Market Shares in Two-Sided Media Industries

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Market Shares in Two-Sided Media Industries

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Abstract

This paper generalizes the frequently used Hotelling model for two-sided markets in order to determine the equilibrium market shares. We show that independent of whether consumers are uniformly or non-uniformly distributed, advertisement levels neither depend on the media price nor on the location of the media firm. An increase in advertising revenues does not change location but only the media price. However, we show that if the distribution is asymmetric, market shares will be asymmetric as well, and that the media firm with the larger market share has the higher media price. Thus, even in absence of any fixed costs, this firm makes a higher profit per reader and in aggregate than its smaller rival.

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1 Introduction

Recent years have seen a huge increase in the literature on two-sided markets (e.g., Armstrong, 2006, and Rochet and Tirole, 2003, 2006). The media industry is one of the most important examples of two-sided markets, and many papers have used Hotelling-inspired models to analyze media firms’ location, price setting on consumer markets and sales of advertising space.\(^1\) However, most of the papers make very specific assumptions about competition for advertising and about consumer heterogeneity. In particular, it is typically assumed that consumers are uniformly distributed along the Hotelling line. This tends to oversimplify location decisions, characteristically resulting in maximum or minimum differentiation, depending on the set-up of the model.

This paper tries to make progress on our understanding of media firms’ location decisions and strategic behavior on the consumer and advertising market by relaxing the assumption that consumers are uniformly distributed. Furthermore, we do not make any specific assumption about the type of competition in the advertisement market. Media firms can compete by prices or by ad space, and we allow for both single-homing and multi-homing.

Within this set-up we show that a non-uniform distribution of consumers implies that the media firms will end up with asymmetric market shares but with the same level of advertising revenue per consumer. We further show that the firm with the smaller market share finds it unprofitable to exercise its market power in the smaller segment by charging higher prices. On the contrary, its equilibrium price will be lower than that of its larger rival. The smaller firm will therefore unambiguously be less profitable than the larger one, measured both in terms of revenue per consumer and in aggregate.

2 The model

We employ a Hotelling model with two competing media firms, \(i = 1, 2\). Media firm \(i\) charges price \(p_i\) and is located at \(x_i\). Without loss of gener-

ality, we assume that \( x_2 \geq x_1 \). The media firms also sell advertising space to producers, and the resulting advertising level is given by \( a_i \). The media consumers may have negative or positive attitudes towards ads, and the net utility level of a consumer located at \( x \) who buys media product \( i \) is given by \( U = v - p_i - t(x - x_i)^2 - d(a_i) \). With this specification the consumers perceive ads as a bad if \( d(a_i) < 0 \) and as a good if \( d(a_i) > 0 \). The constant \( v > 0 \) is assumed to be sufficiently large to ensure market coverage.

Denoting the consumer who is indifferent between buying media product 1 and 2 by \( \tilde{x} \), we find

\[
\tilde{x} = \frac{1}{2} \left( x_1 + x_2 + \frac{p_2 - p_1 + d(a_2) - d(a_1)}{t(x_2 - x_1)} \right),
\]

(1)

Consumers located to the left of \( \tilde{x} \) buy media product 1, while consumers to the right of \( \tilde{x} \) buy media product 2.

The consumers are continuously distributed on \(-\infty \leq a < b \leq \infty\), and the cumulative distribution is denoted by \( F(x) \). We normalize the population size to one, and the density function \( f(x) = F'(x) \) is assumed to be log-concave on \([a, b]\) and twice differentiable. The marginal costs of producing the media product equal \( c \), and for simplicity we set marginal costs of inserting ads to zero, so that the profit functions of the two media firms read as

\[
\Pi_1 = F(\tilde{x})(p_1 - c + A_1(\cdot)),
\]

(2)

\[
\Pi_2 = (1 - F(\tilde{x}))(p_2 - c + A_2(\cdot)),
\]

where \( A_i \) is advertising revenue per consumer. As usual in the literature, aggregate advertising revenues depend linearly on the number of consumers. Otherwise, the model is very general. We allow both single-homing and multi-homing for the advertisers, and assume that ad revenues per consumer depend on the strategic variables \( s_1 \) and \( s_2 \), such that \( A_i = A_i(s_1, s_2) \). Advertisement levels are a function of these strategic variables, such that \( a_i = a_i(s_1, s_2) \). In a simple Cournot setting we have \( s_i = s_i \). But the model

\footnote{See Depken II and Wilson (2004) and Sonnac (2000) for a discussion of whether magazine/newspaper readers consider advertising as a good or a bad.}
also allows for price competition on the ad-market, i.e. it can accommodate competition in strategic substitutes as well as strategic complements.

In the following we consider a two-stage game, where the media firms choose locations before they simultaneously compete for consumers and advertising revenue (setting $p_i$ and $s_i$, respectively). We assume that the profit functions (2) are quasi-concave in $p_i$ and $s_i$, and that solutions are interior. Thereby, we can use the first-order conditions to determine optimal prices and advertising strategies.

As for prices we find that

$$\frac{\partial \Pi_1}{\partial p_1} = F(\bar{x}) + (p_1 - c + A_1)f(\bar{x}) \frac{\partial \bar{x}}{\partial p_1} = 0,$$

$$\frac{\partial \Pi_2}{\partial p_2} = [1 - F(\bar{x})] + (p_2 - c + A_2)f(\bar{x}) \left( -\frac{\partial \bar{x}}{\partial p_2} \right) = 0,$$

and it is straightforward to verify that consumer prices are strategic complements (as is typically the case in Hotelling models).

From equation (1), we derive

$$\frac{\partial \bar{x}}{\partial p_1} = -\frac{1}{2t(x_2 - x_1)} \text{ and } \frac{\partial \bar{x}}{\partial p_2} = \frac{1}{2t(x_2 - x_1)}.$$

The first-order conditions for advertisement strategies are given by

$$\frac{\partial \Pi_1}{\partial s_1} = F(\bar{x}) \frac{\partial A_1}{\partial s_1} + (p_1 - c + A_1)f(\bar{x}) \left[ \frac{\partial \bar{x}}{\partial a_1} \frac{\partial a_1}{\partial s_1} + \frac{\partial \bar{x}}{\partial a_2} \frac{\partial a_2}{\partial s_1} \right] = 0,$$

$$\frac{\partial \Pi_2}{\partial s_2} = [1 - F(\bar{x})] \frac{\partial A_2}{\partial s_2} - (p_2 - c + A_2)f(\bar{x}) \left[ \frac{\partial \bar{x}}{\partial a_2} \frac{\partial a_2}{\partial s_2} + \frac{\partial \bar{x}}{\partial a_1} \frac{\partial a_1}{\partial s_2} \right] = 0.$$

There are strategic interactions between the media firms in the advertising market if the last term in the square brackets of equation (5) is different from zero ($\frac{\partial \bar{x}}{\partial a_j} \frac{\partial a_i}{\partial s_i} \neq 0, i \neq j$). However, we do not have to specify whether the firms compete in strategic complements or strategic substitutes on this side of the market.

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3We have $\frac{\partial a_j}{\partial s_i} = 0$ if the media firms are monopolists in their respective ad markets.
We can now show:

**Lemma 1** Advertisement levels depend only on the marginal disutility of adverts and not on the media price, the location of the media firms or the size of the market.

Proof: See Appendix.

Lemma 1 is closely related to the Anderson and Coate (2005) result. They show that only the ad revenue functions and the (dis-)utility of ads determine equilibrium ad levels per consumer in Hotelling models with uniform distributions. Lemma 1 generalizes this result to arbitrary consumer distributions.

Let the common equilibrium advertisement revenue per media consumer be denoted by $\hat{A}$. Using (3) and (4), we have

\[
p_1 = 2t(x_2 - x_1) \frac{F(\bar{x})}{f(\bar{x})} + c - \hat{A}, \tag{6}
\]

\[
p_2 = 2t(x_2 - x_1) \frac{1 - F(\bar{x})}{f(\bar{x})} + c - \hat{A}.
\]

The difference in the media prices is thus given by

\[
p_2 - p_1 = 2t(x_2 - x_1) \frac{1 - 2F(\bar{x})}{f(\bar{x})}. \tag{7}
\]

The important message from equation (7) is that the media firm with the larger market share charges the higher price; $p_2 > p_1$ if $F(\bar{x}) < 1/2$ and vice versa. This is true even though there are no network effects or other factors which make one firm dominate its rival. The intuition for this result can be seen from equation (3); the first term shows that the gain for each media firm of setting a higher price is proportional to its market share. However, since $A_1 = A_2 = \hat{A}$ both firms face *inter alia* the same reduction in ad sales if they increase the price. Thus, the firm with the larger market share unambiguously benefits most from setting a high price. Not surprisingly, the dominant firm’s ability to set a higher price than its rival is increasing in the differentiation between the media firms, $(x_2 - x_1)$, and in the consumers’ transportation costs, $t$.  

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As in Anderson *et al* (1997) we can now write profits as a function of locations only:

\[
\hat{\Pi}_1 = 2t(x_2 - x_1) \frac{F(\bar{x})^2}{f(\bar{x})},
\]

\[
\hat{\Pi}_2 = 2t(x_2 - x_1) \frac{(1 - F(\bar{x}))^2}{f(\bar{x})}.
\]

Let \( y \) denote the median consumer such that \( F(y) = 0.5 \). We are now able to demonstrate

**Proposition 1** If profit functions (8) are quasi-concave, firm 1 has a higher market share than firm 2 if \( f'(y) < 0 \), and a smaller market share if \( f'(y) > 0 \).

Proof: We can write the location as an implicit function (see (1)):

\[
g(\bar{x}) = \frac{x_1 + x_2}{2} + \frac{1 - 2F(\bar{x})}{f(\bar{x})} = 0
\]

because \( a_1 = a_2 \) and thus \( d(a_2) - d(a_1) = 0 \). Partial differentiation yields

\[
g_{\bar{x}} = -\frac{3f^2 + f'(1 - 2F)}{f^2}, g_{x_1} = g_{x_2} = \frac{1}{2} \Rightarrow \frac{\partial \bar{x}}{\partial x_1} = \frac{\partial \bar{x}}{\partial x_2} = \frac{f^2}{6f^2 + 2f'(1 - 2F)}.
\]

Marginal profits with respect to locations can consequently be written as:

\[
\frac{\partial \hat{\Pi}_1}{\partial x_1} = -\frac{2tf^2}{f} + \frac{\partial x}{\partial x_1} \frac{2t(x_2 - x_1)F(2f^2 - f'F)}{f^2},
\]

\[
\frac{\partial \hat{\Pi}_2}{\partial x_2} = \frac{2t(1 - F)^2}{f} - \frac{\partial x}{\partial x_2} \frac{2t(x_2 - x_1)(1 - F)(2f^2 + f'(1 - F))}{f^2}.
\]

Logconcavity of \( f(x) \) implies \( \partial \bar{x}/\partial x_1 = \partial \bar{x}/\partial x_2 > 0 \) (see Anderson *et al* (1997), p. 107) and \( 2f^2 - f'F > 0, 2f^2 - f'(1 - F) > 0 \). An interior solution to (9) thus satisfies \( x_1^* > a \) and \( x_2^* < b \). Let us evaluate the marginal profits if both firms choose locations such that the median consumer is the indifferent consumer, i.e. if \( \bar{x} = y \). Define

\[
D \equiv 2t(x_2 - x_1) \frac{\partial \bar{x}}{\partial x_1} > 0, \Phi \equiv -\frac{t}{2f(y)} + D.
\]

\footnotetext{4}{For uniqueness and existence in the location game, see Assumptions 1 and 2 in Anderson *et al* (1997).}
Since $\partial \bar{x}/\partial x_1 = \partial \bar{x}/\partial x_2$, marginal profits for $\bar{x} = y$ are equal to

$$\begin{align*}
\frac{\partial \hat{\Pi}_1}{\partial x_1}(\bar{x} = y) &= \Phi - \frac{f'(y)D}{2f^2}, \\
\frac{\partial \hat{\Pi}_2}{\partial x_2}(\bar{x} = y) &= -\Phi - \frac{f'(y)D}{2f^2}.
\end{align*}$$

Suppose that firm 1 has chosen $x_1$ such that its profits are maximized and firm 2 has set $x_2$ such that $\bar{x} = y$ holds. From (10), it follows

$$\frac{\partial \hat{\Pi}_1}{\partial x_1}(\bar{x} = y) = 0 \Rightarrow \frac{\partial \hat{\Pi}_2}{\partial x_2}(\bar{x} = y) = -\frac{f'(y)D}{f^2}.$$ 

Hence, firm 2’s marginal profits are positive if $f'(y) < 0$, and negative if $f'(y) > 0$. Consequently, firm 2 will increase $x_2$ if $f'(y) < 0$, thereby increasing firm 1’s market share, and vice versa. □

Proposition 1 shows that asymmetric distributions lead to asymmetric market sizes. Without loss of generality we have assumed that firm 2 is located (weakly) to the right of firm 1. It thus follows that firm 1 will have a larger market share than firm 2 if and only if $f'(y)$ is negative. The reason is that the location decision affects the behavior of the marginal consumer only. If $f'(y)$ is negative, the distribution is skewed at the median consumer such that firm 2 gains by moving to the right of $F(y) = 0.5$, as illustrated in Figure 1.

![Figure 1](image-url)

**Figure 1:** Firm 2 locates to the right of $F(y) = 0.5$ if $f'(y) < 0$. 7
Note carefully that the market share result holds both for the media market and for the ad market. Since ad revenue per consumer is the same across firms, the media firm with the larger market share ends up with higher mark-ups in the media market and higher total ad revenue. In this sense the two-sidedness of the market tends to favor firms with large market shares, even though there are no economies of scale nor any network effects.

3 Concluding remarks

Our paper has demonstrated that a generalized Hotelling model of two-sided markets behaves like a standard Hotelling model in which ad revenues just reduce marginal production costs. More importantly, we have demonstrated that market shares differ if the distribution of consumers is asymmetric, with the dominant firm charging the higher price. In particular, our model may explain why market shares and profits differ in two-sided media markets even if production costs do not.

4 Appendix

By inserting for \((p - c + A_i)f(\tilde{x})\) from (3) into (5) we have

\[
\frac{\partial \Pi_1}{\partial s_1} = F(\tilde{x}) \left[ \frac{\partial A_1}{\partial s_1} - \left( \frac{\partial \tilde{x}}{\partial p_1} \right)^{-1} \left( \frac{\partial \tilde{x}}{\partial a_1} \frac{\partial a_1}{\partial s_1} + \frac{\partial \tilde{x}}{\partial a_2} \frac{\partial a_2}{\partial s_1} \right) \right],
\]

\[
\frac{\partial \Pi_2}{\partial s_2} = [1 - F(\tilde{x})] \left[ \frac{\partial A_2}{\partial s_2} + \left( \frac{\partial \tilde{x}}{\partial p_2} \right)^{-1} \left( \frac{\partial \tilde{x}}{\partial a_2} \frac{\partial a_2}{\partial s_2} + \frac{\partial \tilde{x}}{\partial a_1} \frac{\partial a_1}{\partial s_2} \right) \right].
\]

Equations (1) and (4) further yield (for \(i \neq j\))

\[
\frac{\partial \tilde{x}}{\partial a_i} \frac{\partial a_j}{\partial s_i} + \frac{\partial \tilde{x}}{\partial a_j} \frac{\partial a_i}{\partial s_i} = \frac{\partial \tilde{x}}{\partial p_i} \left( d'(a_i) \frac{\partial a_i}{\partial s_i} - d'(a_j) \frac{\partial a_j}{\partial s_i} \right).
\]

In equilibrium, \(\partial \Pi_1/\partial s_1 = \partial \Pi_2/\partial s_2 = 0\). Equations (11) and (12) thus imply

\[
\frac{\partial A_1}{\partial s_1} - d'(a_1) \frac{\partial a_1}{\partial s_1} + d'(a_2) \frac{\partial a_2}{\partial s_1} = 0,
\]

\[
\frac{\partial A_2}{\partial s_2} - d'(a_2) \frac{\partial a_2}{\partial s_2} + d'(a_1) \frac{\partial a_1}{\partial s_2} = 0.
\]
Expression (13) implicitly determines the advertising level as a function of the marginal disutility of ads and the ad revenue function. Even though the media firm with the larger market share has the higher total revenue from ads, the ad revenue per consumer is thus independent of the market size and the media price.

References


