} C'(q) + \frac{1}{\delta} (1 - \delta) C'(q) v_\delta \right)
\]

\[
= K'(v) \left( \left( -D'(\delta) (1 - \delta) - D(\delta) \right) \frac{1}{\delta^2} C(q) + 0 \right) v_q
- D(\delta) K''(v) \frac{1}{\delta^2} (1 - \delta) C(q) v_\delta v_q
+ D(\delta) K'(v) \left( \frac{1}{\delta^2} C'(q) + \frac{1}{\delta} (1 - \delta) C'(q) v_\delta \right)
\]

From the definition of \( E(\delta) \) we see that (12) holds. This completes the proof.

**Proof of Proposition 3**
The proof is similar to the proof of Proposition 2. Since $q'(\kappa)$ has the same sign as $F_{q\kappa}$, consider

$$F_{q\kappa}(q, \delta, \kappa) = D(\delta) [-K'_\kappa(v)v_q - K''_\kappa(v)v_\kappa v_q - K'(v)v_{q\kappa}]$$

The partials of $v()$ can be obtained from the constraint. For later reference we consider also here a slightly more general constraint, including an additive term $u(\delta, \kappa)$ as follows:

$$G(q, v; \delta) = q - \frac{1}{\kappa}(1 - (1 - \delta)v)C(q) - D(\delta)K(v) - u(\delta, \kappa) = 0$$

The partials of $v()$ are given by

$$G_q(q, v; \delta, \kappa) + G_v(q, v; \delta, \kappa)v_q = 0$$

$$G_\kappa(q, v; \delta, \kappa) + G_v(q, v; \delta, \kappa)v_\kappa = 0$$

and

$$G_{q\kappa} + G_{qv\kappa} + [G_v] + G_{v\kappa}v_\kappa v_q + G_v v_{q\kappa} = 0$$

Computing the partials of $G()$ and substituting from the last relation into the expression for $F_{q\kappa}$ we obtain, after some algebra (the algebra is included at the end of this proof):

$$\frac{1}{D(\delta)} G_v F_{q\kappa} = -K'_\kappa(v)G_v v_q - K''_\kappa(v)G_v v_\kappa v_q - K'(v)G_v v_{q\kappa}$$

$$= (-K'_\kappa(v)C(q)v_q - K''_\kappa(v)C(q)v_\kappa v_q + K'(v)C'(q)v_\kappa) \frac{1 - \delta}{\delta}$$

Using the equilibrium relation $C(q)v_q = C'(q)(1 - v)$ as in the proof of Proposition 2 we see that the parenthesis in the last line of (13) can be written as

$$(-K'_\kappa(v)(1 - v) - K''_\kappa(v)(1 - v)v_\kappa + K'(v)v_\kappa) C'(q)$$

Since $G_v > 0$ it follows that $F_{q\kappa}$ and hence $q'(\kappa)$ has the same sign as the last expression.

This proves condition (8) in the proposition.

Next consider $v_\kappa$, which is given by

$$G_v v_\kappa = -G_\kappa = D(\delta)K_\kappa(v, \kappa) + u_\kappa(\delta, \kappa)$$

Since $u(\delta, \kappa) \equiv 0$ in the present case, we see that $v_\kappa$ has the same sign as $K_\kappa$.

Finally consider the algebra that verifies the last equality in (13). We have

$$G_v = \frac{1}{\delta}(1 - \delta)C(q) - D(\delta)K'(v, \kappa)$$

$$G_q = 1 - \frac{1}{\kappa}(1 - (1 - \delta)v)C'(q)$$

$$G_\kappa = 0$$

$$G_{qv} = \frac{1}{\delta}(1 - \delta)C'(q)$$

$$G_{vn} = -D(\delta)K'_\kappa(v, \kappa)$$

and thus
\[ G_v v_{qv} = -G_q v - G_q v_{\kappa} - [G_v v_{\kappa} + G_{vv} v_{\kappa}] v_q \]
\[ = 0 - \frac{1}{\delta}(1 - \delta)C'(q) v_{\kappa} - D(\delta) [-K'_\kappa - K'' v_{\kappa}] v_q \]

Substituting into the first line of (13) we have

\[ \frac{1}{D(\delta)} G_v F_{qk} = -K'_k(v) G_v v_q - K''(v) G_{v\kappa} v_q - K'(v) G_v v_{q\kappa} \]
\[ = -K'_k(v) \left[ \frac{1}{\delta}(1 - \delta)C(q) - D(\delta)K'_k \right] v_q \]
\[ -K''(v) \left[ \frac{1}{\delta}(1 - \delta)C(q) - D(\delta) [-K'_\kappa - K'' v_{\kappa}] \right] v_q \]
\[ = K'_k \left( - \frac{1}{\delta}(1 - \delta)C(q) - D(\delta)K'_k \right) v_q - D(\delta)K'_k v_q \]
\[ + (-K'' \left[ \frac{1}{\delta}(1 - \delta)C(q) - D(\delta)K'_k \right] - D(\delta)K'K'') v_{\kappa} v_q \]
\[ -K'(v) \left( - \frac{1}{\delta}(1 - \delta)C'(q) v_{\kappa} \right) \]
\[ = \frac{1}{\delta}(1 - \delta)C(q)v_q \]
\[ -K'' \frac{1}{\delta}(1 - \delta)C'(q) v_{\kappa} v_q \]
\[ + K'(v) \frac{1}{\delta}(1 - \delta)C'(q) v_{\kappa} \]

This accords with the last line of (13) and hence completes the proof.

References


Klein, Benjamin. 1996. Why hold-ups occur: The self-enforcing range of contractual relationships. Economic Inquiry 34: 444-463


