Social Security and Future Generations

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This series consists of papers with limited circulation, intended to stimulate discussion.
Abstract

We survey the effects of social security in the form of mandatory public pension programs on the intergenerational distribution of tax burdens, income, various risks and welfare. The first part considers basic theoretical concepts and highlight how the intergenerational effects of social security hinge on three types of major mechanisms: i) Defined benefit vs. defined contribution, ii) pay-as-you-go financing vs. funding and iii) the strength of the tax-benefit link of the program. The second part of the survey considers advances in the large literature that offer quantitative assessments of social security programs and reform proposals by means of numerical overlapping generations models. In both parts of the survey we distinguish between deterministic models and models that incorporate various stochastic elements.
1. Introduction

Design and reform of social security in the form of public pension programs have a prominent place on the policy agenda in more or less all OECD economies as well as in many emerging markets. According to the OECD, nearly all of their 30 member countries have implemented at least some changes to their pension programs since 1990 and 17 have had major reforms (OECD, 2007). These widespread reforms were the logical, and probably unavoidable, consequence of the general developments. The public pension programs, which essentially are financed on a pay-as-you-go (paygo) basis, matured over several decades and in most cases the generosity of the programs increased along several dimensions. It is probably fair to say that this development in many cases culminated with the implementation of quite liberal early retirement programs during the 1970s and 1980s. Then, during the last two or three decades, a growing awareness of how the financial viability of the programs is threatened by (mainly) ageing populations, ignited the current wave of reforms. Given the widely perceived seriousness of the financial problems and the intensity of the ongoing debate, a series of additional reforms in the years to come is a safe prediction.

Assessments of public pension programs must, as in the case of any other tax-transfer program, consider the trade-off between the gains caused by intended distributional effects associated with protection and income maintenance for the old and, on the other hand, the costs due to the induced distortions. These trade-offs tend to be quite complex, reflecting that any program or reform proposal have both inter- and intragenerational distributional effects and distortions that must be assessed by means of an intertemporal framework. At the current stage, the typical OECD economy recognizes in particular the need for reforms in order to prevent sharply escalating tax burdens for the young and future generations. This focus is reflected in the following. We consider the effects of pension reforms on the intergenerational distribution of welfare and tax burdens - and we highlight the assessments of such effects from reforms that intend to scale down unfunded pension programs in order to improve their financial viability.

In order to illustrate the severity of the expected ageing, it is useful to look at some data from the OECD economies. Table 1.1 presents the dependency ratios defined as the ratio between the number of people above the age of 64 to the number of people aged 15-64. The pattern is striking and obviously critical for the financing of paygo financed pension programs. In the average OECD economy each individual above the age of 64 was in year 2000 supported by approximately 4 individuals in the regular working age. In 2045 this number is expected to drop to approximately 2.

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1 The existence of welfare gains of a public pension system is contingent on underlying motives like i) an explicit desire for income redistribution, ii) an ambition to correct market failures or iii) paternalism, see Diamond (1977) for a discussion.

2 This focus is well known from any OECD economy and also several emerging market economies like several Latin American countries. It does not apply, however, to some other emerging market economies without significant social security programs (like, for example, China).
This development reflects a combination of strongly increased longevity and (particularly in some countries) low fertility rates.\(^3\)

In addition, the financial burden of social security also escalates as a consequence of more early retirement over time, see Table 1.2.\(^4\) Many OECD countries also experience an increasing number of exits from the labor force and into disability programs or equivalent schemes. These developments imply that the median real retirement age has been on a falling trend during the last decades.\(^5\) Accordingly, the tax base for social security contribution rates deteriorates even more than suggested by the drop in dependency ratios.

*** Table 1.1, dependency ratios ***

*** Table 1.2, early retirement ***

Describing and comparing various public pension programs and associated reform packages, we find it useful to have in mind a three dimensional classification inspired by Lindbeck and Persson (2005). The first dimension refers to whether contribution rates or benefit rates adjust in response to fluctuations in variables that determine the financial opportunity set of the pension program. Theory focuses on the cases of pure defined benefit (DB) or defined contribution (DC) programs. While the benefit level is exogenously given and the contribution rate adjusts accordingly in a DB program, the opposite characterises a DC program. In reality many programs are a mixture of these strict alternatives, see Wagener (2004). As discussed in more detail below, the DB vs. DC dimension is crucial for the risk sharing properties of the pension programs.

The second dimension relates to the degree of funding, ranging from a pure paygo program (zero funding) to a fully funded program. Paygo funding means that pensions to the old are financed by contributions from the working generation period by period. Funding, on the other hand, implies that pensions to the old are financed by previously accumulated contributions (by the same generation while participating in the labor market) plus interests.

The third dimension captures individual incentives in the sense of the strength of the tax-benefit link of the actual program, ranging from a completely flat benefit program to a perfectly actuarial program. We note that this dimension, in terms of the definitions offered by Lindbeck and Persson, focuses on the “degree of actuarial fairness” facing each individual and influencing their

\(^3\) According to the UN, life expectancy is expected to rise approximately 0.2 years per year over the next 5 decades, see UN (2004). The fertility rates vary a lot between countries, from a reasonably high 1.8 – 2.0 child per female in countries like Sweden, Norway and the US to a worrying low 1.3 level in countries like Germany and Italy.

\(^4\) To some extent the escalation of early retirement may reflect an income effect in the sense that economic growth over time increases the demand for more leisure spent in retirement. Still, it seems fair to conclude that available evidence shows that the incentive effects of generous early retirement program play a major role in most economies see for example Gruber and Wise (1999) and Fenge and Pestieau (2005).

\(^5\) See for example Fenge and Pestiau (2005) for the trend in the real retirement age for Germany.
labor supply decisions, and not “the actuarial balance” of the program which characterises the aggregate financial viability of the program.  

Figure 1.1 clarifies the relationship between the two latter dimensions. As shown by the solid arrow, many OECD economies with (almost) pure paygo programs experienced a weakening of the tax-benefit link during the 1970s and 1980s due to different types of reforms including implementation of early retirement programs.

The dotted arrows refer to three possible reform strategies. Reform A can be interpreted as basically a fiscal policy strategy. In this case the reform does not alter the rules and incentives of the pension programs facing the individuals, but the government accumulates more capital (in, say, a buffer fund), for example motivated by an intention to counteract potential increases in future contribution rates. Reform B, on the other hand, maintains the pure paygo financing, but introduces improved incentives towards higher labor supply and/or a longer working career. Finally, reform C involves both more funding and an improved tax-benefit link. The typical clear cut example of such a reform is the introduction of real, individual (privatized) accounts as a part of the mandatory pension program. Implementation of this type of reform is well known from for example Chile and Sweden, see respectively Mesa and Mesa-Lago (2006) and Persson (2000).

*** Figure 1.1, dimensions of pension program ***

The next section explains the main theoretical principles related to the study of intergenerational effects of pension programs and reforms. Emphasis is devoted to the clarification of the different mechanisms that is underlying the three dimensions of pension reform that is presented above. First we consider basic key insights based in a deterministic framework. Then we move on to stochastic economies, which have received much attention recently. In section 3 we turn to numerical assessments based on simulation models that feature several extensions necessary to provide realistic magnitudes - but still build on the same essential mechanisms as the theoretical models. Recent advances in the rich literature dealing with large-scale deterministic models in the tradition of Auerbach and Kotlikoff (1987) are given a broad treatment. Finally, the smaller, but quickly growing literature on stochastic numerical models is given ample attention. Section 4 concludes.

At the outset, we will mention that it is obviously not possible to cover all parts of the enormous literature on intergenerational effects of social security. We will particularly point at the fact

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6 The actuarial balance issue is captured by the discussion of our second dimension, paygo vs. funding.
7 As an example, fiscal policy in the oil-rich Norwegian economy during the last decade can be given such an interpretation. Disciplined fiscal policy has led to an accumulation of a fund that exceeds 100 per cent of GDP. This fund is officially referred to as “the State Pension Fund” despite the lack of any formalized link between the size of the fund and future pension liabilities.
8 As discussed and exemplified by Thøgersen (2001), the term funding is often used in an imprecise manner in the social security literature, i.e. it sometimes refers to reforms of type A in figure 1.1 and sometimes to reforms of type C. In the literature related to the US policy debate, the terms funding and privatization are often used interchangeably and indicate an introduction of real, individual pension accounts (i.e. a type C reform).
that the highly interesting literature on political aspects of social security reform is not covered, the
reason being that such a topic warrants a separate article. Moreover, we will only very briefly refer to
the econometric literature on labor supply and retirement responses to tax-transfer policies.

2. Theoretical principles

Attempting to explain theoretical principles in a clear and transparent way, this section relies on a
stylized overlapping generations framework where two generations are present in any period and the
only objective of the government is to run the pension program. The young generation participates in
the labor market, while the old generation is retired. The representative member of the young
generation in period $t$ (i.e. generation $t$) supplies $l_t$ units of labor and receives a net wage equal to
$w_t(1 - \tau_t)$ per unit. Here $w_t$ is the gross wage, and $\tau_t$ is the contribution rate of the pension program.

In period $t + 1$, the representative generation $t$ individual is old and receives a pension benefit given by
$p_{t+1}$. Introducing $\theta_{t+1}$ as the replacement rate of the pension program, we have that $p_{t+1} = \theta_{t+1}w_t l_t$.
The size of generation $t$ is given by $N_t$ and $n_{t+1}$ is population growth from period $t$ to $t + 1$, i.e.

$N_{t+1} = (1 + n_{t+1})N_t$. Wage growth reflects productivity growth in our long run setting and is given by
$\lambda_t$, $w_{t+1} = (1 + \lambda_{t+1})w_t$.

The basic accounting identities of the pension programs reveal the key variables that
determine their attractiveness and effects. A paygo program transfers resources between generations in
the sense that total contributions from the young generation must per definition be equal to total
pension benefits to the old. Thus,

$N_{t+1} \tau_{t+1} w_{t+1} l_{t+1} = N_t \theta_{t+1} w_t l_t$.

Substituting for $N_{t+1}$ and $w_{t+1}$, we obtain

$\theta_{t+1} = (1 + g_{t+1}) \tau_{t+1}$, \hspace{1cm} (1 + g_{t+1}) = (1 + \lambda_{t+1})(1 + n_{t+1}) \frac{L_{t+1}}{L_t}$,

where $g_{t+1}$ is the implicit return of the paygo program. It follows that this return is given by the
growth of the aggregate wage sum which in turn is determined by three key factors: $n_{t+1}$, $\lambda_{t+1}$ and the
developments in labor supply.

In the case of a funded program, resources are transferred over the lifetime of each generation
and the representative individual receives a pension which is given by own contributions plus interests
earned from the fund.\(^{10}\) We have that 
\[ p_{t+1} = (1 + r_{t+1})\tau_t w_t l_t = \theta_{t+1} w_t l_t, \]
where \(r_{t+1}\) is the relevant real interest rate. It follows that
\[ \theta_{t+1} = (1 + r_{t+1})\tau_t. \]

We start out with a deterministic framework. Then the implicit return of the paygo program as well as the market return of the funded program are known for all decision makers and individuals at all points of time, and the distinction between DB and DC programs do not matter. This allows us to focus exclusively on the tax-benefit link and the choice (or mixture) between funding and paygo financing. Turning subsequently to a stochastic environment where the key variables \(\lambda_{t}, n_{t}\) and \(r_{t}\) are subject to shocks, we consider the intergenerational redistribution of risks by means of pension programs and how this depends crucially on the DB vs. DC dimension.

2.1. Basic insights - deterministic framework

Effects of funding vs. paygo
Consider a completely deterministic model where \(n_{t} = n\) and \(\lambda_{t} = \lambda\) for all \(t\). To obtain the pure intergenerational effects of paygo financing versus funding of the pension program, we assume that the labor supply of any young generation is inelastic and equal to unity, i.e. \(l_{t} = 1\). These assumptions imply that \(g_{t} = g\). The welfare of generation \(t\) is given by a standard utility function \(U_{t} = u(c_{1,t}, c_{2,t+1})\), where \(c_{1,t}\) is consumption in the first period of life (which takes place in period \(t\) for generation \(t\)) and \(c_{2,t+1}\) is consumption in the second period of life. Both \(c_{1,t}\) and \(c_{2,t+1}\) are normal goods, which implies that the sign of the welfare effects are given by the effects on the net lifetime income, \(b_{t}\),
\[ b_{t} = (1 - \tau_{t}) w_{t} + \frac{1}{1 + r_{t+1}} \theta_{t+1} w_{t}. \]

Clearly, the case of no pension program is characterized by \(\tau_{t} = \theta_{t} = 0\) and implies \(b_{t} = w_{t}\).

When a paygo program is implemented, say in period \(t = 1\), the initial old generation (generation 0) receives a windfall in the form of a pension benefit from the succeeding generation. This, of course, leads to an increase in this generation’s consumption and welfare. All succeeding generations face both contributions while young and benefits while old. Assuming \(\tau_{t} = \tau\) and \(\theta_{t} = \theta\) for \(t = 1, 2, 3, \ldots\), it follows from (2), (4) and the assumptions above that

\(^{10}\) To what extent this holds for all individuals within each generation depends on the tax-benefit link of the program, see Figure 1.1. If, for example, the program is characterized by a weak tax-benefit link, there is an element of intragenerational redistribution. On the other hand, if the program is fully actuarial, (3) holds for all individuals.
In the small open economy case factor prices are given and the net effect on $b_t$ is given by the last term on the RHS. The sign of this effect, and in turn the sign of the welfare effect, hinges on the sign of $r_{t+1} - g$. The latter sign is strictly positive in the empirically interesting case of dynamic efficiency, implying that the paygo program reduces $b_t$ and welfare for all generations except the initial old generation. This reflects that the paygo program reduces the need for private saving, which in a small open economy leads to a deterioration of the net foreign asset position and correspondingly a lower steady state net import volume.

If general equilibrium effects are taken into account (i.e. the closed economy case), it follows that the reduction in private saving reduces the capital intensity of the economy and implies a lower wage level and higher real interest rates. Given dynamically efficiency, the paygo program pushes the economy further away from a golden rule growth path and welfare is reduced, see Samuelson (1975) for the original analysis or Heijdra and Ploeg (2002, ch. 17) for a textbook treatment.

It follows that the effects of a paygo program are essentially equivalent to the effects of public debt, see Diamond (1965) for the seminal analysis in a closed economy and Persson (1985) for open economies. This is highlighted by Auerbach and Kotlikoff (1987, p. 149-150) who show how the implementation of a paygo program can be modeled as a type of debt policy where the future pension benefits to the young generation are an implicit social security debt.

In contrast to a paygo program, a funded pension program has no real effects in this framework. Substituting (3) into (4), we immediately see that $b_t = w_t$, just as in the case of no pension program. This reflects that the funded program is a perfect substitute for private saving - as long as we assume that the credit market is perfect and all agents in the economy face the same interest rate. The capital accumulation in the pension fund will exactly offset lower ordinary private saving and national saving will not be affected.

**Transition from paygo to fund**

Ageing and early retirement have led to a decline in the implicit return of the paygo programs and therefore a widespread ambition to implement transitions to (more) funded programs. A direct and immediate transition is obviously hard to imagine, because this implies that the old generation in the transition period would not receive any pension - or alternatively that the young generation in this period have to pay both the pension benefits to the old as well as its own contribution to the new pension fund. Neither of these brutal options seems feasible. The issue is therefore whether it is

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11 The importance of the sign of this expression for the effects of pension programs was first highlighted by Aaron (1966). In social security settings, the condition $r_{t+1} - g > 0$, i.e. dynamic efficiency, is often referred to as the “Aaron condition”.

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possible to implement a transition accompanied by transfers between generations in a way that leads to a
directly improving outcome. As shown by Breyer (1989) such a combined transition and transfer
scheme does not exist. This result is intuitive when we note that the transition does not lead to any
aggregate income gain in the sense that income loss of the old generation in the transition period that
does not receive its expected pension, is equal to the capital value of all future generations’ income
gain, see Lindbeck and Persson (2003) and Sinn (2000).

The conclusion that a directly improving transition is not feasible, relies on a framework that
disregard distortions associated with the pension program and related behavioral responses, i.e. we
have considered transitions of the “reform A” type in Figure 1.1. This conclusion is altered if the
reform package consists of both a switch from paygo to fund and a tax reform that reduces the
distortions caused by the contribution rate. In order to capture the latter effect, the model framework
must be extended somewhat, typically by introducing endogenous labor supply in the first period of
life. In this case, the utility of generation \( t \) is given by
\[
U_t = u(c_{1t}, c_{2,t+1}, l_t),
\]
and it is assumed that the

12 This result also appears in Verbon (1989) and Fenge (1995).
13 Pareto optimal transitions may also reflect the abolishment of other types of distortions associated with the
paygo program. This includes distortions in the labor market, see Demmel and Keuschnigg (1999) or Corneo and
Marquardt (2000), and in the capital market, see Befan et al. (1998).
transfer scheme to implement pareto optimal transitions. In section 3 we return to the interaction between intra- and intergenerational redistribution when we look at richer simulation models.

With a view to the current reform agenda in many countries, it is clear that policymakers rarely have the luxury to implement pareto optimal reforms. The real issue is normally to choose between alternatives which in different ways benefit some generations (and/or subgroups within each generation) and hurt others. Comparative assessments of the various alternatives must then be based on a social welfare function. Disregarding intragenerational considerations and assuming that the utility of the representative generation $t$ individual is additive separable, we can introduce the social welfare function

$$V_t = u(c_{2,t}) + \sum_{s=t}^{\infty} \frac{1}{(1 + \theta)^{s-t}} \left( u(c_{1,t}, I_t) + \delta u(c_{2,t+1}) \right),$$

which weight together the utility of the current and all future generations (the first term on the RHS captures the utility of the old generation in the initial period $t$). We note the distinction between the individual time preference rate, $\delta$, and the intergenerational social discount rate, $\theta$.

The crucial issue for the assessments of reforms which impact the intergenerational distribution, is obviously the specification of $\theta$. We might imagine upfront that $\theta \geq 0$, or we can alternatively adopt a “Benthamite” specification and weight utility by the size of each generation, i.e. $\frac{1 + \theta}{(1 + n)^{-1}}$. Then the sign of $\theta$ will be the opposite of $n$. Generally, the specification of $\theta$ is a major topic in the social choice literature, see for example Portney and Weyant (1999) for surveys of the major issues involved.

**Endogenous retirement**

The observed escalation of early retirement implies that the probably most important aspects of interaction between labor supply and the pension program are related to the retirement decisions of the individuals. The simplest way to capture this explicitly in theoretical models is simply to assume that $I_t = 1 - x_t$, where $I_t$ and $x_t$ are interpreted as the fractions of the first period of life that is spent in respectively the labor force and as a pensioner on presumably an early retirement scheme.

Alternatively, several papers specify the utility function as $U_t = u(c_{1,t}, c_{2,t+1}, \alpha_{t+1})$, where $\alpha_{t+1}$ is the fraction of generation $t$’s second period of life (i.e. of period $t+1$) that is spent in the labor force, see for example Hu (1979) and Andersen (2005).

As long as the contribution rate is not levied as a lump-sum tax or the tax-benefit link is fully actuarial, the pension program distorts the retirement decision. Most paygo financed pension programs and early retirement schemes have an ambiguous effect on labor supply because the income effect caused by the reduction in $b_t$ in the normal case of dynamic efficiency contribute to a higher retirement age, while the substitution effect has the opposite sign. Given available evidence, see for example Gruber and Wise (1999) and Fenge and Pestiau (2005), it seem clear that the substitution
effect dominates in the overwhelming majority of real world programs, reflecting in particular very strong incentives for early retirement for low income individuals.

Accepting the conclusion that the retirement age is lowered, individuals might choose to increase their life cycle savings in response to more years spent in retirement. This induced retirement effect on saving was first highlighted in a well known paper by Feldstein (1974). The existence of this effect raises the empirical question of whether, or to what extent, the first order effect of public pensions replacing private savings (for a given retirement age, see above), is counteracted.\footnote{The first order effect of public pensions replacing private saving, might also be counteracted by prudent behavior in the sense that individuals perceive promised future pension benefits as risky, due for example to fears about the financial viability of the pension programs.} Available evidence, including Feldstein’s own, indicates that this is clearly not the case, however. Calculations by Feldstein (1974, 1996), suggest that social security reduces private savings in the US by as much as 60 per cent.

2.2. Intergenerational risk sharing

Turning to intergenerational risk sharing issues, we assume in this subsection that one or more of the key variables $\lambda$, $n$, and $r$ are stochastic. In a DC program ($\tau = \tau$), fluctuations in these variables translate into fluctuations in the replacement rate, $\theta$, according to the budget constraints (2) and (3) for respectively paygo and funded programs. Correspondingly, in a DB paygo program $\theta = \theta$ and $\tau$ fluctuates. Focusing on risk sharing, we abstract from issues related to the potentially distorting nature of public pension schemes in most of the following theoretical analysis. Thus, unless otherwise is explicitly stated, we assume that the representative individual in each generation supplies inelastically one unit of labor in the first period of life.

The potential for improved intergenerational risk sharing by means of public pension programs rests mainly on capital market imperfections, most notably the non-marketable of human capital. Because a paygo program in effect is a government created asset that allows one generation to trade in the human capital returns of the next, such a program may serve to correct for incomplete financial markets if the correlation between the implicit return of the paygo program and other capital returns are less than perfect, see Merton (1983), Richter (1990), Persson (2002) and Matsen and Thøgersen (2004). An additional issue is related to the fact that large fractions of the households have zero or negligible stocks holdings in their portfolios, even in the US, see Poterba (2000). This might reflect factors like credit constraints (particularly early in life) and/or information asymmetries. Under such circumstances, a properly designed funded pension program might provide individuals with a welfare-improving stock market exposure.
Figure 2.1 is a useful tool for clarifying the various intergenerational risk sharing effects of a paygo program and how their relevance depends on the appropriate risk concept. The social security literature deals essentially with two alternative risk concepts, see Ball and Mankiw (2001), Wagener (2003) and Thøgersen (2006). One is the “rawlsian”, ex-ante perspective that considers individuals in a pre-birth position. The other is the “interim” perspective that considers individuals’ position contingent on realized wages in their first part of life.

At the outset the representative generation $t$ individual is exposed to a wage shock in period $t$ and a capital return shock in period $t+1$. The existence of a DC paygo program on the one hand transfer some of the wage shock to generation $t-1$, but on the other hand provides an exposure to the wage shock of the succeeding generation $t+1$. Thus, looking at Figure 2.1, this program pools generation $t$’s capital income risk ($r_{t+1}$) with the wage income risk of generation $t+1$ ($\lambda_{t+1}$) and population risk ($n_{t+1}$). This effect is relevant under both risk concepts. In addition, there is an intertemporal sharing of wage income and population risks, i.e. the pooling $\lambda_t$ with $\lambda_{t+1}$ and $n_{t+1}$.

This effect is only relevant under rawlsian risk sharing.

In a DB paygo program, the realization of $\lambda_t$ determines both the first period wage and the pension benefit. Consequently, the interim perspective on intergenerational risk sharing is not relevant here. In the rawlsian case, this version of the program implies a pooling of $\lambda_t$ and $n_t$, however.

**Intergenerational income risk sharing**

Early contributions to the intergenerational risk sharing effects of paygo programs are provided by Enders and Lapan (1982, 1993) and Gordon and Varian (1988). Utilizing overlapping generations models with somewhat different features, they demonstrate that paygo programs might provide welfare gains as a consequence of intergenerational income risk sharing corresponding to a pooling of $\lambda_t$ with $\lambda_{t+1}$ in the above framework. Thøgersen (1998) builds closely on Gordon and Varian’s analysis and shows how the risk sharing properties of the paygo program in the case of rawlsian risk sharing depends crucially on whether it is a DC or a DB program.

In the stylized framework of Thøgersen, the gross wage of generation $t$ is given by $w_t = w + \epsilon_t$ where $\epsilon_t$ is a mean zero shock with a variance equal to $\sigma^2$. Both population growth and the real interest rate are constant, $r_{t+1} = r$ and $n_{t+1} = n$. Introducing a pension program with a fixed contribution rate, $\tau_t = \tau$ and $0 < \tau < 1$, we derive – consistent with (2), (4) and the fact that

\[(1 + \lambda_t) = \frac{w + \epsilon_{t+1}}{w + \epsilon_t} \text{ in this specific set-up } \]  

\[(7) \quad b_t = w - \left(\frac{r - n}{1 + r}\right) \delta w + (1 - \tau)\epsilon_t + \frac{1 + n}{1 + r} \tau \epsilon_{t+1} . \]
Preferences are given by a mean-variance specification, i.e. $U_i = u(E(b_i)) - v(Var(b_i))$ where $u' > 0$ and $v' > 0$. The risk sharing properties of this program are obvious if we consider for a moment the case of $r = n$. Then $E(b_i) = w$, just as in the case of no program at all. The variance has been reduced, however. It follows from (7) that $Var(b_i) = (1 - \tau)^2 + \tau^2 \sigma^2 < \sigma^2$. Thus, $Var(b_i)$ is minimized for $\tau = \frac{1}{2}$, implying that $Var(b_i) = \frac{1}{2} \sigma^2$. In the case of $r > n$, (7) implies intuitively that the gains of a lower $Var(b_i)$ comes at the costs of a reduced $E(b_i)$.

Looking at a DB version of the paygo program, i.e. the case of $\theta = \theta$, $0 < \theta < 1$, the conclusions are altered dramatically. Using (2) and (4), we now obtain

$$b_i = w - \frac{r - n}{(1 + n)(1 + r)} \theta v + \left(1 + \frac{1}{1 + r} \theta\right) \varepsilon_t - \frac{1}{(1 + n)} \theta \varepsilon_{t-1},$$

and it follows that $Var(b_i) = \left[(1 + \frac{1}{1 + r} \theta)^2 + \left(\frac{1}{1 + r} \theta\right)^2\right] \sigma^2 > \sigma^2$. Thus, given the rawlsian perspective, this version of the paygo program increases the variance, while $E(b_i) \leq w$ in the relevant case of $r \geq n$.

A crucial issue related to the potential risk sharing effects of paygo programs is related to the specification of the long run wage process. As pointed out by Thøgersen (2006), a random walk assumption (i.e. permanent shocks) rather than the deterministic trend assumption in the example above implies that the favorable risk sharing effects of a DC paygo program vanishes. Intuitively, this reflects that the period $t$ wage shock is also fully reflected in the wage and DC paygo transfer in period $t + 1$. While available evidence on the stochastic properties of wage growth mainly sheds light on much more short run developments, it still seems fair to argue that evidence is tilted towards a specification with deterministic long run wage growth, see De Menil et al. (2006) and Thøgersen (2006).

Comparisons between DB and DC paygo programs are also provided by Wagener (2003, 2004). Adopting an interim risk sharing perspective, he shows that the conclusions are altered significantly as compared to rawlsian risk sharing. In the interim case the wage shock in the first period of life is not relevant. The key issue is whether the paygo program implies uncertain pension benefits while old. A DC program adds an exposure to the succeeding generation’s wage shock that translates into uncertain pension benefits. Intuitively, this is not the case for a DB version of the program, which therefore is favoured.

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15 The specification of $b_i$ as the argument in the utility function is equivalent to the assumption that consumption takes place in (only) the second period of life. This simplification is common in the literature on social security and intergenerational risk sharing, see for example Gordon and Varian (1988) and Shiller (1999). While this approach leads to analytical tractability, a problem related to this approach is the fact that precautionary saving issues are disregarded.
A portfolio choice approach

As discussed in detail by Person (2002) and also by Merton (1983), a paygo program can be interpreted as a “quasi-asset”. The optimal investment in this asset, as expressed by the optimal contribution rate, will depend on its stochastic properties, notably the expected value and variance of the implicit return, $g_t$, and its covariance with alternative financial assets. Given a below-unity correlation between the return on alternative financial assets and $g_t$, the paygo asset may well be a part of individuals’ optimal portfolio of pension savings.

While Person presents some calculations based on Swedish data that suggest that a paygo program should enter the optimal total portfolio of Swedish citizens, more formalized models are provided by Dutta et al. (2000), Matsen and Thøgersen (2004) and Thøgersen (2006). Dutta et al. (2000) consider intergenerational risk sharing based on what we have defined as interim risk sharing. Employing a mean-variance specification of preferences, they derive simple closed form solutions for the optimal paygo program, i.e. the optimal portfolio share of individuals’ gross wage income invested in the paygo-asset. A problem related to the interpretation of these solutions is the fact that they depend on the magnitude of the realized wage in the early part of life. The wage will change over time and the solutions are time-inconsistent in the sense that the optimal paygo program of generation $t$ is not optimal for generation $t + 1$.

Adopting a somewhat extended framework that combines an overlapping generations structure with a modelling of portfolio choice based on isoelastic preferences and loglinear approximations (see Campbell and Viceira, 2002), Matsen and Thøgersen (2004) and Thøgersen (2006) derive closed form solutions for the optimal DC paygo program under both interim and rawlsian risk sharing – and in the latter paper also under alternative assumptions for the stochastic properties of wage income growth. As compared to a mean-variance specification, isoelastic preferences yield optimal portfolio shares that are independent of the wage realization in the first period of life. In our setting, this implies that the optimal paygo program of generation $t$ is also optimal for succeeding generations.

Formally, the model assumes that consumption takes place in the second period, implying that the expected utility of generation $t$ is given by $U_t = E_t \left( \delta \frac{1}{1+r_t} c_{2,t+1}^{1+\gamma} \right)$, where $\gamma$ is the coefficient of relative risk aversion. Financial savings, which in this framework is the full net wage income in the first period of life, can be allocated to risk free bonds or risky stocks. Their respective returns are $r_f$ (which is constant) and $r_{t+1}$, $r_f < E_t(r_{t+1})$. It follows that

$$c_{2,t+1} = (1-\tau)w_t \left[ 1 + r_f + \omega(r_{t+1} - r_f) \right] + \theta_{t+1}w_t,$$

where $\omega$ is the portfolio share of net wage in stocks. Substituting for $\theta_{t+1}$, see (2) when $\tau$ is fixed and labor supply is exogenous and equal to unity, we can rewrite (9) as

$$c_{2,t+1} = w_t \left[ 1 + r_f + \tau(g_{t+1} - r_f) + (1-\tau)\omega(r_{t+1} - r_f) \right].$$
We observe that the effective portfolio shares in the paygo-asset, stocks and bonds are, respectively, \( \tau \), \((1 - \tau)\omega \) and \(1 - \tau - (1 - \tau)\omega \). Assuming perfect financial markets and no frictions, individuals choose the optimal \( \omega \) given \( \tau \), while the government derives the optimal \( \tau \) given knowledge about how \( \omega \) will depend on \( \tau \). An alternative approach is to consider the case where the representative individual does not participate in stock market. Then the government can optimize both \( \omega \) and \( \tau \), and accordingly run a mixed paygo and funded pension program.

Utilizing log-linear approximations along the lines of Campbell and Viceira (2002), it turns out that it is possible to derive very illuminating closed form solutions for the optimal \( \tau \) and \( \omega \). Intuitively, the key factors are a measure the trade-off between each asset’s mean return and variance – in addition to the covariance between these returns, \( \sigma_{rg} \). While straightforward general equilibrium models suggest that this covariance is high, reflecting a model prediction of perfect correlation between capital returns and economic growth, available numerical evidence gives another impression even for long time spans relevant for social security analyses. Particularly, when stock market returns are considered rather than broader measures of capital income, this correlation tend to be low for any small or medium sized economy and modest also for the US, see Matsen and Thøgersen (2004) for a discussion. Given this insight, the basic message is that a low-yielding paygo program can be rationalized.

**Factor price responses**

In closed economies, the risk sharing effects of social security are accompanied by factor price responses. An early contribution that highlighted these responses is Smith (1982) who considered stochastic demographic changes. Building on the basic overlapping generations model of Diamond (1965), he noted that individuals which are born into large generations face disadvantages compared to individuals born into smaller generations. First, the gross wage received in the first period of life will be relatively low as a consequence of the abundance of labor (i.e. a high labor-capital ratio). Second, the real interest rate earned on savings brought into the second period of life will (if the succeeding generations is relatively small) be low because of a high capital-labor ratio in this period.

Smith then considered the effects of respectively DB and DC types of paygo pension programs. Intuitively, a DB program will under reasonable assumptions provide welfare gains (as seen from a rawsian perspective as defined above). This reflects that a DB program will transfer part of the wage disturbance of the young generation to the pensioners. This will also partly offset the factor price responses to the disturbance because the DB program counteracts the effects of the disturbance on saving.

Turning to the analysis of productivity risk, which in turn lead to fluctuations in gross income and factor prices, the issue is whether, and to what extent, the gains of the direct risk sharing effects (which we have analyzed in a partial equilibrium, small open economy framework above) are offset by
the crowding out effects of the paygo program. A meaningful analysis of this trade-off calls for numerical analyses, which will be the topic for the remaining part of this paper.

3. Simulation models to assess social security policies

Since the pathbreaking work of Auerbach and Kotlikoff (1987), social security programs and policies around the world have been analyzed quantitatively with dynamic general equilibrium models that feature overlapping generations. In the following section we first describe the general structure of this model and then discuss recent developments and innovations. In order to structure our presentation, we distinguish between recent innovations with deterministic and with stochastic OLG models that focus on the social security debate.

3.1. Basic set-up and implications of the Auerbach-Kotlikoff model

The Auerbach and Kotlikoff model (the AK model) builds on the discrete time overlapping generations structure utilized in the theoretical analyses above, but extends it along several dimensions in order to obtain realism with respect to quantitative assessments. Intuitively, as soon as more than two generations are taken into account, analytical aggregation of consumer choices becomes hardly possible and the model has to be solved by numerical simulations. The original AK model distinguishes between 55 overlapping generations (i.e. ages 21 to 75), the preference structure of a “newborn” agent who just enters the labor market is represented by a time-separable, nested CES utility function:

$$U = \frac{1}{1-1/\gamma} \sum_{a=1}^{55} \delta^{a-1} \left[ c_a^{1-1/\rho} + \beta x_a^{1-1/\rho} \right]^{1-1/\gamma},$$

where \( c \) and \( x \) denote consumption and leisure, respectively. The parameters \( \delta, \rho, \gamma \) and \( \beta \) represent the time preference rate, the intra- and intertemporal elasticity of substitution, and the leisure preference, respectively. “Newborn” agents maximize their utility function subject to the inter-temporal budget constraint

$$\sum_{a=1}^{55} \left[ (1-x_a) h_a w - c_a - \Gamma_a \right] (1+r)^{1-a} + \sum_{a=a_o}^{55} p_d (1+r)^{1-a} = 0,$$

where the time endowment is normalized to unity, \( r \) denotes the pre-tax return on savings, \( h_a \) defines the efficiency (or human capital) of the agent at age \( a \) and \( \Gamma_a \) represent the individual tax liabilities including social security contributions. Finally, \( p_d \) represent the payments from the pension program.
at age $a$ after retirement at age $a_R$. Of course, agents are restricted in their leisure consumption by their time endowment, i.e. $x_a \leq 1$.

At corner solutions, where the time constraint bites, a shadow price of labor is computed to make the corner solution satisfy the first-order condition. The utility function and the budget constraint already highlight the central assumptions of the original model. First, each agent faces a certain life span of 55 years, is endowed with an exogenous age-specific productivity profile $h_a$ and receives retirement benefits after they pass a government-specified retirement age $a_R$. Second, the original model includes no bequest motive and abstracts from inheritances or other private intergenerational transfers. Third, agents face no liquidity constraints, i.e. they might accumulate debt during young age which they pay back later in life. Finally, individual variables are only indexed by age, there is no disaggregation within a cohort according to income class or sex. Consequently, the model can not address distributional issues within a generation. On the other hand, the model is able to replicate the tax and social security system in quite some detail by the specification of individual tax payments and pension benefits. Since at the end of each period the oldest cohort alive dies, the remaining lifetime is varying across cohorts living in a specific year. Consequently, fiscal reforms have a different impact on the budget constraints of the existing and future cohorts who are “born” (i.e. enter the labor market) after the reform year.

Typically, the initial long run equilibrium is calibrated to represent the existing fiscal system. After a policy reform is announced or enacted, the model computes a transition path to the new long run equilibrium. Given the model’s solution, researchers are able to evaluate the growth effects for the macroeconomy as well as the distributional consequences of the considered reform for different current and future generations. In addition, it is also possible to quantify the aggregate efficiency consequences of a specific reform. For this purpose, Auerbach and Kotlikoff (1987) introduce the so called “Lump sum redistribution authority” (LSRA) which compensates existing generations after the reform with lump-sum transfers and taxes so that they end up at their pre-reform welfare. If the LSRA ends up after compensation with positive assets, the reform has improved the resource allocation of the economy. If the LSRA ends up with debt, economic efficiency has deteriorated.

In order to understand the economic implications of the model, it is useful to keep in mind that the basic effects of alternative pension financing remain as in the stylized theoretical models above. Due to the absence of bequest and liquidity constraints, individuals can perfectly provide for their old-age consumption by means of private saving. Consequently, a mandatory individualized funded pension program would only replace private savings with saving in the public pension fund but would have no real effects in the economy. On the other hand, a paygo program crowds out the capital stock and redistributes resources from young and future generations towards the elderly. Consequently, a move from a paygo to a funded program will always increase the capital stock of the economy and redistribute resources from current towards future generations. Various studies that have been carried...
out with the original model during the 1990’s are discussed in Kotlikoff (2000). In the following we focus on the progress which has been made since then.

3.2. Recent developments with deterministic models

The demographic transition, its intra-generational consequences and its international context

Already Auerbach and Kotlikoff (1987) as well as Auerbach et al. (1989) present initial calculations which quantify the impact of population ageing for the government and the macroeconomy in the US, Japan, Sweden and Germany. These studies clearly pointed out the dramatic increase in social security contribution and tax rates as well as a rise in wages and fall in interest rates due to ageing. However, within the structure of the original single country model, population ageing could only be captured by an exogenously specified population vector which changes from year to year. Consequently, ageing did not change the individual consumption, saving and bequest behavior, nor did the analysis include international repercussion effects. In the following years various studies have included age-specific survival probabilities and an uncertain life span, see Broer and Lassila (1997). However, since these studies assume perfect annuity markets, the remaining assets of those who have died are distributed to the surviving members of the respective cohort. Consequently, the economic effects of these models did not really change compared to the model with certain life span.

While the original model completely ignores intra-cohort effects of government policies, a natural idea was to distinguish various productivity (i.e. human capital) profiles within a specific cohort. This innovation allows one to address new policy issues. Since agents’ welfare is evaluated from an ex-post point of view (i.e. after his/her specific human capital profile has been revealed), the computed welfare changes after a policy reform capture the intra-generational distribution effects of that policy, see Fehr (2000), Hirte (2001) or Beetsma et al. (2003). For example, Fehr (2000) simulates an increase in the eligibility age and a partial tax financing of pension benefits and compares the welfare and efficiency consequences of a reduction in the pension level accompanied either by a tightening or an elimination of the tax-benefit linkage. While the reduction of the pension level is in favor of future generations at the cost of current, the switch towards flat benefits clearly favors low and burdens high income agents and also entails dramatic efficiency losses due to the increased labor supply distortions. Consequently, these models indicate that pension policy should always keep a very tight tax-benefit linkage. Recalling Figure 1.1. above, the message is that reform types B and C are favored on efficiency grounds.

Kotlikoff et al. (2007) also allow for intracoort heterogeneity and analyze quite similar moderate and radical reform proposals for the U.S. as Fehr (2000) for Germany. However, their framework features a much more detailed mapping of the demographic process. During their child-bearing years, agents give birth each year to fractions of children. This means of finessing marriage and family formation permits to incorporate changes through time in age-specific fertility rates and to
closely line up the model's age-specific population shares to the official population forecasts. In addition, they also assume that agents care about their children's utility when the latter are young and, as a consequence, make consumption expenditures on behalf of their children. Therefore, the model delivers the hump in the consumption profile that appears during child-rearing years in the actual data. As previous studies they include utility from leaving bequest and realistic mortality probabilities for agents. However, agents fail to annuitize their assets in old age. Consequently, agents gradually reduce their consumption in old age due to the uninsurable lifespan uncertainty and leave desired and undesired bequests to their children when they die. While agents die at different ages and have children of different ages, their heirs also inherit at different ages. Agents who were born when their parents were young receive inheritances later in their life than do their younger siblings. Kotlikoff et al. (2007) find that the burden from ageing is most evenly spread across current and future generations by a strategy of pre-funding social security with consumption taxes. However, since their model already features a lot of computational complexities, they do no compute the aggregate efficiency consequences of such a reform.

The multiple-region models of Fehr et al. (2004, 2005, 2007) build directly on Kotlikoff et al. (2007). Besides the explicit provision for immigration, the demographic transition at the national level and the household decision problem is modelled in a very similar fashion. However, the ageing processes of U.S., Europe and Japan are now interlinked via the international capital market. One would expect that countries with less ageing such as the US will experience capital inflows in the future which drive up asset prices and growth. In order to isolate the quantitative impact of the open economy assumption, Table 3.1 compares the effects of aging on asset prices in a model with closed and open economies.

*** Table 3.1, asset prices in closed and open economies ***

Each simulation starts from a temporary equilibrium which captures the population structure of the respective countries in year 2000. Due to the diverging population dynamics and fiscal systems, the growth paths as well as the asset prices of the three regions considered differ. Table 3.1 mainly makes four points. First, the figures replicate the so called “asset meltdown”, i.e. due to ageing, asset prices increase in all countries initially and then drop afterwards. Second, the stronger is the ageing process, the lower is the level of asset prices during the next half century. Third, due to aging capital will flow out of Japan and increases asset prices there. In the short run, mainly Europe will benefit from capital inflows, in the medium and long run mainly the US will benefit. However, the effects of capital flows on asset prices are rather small. Forth, as can be seen from the last column, aging increases world interest rate. Especially the latter result stands in stark contrast to some other recent multi-country OLG models such as Brooks (2003), Saarenheimio (2005) or Börsch-Supan et al. (2006). These studies predict that aging will reduce the rate of return on the international capital market during the next 50
years. In the model of Fehr et al. (2005), effective labor supply is improved by productivity growth while at the same time savings are dampened by rising tax payments and social security contributions.

Fehr et al. (2004, 2005, 2007) also account for intracohort heterogeneity. In addition to three different skill levels, they also distinguish between native and foreign individuals. Immigrants are also split into these income classes permitting them to simulate the arrival of immigrants with different stocks of human and physical capital. Since especially high-skilled immigrants are known to be net tax payers to the public system, selective immigration is often offered as a solution to the demographic transition under way in the industrialized world. However, the simulations in Fehr et al. (2005) show that even a significant expansion of immigration, whether across all skill groups or among particular skill groups, will have only a minor impact on the major capital shortage and tax hikes that can be expected along the demographic transition.

While the prospects with respect to immigration seem to be quite frustrating, the model’s predictions are dramatically altered when China is added to the picture. Even though China is aging rapidly, it’s saving behavior, growth rate, and fiscal policies are currently very different from those of developed countries. As Fehr et al. (2007) demonstrate, China might eventually become the developed world's savior with respect to its long-run supply of capital and long-run general equilibrium prospects, if successive cohorts of Chinese continue to save like current cohorts, if the Chinese government can restrain growth in expenditures, and if Chinese technology and education levels ultimately catch up with those of the West and Japan.

Advances in Modelling the Retirement Choice

Increases in the retirement ages are often seen as an alternative to tax increases or benefit cuts. Consequently, among others, Fehr (2000) as well as Kotlikoff et al. (2007) also quantify the macroeconomic and distributional consequences of an increase in the eligibility age for social security. However, the retirement choice in these models is very artificial. Given an exogenous age when they start receiving pension benefits, agents can only decide at what age they quit working. In order to achieve retirement exactly at the eligibility age for social security, either a significant drop in productivity or a dramatic increase in marginal labor income taxes is assumed at the eligibility age. This approach has mainly two disadvantages. First, the drop in individual productivity around retirement is at odds with empirical evidence, which shows only a modest decline in productivity between ages 60 and 70, see French (2005). Second, and even more important, since agents have no choice when to claim social security, social security rules which affect early retirement can’t be captured by these models.

Consequently, recent studies have introduced models where individuals have a labor-leisure choice in each working year, but also optimize the retirement age when they quite working and start to

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16 Storesletten (2000) has already evaluated the positive impact of immigration in the U.S. without taking into account the demographic transition.
receive their pensions. Technically, the household optimization problem is solved in two stages. Given a price vector from the supply side of the economy, individuals first compute their optimal consumption and leisure path for alternative retirement ages. Then the retirement age which yields the highest utility level is selected on the second stage. Due to the evaluation of various alternatives and the discrete jumps in the aggregate variables, the computation is quite complicated.

Hirte (2001) was probably the first who modelled the retirement choice in such a detail. He simulated various pension reform options for Germany which also affect the retirement choice. However, his model does not capture intra-generational heterogeneity. Therefore, the identified early retirement incentive effects are only cursory. Fehr et al. (2003) analyze early retirement incentives of the Norwegian pension system in a model that distinguishes five income classes within a generation. The Norwegian pension system is especially interesting since it consists of a flat tier and an earnings-based supplementary benefit and includes a very generous early retirement scheme. The initial equilibrium which represents the existing system is calibrated so that retirement ages increase with rising income level from age 62 to age 68. Next various reforms of the early retirement scheme and the benefit formula are introduced and the long-run impact on the retirement behavior and the welfare of the five income classes is computed. The simulations indicate that reforms which increase the retirement age also have a positive long run welfare impact.

Finally, Eisensee (2005) develops a framework where fiscal sustainability and the retirement decision in the US are jointly determined. The model distinguishes within a cohort between low, medium and high skilled labor and extends Fehr et al. (2003) by including population ageing and the transition path, by considering differential mortality among the income classes and by simulating a closed economy. The study analyzes three policy options for the US which all would retain fiscal sustainability despite population ageing: a tax increase by 5.5 percentage points, a reduction of the replacement rate by 42 per cent and an increase in the full retirement age (FRA) from 65 to 73. As shown in Table 3.2, all three alternatives increase the retirement age substantially throughout the transition.

*** Table 3.2, sustainable policy options and retirement age ***

Consequently, while tax increases only modestly increase retirement ages, the necessary adjustment of the full retirement age triggers a dramatic delay in retirement. The study disaggregates a direct effect and a general equilibrium effect on retirement behavior. While the direct effect measures the impact of the policy in a partial equilibrium framework, the indirect effect captures the changes in interest rates and wages. It turns out that in the first two options considered the indirect effect increases retirement substantially stronger than the direct effect. While this finding explains why partial equilibrium studies often predict only a modest response of retirement behavior it also indicates that these studies could be quite misleading.
Eisensee (2005) also computes the required policy adjustments in a traditional model where the retirement ages are kept at the initial levels. In this case, taxes have to be increased by 6 percentage points and replacement rates are reduced by 51 per cent. Consequently, a traditional model with exogenous retirement would substantially underestimate the required adjustment. On the other hand, with exogenous retirement the full retirement age only has to increase to 69 years instead of 73 years. The latter might come as a surprise on first sight, but one has to remember that individuals increase their retirement age in order to receive higher pension benefits afterwards. With fixed retirement ages they have to accept dramatic reductions in pension benefits after the increase in the full retirement age. It is noteworthy that – in contrast to Fehr et al. (2003) - Eisensee (2005) does not compare the welfare consequences of alternative policy options with and without endogenous retirement. The study mainly indicates that it is important to include endogenous retirement in policy analysis and offers important innovations in modelling and analyzing retirement choice for future work.

Endogenous human capital and growth

Public financed social security and education systems can be both viewed as intergenerational contracts. Each generation receives education transfers when young and social security benefits when old and has to finance these transfers at middle age via taxes and social security contributions. Due to the similar financing, human capital accumulation and the social security system are strongly linked in both directions. On the one hand, contributions to social security will in general reduce the incentive to acquire human capital especially if pension benefits are flat and the earnings history is not taken into account. On the other hand, it is often argued that human capital investment will improve the viability of the social security system during the ongoing demographic transition. Due to the decreasing share of youth public spending on education will fall which in turn, at least partially, compensates for higher pension outlays. In addition, investments in human capital will increase the efficiency of future workers and thus compensate their declining numbers. However, rising human capital could also undermine the viability of the pension system, if one takes into account that well educated people live longer and have fewer children.

Consequently, it is not surprising that the interaction of the education and the social security system has become an important research area in recent years. In order to introduce human capital accumulation in the original AK-model, the exogenous age-specific productivity profile \( h_a \) in equation (12) becomes endogenous, i.e.

\[
(13) \quad h_{a+1} = \Phi(h_a, \bar{h}, s_a),
\]

where \( s_a \) defines the time spent for education at age \( a \) and \( \bar{h} \) defines the average human capital (or state of knowledge) of the society. Of course, the time spent on education has to be subtracted from the total time endowment and can’t be used for labor. In the simplest formulation of a one-skill model there is no endogenous growth and education is simply modelled as “on-the-job training”, see Perroni
(1995). Consequently, future (individual) productivity $h_{t+1}$ only depends on the existing (own) productivity $h_t$ and the time spent for on-the-job training $s_t$. Since in this model the initial productivity is specified exogenously as before, it is not really an innovation compared to the original model with exogenous human capital. Accordingly, more recent work has extended the human capital model in two different directions.

On the one side, Heckman et al. (1998) keep the exogenous growth assumption but introduce different skill levels and heterogeneity in endowments and the human capital production technology within cohorts. Consequently, they distinguish between schooling at the beginning of the life cycle and on-the-job training later on. However, their model does not include demographic characteristics of individuals and a social security system. These features are addressed by Rojas (2004). He introduces an overlapping generation model where individuals in each cohort have to decide in the initial period whether or not to acquire education. If they do, they receive higher wages, experience a longer lifetime and have fewer kids later on. Educational subsidies that increase the educational attainment of the population therefore generate a demographic change which increases the financial pressure of the social security system. The model is calibrated in order to quantify the doubling of education subsidies in Spain during the 1990’s. The simulations indicate a clear and stable intergenerational pattern of the policy reform. While elderly generations are worse of since they have to finance the higher education spending, generations that benefit directly in the initial years of the reform are the main winners. In the medium- and long-run, higher social security contributions dampen the welfare increase for future generations substantially.

Whereas in Rojas (2004) education does not affect the long-run growth rate of the economy, various authors have developed human capital models with endogenous growth. Technically, this can be achieved by including a human capital externality $\overline{h}$ either in the production function of human capital (3) and output. On the other hand, endogenous growth models do not include a schooling choice, i.e. human capital acquisition is interpreted as on-the-job training in order to keep the model dimension traceable. Endogenous and exogenous growth models sometimes come up with quite different policy conclusions. For example, an increase in life expectancy increases human capital accumulation in Rojas (2004) and, therefore, positively affects the economy. Quite the opposite happens in the endogenous growth model of Echevarria (2003, 2004). Since $\overline{h}$ captures the per-capita human capital endowment of the society, an increase in life expectancy reduces $\overline{h}$ which in turn negatively affects the economy.

Echevarria (2003) estimates that a one year increase in life expectancy reduces the long-run growth rate by 0.02-0.05 percentage points. The opposite happens, when the retirement age is increased. Since in this case returns from human capital investment rise, individuals will increase their

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17 Heckman et al. (1999) apply this model in order to quantify the costs and benefits of tuition subsidies.
on-the-job training which in turn improves the long-run growth rate. For specific parameter constellations, the positive effect from a one year increase in the retirement age dominates a simultaneous one year increase of life expectancy. However, the retirement age is set exogenously and it is not clear how strong rising life expectancy affects the individual retirement choice. Consequently, Echevarria (2004) extends his previous model by including an endogenous retirement age. Now an increase in life expectancy also increases the retirement age. However, for all considered parameter constellations the (negative) direct effect of the reduced per-capita productivity dominates the (positive) effect of the higher retirement age.

*** Table 3.3, schooling, retirement and growth ***

Table 3.3 is taken from Echevarria (2004) and compares the impact of an increase in life expectancy by 3 years on schooling, retirement and growth for depreciating and non-depreciating physical and human capital. Note that it is assumed that human capital only depreciates during schooling, not during working time! Consequently, the growth rate is lower, and people leave school earlier and retire later compared to the situation without depreciation. It both calibrations considered, the exogenous increase in life expectancy induces agents to leave school later and retire later. Nevertheless, the growth rate still decreases by roughly 0.05 percentage points. This negative effect of higher life expectancy on economic growth rates seems to be in stark contrast with the historical observation that the rising life expectancy during the industrial revolution of the 19th century had a positive growth effect. However, when life expectancy is low, people don’t retire and work their whole life. Consequently, increases in life expectancy also induce an equal increase in the working periods which in turn improves economic development. When life expectancy passes a certain level, people start to retire before they die. This is the turning point since from now on further increases in life expectancy have a less pronounced effect on the retirement age. As a consequence, economic growth rates will decline.

One central problem of endogenous growth models is that the specification of $\tilde{h}$ is rather arbitrary but has an enormous impact on the models dynamics. For example, Bouzahzah et al. (2002) assume that each newborn generation inherits the human capital of the previous generation. Of course, with such a specification increases in life expectancy do not have such negative consequences for economic growth! Bouzahzah et al. (2002) compare the economic effects of various policy reforms in endogenous and exogenous growth models. With respect to the pension system they simulate a switch to a funded system and an increase in the retirement age. Since the economic consequences of both reforms are very similar in both endogenous and exogenous growth models, they conclude that for pension reforms endogenous growth does not really matter.
3.3. Recent developments with stochastic models

During the last decade, a new direction of quantitative research has extended the traditional AK-model by considering various sources of idiosyncratic and aggregate economic risk, see Krüger (2006). This section presents the most important contributions and discusses their central findings.

Idiosyncratic risk and liquidity constraints

Already Hubbard and Judd (1987) extend the AK model by including life span uncertainty and liquidity constraints. Since private annuity markets are missing, social security provides an insurance against longevity risk but at the same time increases welfare losses due to borrowing constraints. However, Hubbard and Judd (1987) only provide an introduction to insurance analysis. Since their model abstracts from labor income uncertainty, precautionary savings are neglected so that liquidity constraints are relevant for all young individuals. Since a meaningful analysis of the insurance and liquidity effects of social security has to include income risk, various stochastic general equilibrium models with overlapping generations have been developed recently.

İmrohoroğlu et al. (1995) where the first, who examine the optimality of and the welfare effects from alternative social security arrangements in a framework with stochastic employment opportunities. Agents supply labor inelastically when they are given the opportunity to work and otherwise receive unemployment benefits. After the mandatory retirement age, individuals rely on flat-rate pension benefits. In this framework, the welfare consequences of social security reflect the trade-off between the (positive) insurance provision against income and longevity risk and the (negative) effects of stronger binding liquidity constraints. While already the institutional setup is very favourable for social security, the calibrated initial equilibrium (without social security) lacks dynamic efficiency. When the growth rate of the economy exceeds the interest rate, it is not surprising that the introduction of social security increases the resources of all generations.

In their follow up study, İmrohoroğlu et al. (1999) eliminate dynamic inefficiency by incorporating land as a fixed factor of production. In this setting, the introduction of social security has again positive insurance and negative liquidity effects, but it also redistributes income across generations. The simulations indicate that an economy without social security provides the highest welfare for individuals. However, this result might only reflect the negative income effects of social security for future generations. This argument also applies to a recent study by Hong and Rios-Rull (2007) who develop an OLG-model with two sexes and family formation. In this set-up, marriage provides some kind of private insurance against income uncertainty which may weaken the case for public insurance programs. However, the computed long-run welfare losses due to the introduction of social security are probably again mainly due to intergenerational redistribution.
Of course, the same critique also applies to studies that analyze the long-run consequences of gradual social security reforms. For example, Huggett and Ventura (1999) quantify the distributional consequences if the current U.S. pension system would be substituted by a two-tier system where the first tier is strictly connected to former contributions and the tax-financed second tier would guarantee a minimum pension for all households with low income. Since their model allows for endogenous labor supply, the induced labor supply distortions weaken the case for social security. In addition, agents are distinguished according to ability levels within a generation, so that intragenerational distributional effects of social security arrangements could be quantified. Their simulations suggest that both high- and low-ability agents would benefit from a switch to the two-tier system while median-ability agents would lose. Since the majority of the agents in the economy is close to median, the considered reform would yield a very robust aggregate welfare loss.

Storesletten et al. (1999) consider the long-run effects when the current U.S. pension system is either replaced by a two-tier system of personal saving accounts or completely eliminated. In contrast to Huggett and Ventura (1999), their two-tier system delivers a long-run welfare gain, which is even larger than the gain from privatization. Since their model abstracts from variable labor supply and they try to neutralize the induced intergenerational income effects by appropriate government transfers, the welfare gains are mainly due to insurance effects. Consequently, the assessment of the long-run consequences of social security may change dramatically when the implied intergenerational income effects are eliminated.

Fuster et al. (2003) present a model where individuals are linked across generations by two-sided altruism. Of course, in this framework, the intergenerational redistribution induced by public policy is (at least partly) neutralized by intervivos transfers and bequests. In addition, borrowing constraints are less binding so that social security mainly provides an insurance against uninsurable “labor ability” shocks at birth. For this reason, Fuster et al. (2003) find that social security increases long-run welfare for most of their considered households. A final argument in favour of social security is explored by Imrohoroğlu et al. (2003). They introduce quasi-hyperbolic discounting in the framework already discussed above and show that social security may raise long-run welfare for individuals with time-inconsistent preferences if the short-term discount rate is sufficiently high. Of course, social security may always serve as a commitment device for myopic individuals who do not adequately save for their retirement.

All studies with stochastic economies discussed so far share a common deficiency which was already mentioned above. Since they only consider the long-run effects of social security, the consequences for transitional generations are completely neglected. Therefore, the computed long-run welfare changes could be simply due to intergenerational redistribution. In order to provide a complete assessment of social security, one has to compute the transition path between steady states and separate intergenerational distribution from efficiency effects. A first study which addresses this critique is Huang et al. (1997) who compare two experiments where the existing unfunded social
security system is eliminated and a private or a mandatory state-run funded system is introduced with all existing and transitional generations compensated virtually. While both experiments yield a significant aggregate efficiency gain, the government-run funding scheme is preferred to privatization due to its superior insurance properties. De Nardi et al. (1999) extend this model by including realistic US demographics and variable labor supply. The latter allows them to analyze reforms where the tax-benefit linkage of the pension system is improved, which increases welfare in their framework. Conesa and Krueger (1999) simulate an immediate, a gradual and an announced elimination of social security and compute the political support for the three proposals in the initial year. Although for all cases considered agents would prefer to be born into the final steady-state, no proposal receives an initial voting majority in the closed economy case. The political support is declining when intra-cohort heterogeneity is increasing due to the rising insurance gains from flat pensions.

While Conesa and Krueger (1999) can explain why pension reforms are delayed in democratic systems, their study does not include efficiency calculations. If many individuals receive small welfare losses while the (fewer) winners receive enormous welfare gains, it might be possible that the reform receives no political support although it delivers aggregate efficiency gains. A very similar problem arises in Fuster et al. (2007) who extend their two-sided altruism model by incorporating variable labor supply and the transition paths across steady states. As before the family insurance substitutes the missing market insurance but now the social security contributions distort the labor supply choice. The latter is reinforced by the fact that the payroll tax comes on top of personal income taxes. Consequently, they find that the majority of individuals are better off with the elimination of social security in all privatization schemes considered. However, since the resulting welfare changes are not aggregated across individuals and generations, the overall efficiency effect is not explicitly determined.

The latter is done by Nishiyama and Smetters (2007) who simulate a stylized 50-percent privatization of the US social security system. Again, the considered reform reduces the labor supply distortions but also the insurance provision of the social security system. In order to isolate the overall efficiency effects, the authors follow Auerbach and Kotlikoff (1987) by introducing a Lump-Sum Redistribution Authority (LSRA) which compensates initial agents and distributes the accumulated assets (i.e. efficiency gains) or debt (i.e. efficiency losses) to newborn and future agents. They find efficiency gains from privatization which amount to $21,900 (in 2001 growth adjusted dollars) per household, if wage shocks could be insured privately. Consequently, if income uncertainty is perfectly insured, the aggregate efficiency effect of social security is dominated by the distortions of the labor/leisure choice. However, if wage income shocks could not be insured, the overall efficiency effect from privatization turns into a loss of $5,600 per household.

Table 3.4 reports the welfare changes (i.e. without compensation) and the efficiency effects (i.e. with compensation payments for elderly) for different cohorts and income classes. In the long run, the welfare gain from privatization ranges from $52,400 for low income households to $84,300 for top
income households. The higher gains for rich households simply indicate the redistributive features of the U.S. social security system. Similar figures are also reported by the studies discussed above which only compute the long-run consequences of social security reforms. However, thses gains are mainly due to income redistribution, since elderly cohorts at the time of the reform realize significant welfare losses due to higher income or consumption taxes which are required to finance the existing pension claims. Table 3.4 shows that 60-year olds at the time of the reform lose between $27,600 and $361,800 per household. If the LSRA is introduced and all existing generations at the time of the reform are compensated by transfers, privatization would require each future household that enters the economy in the year after the reform or later to pay $5,600 to service the debt of the LSRA\textsuperscript{18}. In other words, the partial privatization of the U.S. social security system does reduce economic efficiency. This clearly indicates that the (positive) insurance effects of the US social security system dominate the distortionary effects on labor supply.

*** Table 3.4, welfare and efficiency effects from social security privatization (in $ per household)***

Fehr and Habermann (2005) reach a similar conclusion for the German social security system. In contrast to the US system, benefits in the German system are strongly linked to former contributions. On the one hand, this institutional feature minimizes labor supply distortions but at the same time it also reduces the insurance provision against income shocks. Simulations show that a more progressive system would yield a significant aggregate efficiency again, if all initial generations are compensated by LSRA transfers.

*Aggregate risk and social security*

Up to now the discussion assumed that risk is specific to the individual household. In this case *intragenerational* risk sharing arrangements such as progressive taxes or social security systems have the potential to improve the wellbeing of individuals. Turning to quantitative studies that consider aggregate risks, only *intergenerational* risk sharing arrangements could provide insurance, however.

The literature has looked especially at two sources of aggregate risk: demographic risk and productivity risk. As discussed above, someone that is born into a large (small) cohort, can expect lower (higher) wages during the working periods while the previous generation experiences higher (lower) interest rates during their retirement. Depending on the design of the social security system, theses factor price effects are either dampened or reinforced by adjustments of contribution rates and benefits. In order to quantify such effects, various contributions in Ahlo et al. (2007) combine a

\textsuperscript{18} Since the LSRA accumulates debt or assets during the transition, the market interest rate is implicitly used to weight the utility of different generations during the transition. Of course, such transfers are not possible in practice, but the computed potential efficiency effects allow us to compare different policy experiments without explicitly specifying a social welfare function and social discount rates.
detailed stochastic population model with a traditional (i.e. deterministic) overlapping generation model. This allows to model the pension system in detail and to compute distributions for all future macroeconomic and social security variables. On the other hand, this approach has a central shortcoming: When agents decide about savings, labor supply etc., they do not take into account the demographic risk, i.e. there is no precautionary behavior. Consequently, the computed welfare effects have to be interpreted with caution.

In a consistent model, the actions of individuals have to reflect the existing aggregate uncertainty. Bohn (2001) presents such an analysis, however, his framework only considers two overlapping generations. He compares the intergenerational risk sharing implications of DB and DC pension systems when there is demographic risk. Capturing the same mechanisms as in the stylized theoretical model of Smith (1982) discussed above, Bohn shows that DB paygo pension programs are able to spread demographic risk across generations.

Sanchez-Marcos and Sanchez-Martín (2006) extend the approach of Bohn (2001) by analyzing a model with four overlapping generations and simulating the short- and long-run implications of unfunded DB systems. It turns out that now the improved intergenerational risk sharing can not compensate the long-run crowding-out of the capital stock due to the introduction of social security. However, the simulations also show clear welfare improvements for those generations living in the short-run. As before, it would be interesting to neutralize the intergenerational income redistribution in order to isolate the welfare effects from intergenerational risk sharing, but this is not done in this study.

Other recent papers capture aggregate uncertainty by productivity shocks which affect the production function of the economy. In addition, the depreciation rate is also assumed to be stochastic, in order to allow the return from capital to vary independently from the wage rate. Without social security, wage and interest rate shocks affect young and old generations separately. In accordance with the theoretical portfolio choice approach to social security design examined above, it may be optimal to have a low-yielding paygo program in an optimal portfolio. Krüger and Kubler (2006) simulated such a model with nine overlapping generations in order to analyze whether the welfare gains from improved intergenerational risk sharing dominate the welfare losses due to the long-run crowding-out of the capital stock. They are able to show that in a small open economy social security may improve long-run welfare if individuals are very risk averse.

In a closed economy the crowding out effect of the capital stock seems always to dominate the improved risk sharing, however. While in the model of Krüger and Kubler (2006) hedging against aggregate risk was very indirect, Olovsson (2004) explores a direct mechanism to transfer aggregate risk across generations. In his three-period model, the pension benefits are indexed either to wage or to capital returns. The long-run consequences are compared to an equilibrium which reflects the existing US social security system and to a laissez-fair equilibrium without social security. It turns out that the economies with highly volatile pension benefits may in fact neutralize the crowding-out of the capital stock.
stock and improve long-run welfare significantly compared to the existing social security system. However, the elimination of social security would still yield the highest long-run welfare gains.

Summing up, the quantitative results of these few papers that deal with aggregate risk in overlapping generations models indicate that in the long-run the benefits from intergenerational risk sharing will not dominate the cost from intergenerational redistribution. A natural venue for future work will therefore be to design transition strategies that allow to minimize the intergenerational redistribution and to isolate the aggregate benefits from intergenerational risk sharing.

4. Final remarks

Surveying the intergenerational distribution effects of social security, we have attempted to cover basic theoretical insights as well as a broad selection of recent policy relevant applications based on numerical overlapping generations models. Given the formidable size of this literature, our survey is selective and we have not covered several issues which are important for the theoretical discussion and the social security reform process in many countries. In closing this survey we stick to our broad focus and highlight two important issues that are likely to shape the next steps in this field.

First, the discussion in the two previous sections shows a clear distinction between the risk analysis in theoretical and numerical models. Whereas the former focuses on aggregate risk and intergenerational risk sharing, the latter concentrates on idiosyncratic risk and intragenerational risk sharing. To a large extent this reflects that numerical models in the past were restricted to idiosyncratic risk mainly for technical reasons. Advances in computer technology have recently made it possible to simulate more sophisticated overlapping generation models with aggregate risk, however. As soon as it is possible to include a transition path in models with aggregate risk, the efficiency effects of social security could be quantified from a new perspective. This will allow the theoretical and the empirical discussions of risk and risk sharing issues in social security to be more closely connected.

Second, despite all differences, both the theoretical and the numerical studies with aggregate and idiosyncratic uncertainties share a common characteristic. They both highlight the insurance properties of social security systems and, for this reason, social security tends to appear more favorable in terms of economic efficiency than in older studies based on deterministic models. We conjecture that future quantitative studies which include aggregate risk will strengthen this general trend.
References


Ball, L. and N.G. Mankiw (2001): Intergenerational risk sharing in the spirit of Arrow, Debreu, and Rawls, with applications to social security design, manuscript, Johns Hopkins University.


Table 1.1: Old age dependency ratios
Ratio of population aged 65 and above to population aged 15 – 64, selected countries, per cent

<table>
<thead>
<tr>
<th>Country</th>
<th>2000</th>
<th>2015</th>
<th>2030</th>
<th>2045</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>36.4</td>
<td>43.9</td>
<td>59.3</td>
<td>67.7</td>
</tr>
<tr>
<td>Germany</td>
<td>33.3</td>
<td>40.2</td>
<td>56.6</td>
<td>65.3</td>
</tr>
<tr>
<td>Japan</td>
<td>25.3</td>
<td>45.4</td>
<td>58.8</td>
<td>75.9</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>30.8</td>
<td>35.2</td>
<td>46.5</td>
<td>53.2</td>
</tr>
<tr>
<td>United States</td>
<td>21.6</td>
<td>24.6</td>
<td>36.4</td>
<td>40.5</td>
</tr>
<tr>
<td>OECD total</td>
<td>24.8</td>
<td>30.5</td>
<td>42.2</td>
<td>51.7</td>
</tr>
</tbody>
</table>

Source: OECD

Table 1.2: Escalation of early retirement
Per cent of male population aged 55 – 64 in the workforce, selected countries

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>65.3</td>
<td>43.0</td>
<td>38.4</td>
<td>38.5</td>
</tr>
<tr>
<td>Germany</td>
<td>64.1</td>
<td>52.0</td>
<td>48.2</td>
<td>48.2</td>
</tr>
<tr>
<td>Japan</td>
<td>82.2</td>
<td>80.4</td>
<td>80.8</td>
<td>78.4</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>62.6</td>
<td>62.4</td>
<td>56.1</td>
<td>59.8</td>
</tr>
<tr>
<td>United States</td>
<td>69.7</td>
<td>65.2</td>
<td>63.6</td>
<td>65.6</td>
</tr>
</tbody>
</table>

Source: Fenge and Pestieau (2005, page 6), OECD

Table 3.1: Asset prices in closed and open economies

<table>
<thead>
<tr>
<th>Year</th>
<th>Closed Economies</th>
<th>Open Economies</th>
<th>Interest rate (per cent)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>USA</td>
<td>EU</td>
<td>Japan</td>
</tr>
<tr>
<td>2000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>2010</td>
<td>1.086</td>
<td>1.066</td>
<td>1.026</td>
</tr>
<tr>
<td>2020</td>
<td>1.109</td>
<td>1.095</td>
<td>1.012</td>
</tr>
<tr>
<td>2030</td>
<td>1.078</td>
<td>1.044</td>
<td>1.004</td>
</tr>
<tr>
<td>2050</td>
<td>1.125</td>
<td>1.009</td>
<td>0.959</td>
</tr>
</tbody>
</table>

Source: Fehr et al. (2005).
### Table 3.2: Sustainable policy options and retirement age

<table>
<thead>
<tr>
<th></th>
<th>Low skilled</th>
<th>Medium skilled</th>
<th>High skilled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial period</td>
<td>62</td>
<td>65</td>
<td>66</td>
</tr>
<tr>
<td>Tax increase</td>
<td>65</td>
<td>67</td>
<td>69</td>
</tr>
<tr>
<td>Benefit reduction</td>
<td>65</td>
<td>69</td>
<td>70</td>
</tr>
<tr>
<td>FRA increase</td>
<td>71</td>
<td>73</td>
<td>74</td>
</tr>
</tbody>
</table>

Source: Eisensee (2005)

### Table 3.3: Schooling, retirement, and growth

<table>
<thead>
<tr>
<th></th>
<th>Depreciation rates</th>
<th>Life expectancy</th>
<th>Schooling years</th>
<th>Retirement age</th>
<th>Growth rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Physical capital</td>
<td>10 %</td>
<td>80</td>
<td>17.33</td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td>Human capital</td>
<td>8.5 %</td>
<td>77</td>
<td>62.0</td>
<td>1.95</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>80</td>
<td>63.3</td>
<td>4.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>24.55</td>
<td>55.3</td>
<td>4.63</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>24.91</td>
<td>56.2</td>
<td></td>
</tr>
</tbody>
</table>

Source: Echevarria (2004), Case I &II

### Table 3.4: Welfare and efficiency effects from social security privatization (in $ per household)

<table>
<thead>
<tr>
<th>Age in reform year/Birth year</th>
<th>Welfare effects for selected productivity classes</th>
<th>Efficiency effects</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Class 1</td>
<td>Class 3</td>
</tr>
<tr>
<td>79</td>
<td>-4.800</td>
<td>-5.700</td>
</tr>
<tr>
<td>60</td>
<td>-27.600</td>
<td>-43.500</td>
</tr>
<tr>
<td>40</td>
<td>-18.700</td>
<td>-46.700</td>
</tr>
<tr>
<td>20</td>
<td>2.200</td>
<td>-1.500</td>
</tr>
<tr>
<td>0</td>
<td>32.800</td>
<td>33.700</td>
</tr>
<tr>
<td>∞</td>
<td>52.400</td>
<td>56.700</td>
</tr>
</tbody>
</table>

Source: Nishiyama and Smetters (2007), Table 9.
Figure 1.1: Dimensions of pension reform

Tax-Benefit Link

Perfectly actuarial

"1970"

"Today"

Reform A

Reform B

Reform C

Flat Benefit

Pure paygo

Fully funded

Degree of funding

Figure 2.1: Shocks to labor income and stock market returns in the model economy

Generation $t$ is considered from time “rawls” in the case of rawlsian risk sharing and from time “interim” in the case of interim risk sharing.

Generation

$t + 1$

$t$

$t$

$t + 1$

$t + 2$

Period

Young

Exposure to $\lambda_{t+1}$

Old

Exposure to $r_{t+2}$

Young

Exposure to $\lambda_t$

Old

Exposure to $r_{t+1}$