FDI, R&D and Endogenous Competitiveness

BY
ARMANDO J. GARCIA PIRES

This series consists of papers with limited circulation, intended to stimulate discussion.
FDI, R&D and Endogenous Competitiveness

Armando J. Garcia Pires *
Norwegian School of Economics and Business Administration

January 10, 2008

Abstract

We analyze the influence of endogenous competitiveness on multinational activity. Competitiveness is endogenized by assuming that firms differ on R&D commitment power, i.e.: some firms are leaders in R&D. We show that firms with higher commitment power tend to invest more in R&D and consequently also tend to be more competitive than rivals that lack such capability. As a result, firms with higher commitment power have higher propensity to become multinationals than firms with lower commitment power. In addition, the former use the R&D leader advantage to compel the latter to not enter the market or, in case of entry, to force them to adopt the domestic strategy. Therefore, in addition to the proximity-concentration trade-off, we identify another FDI determinant: strategic technological competitiveness.

Keywords: Market Structure, R&D Investment, Multinationals, Endogenous Asymmetric Firms.

JEL Classification: F23, C72, L11.

*This paper has benefited greatly from discussions with Gianmarco Ottaviano, Peter Neary and Renato Flôres. I am also grateful to Dermot Leahy, Francesca Sanna-Randaccio, Jan I. Haaland, Leo Grünfeld, Paula Fontoura, Pedro Pontes and Victor Norman for helpful comments during the preparation of this work. The usual disclaimer, however, applies.

†Address for correspondence: Norwegian School of Economics and Business Administration, Department of Economics, Helleveien 30, N-5045 Bergen, Norway. Tel: +(47)55959622, Fax: +(47)55959543; E-mail: armando.pires@nhh.no.
1 Introduction

According to Markusen (1995, 2002) one of the stylized facts on foreign direct investment (FDI) is that multinational activity tends to be more important in industries and firms that have high levels of R&D to sales. In other words, R&D investment and FDI are positively correlated (see also Grubaugh, 1987; Morck and Yeung, 1992 and Lin and Yeh, 2005). This is confirmed by Kravis and Lipsey’s (1992) empirical study on US multinationals. They show that high R&D investment by individual firms is associated with high multinational shares and that the international competitiveness of US multinationals is determined by the level of investment in firm specific assets, such as R&D. Not surprising then that in the US, multinational firms account for 80% of total R&D expenditure by private firms (Graham, 1996).

The motivation for this paper comes from the evidence discussed above on the interrelations between FDI and technological competition. In particular, we focus on three closely related issues: (1) firms’ choice of mode of foreign entry (export versus horizontal FDI), (2) investment in R&D and (3) endogenous competitiveness asymmetries between firms. These issues have so far been dealt with separately in the FDI literature. For example Horstmann and Markusen (1992) and Rowthorn (1992) look at the decision between exporting and FDI; Petit and Sanna-Randaccio (2000) jointly determine FDI and R&D; and Helpman et al. (2004) link FDI with exogenous firm heterogeneity.

In Horstmann and Markusen (1992) and Rowthorn (1992) firms decide between exporting (domestic strategy) and establishing a plant in the foreign market (multinational strategy). It is assumed that there are fixed costs at firm level (R&D, blueprints, patents and so on), plant-specific fixed costs and increasing returns in production. The choice of the mode of foreign entry depends on a trade-off between concentration of production (“economies of scale” effect) and proximity to consumers (“firm size” effect, i.e.: multinationals are by assumption bigger, because by avoiding trade costs they can have higher sales in the destination market). This trade-off is then determined by the interplay between trade costs and the fixed cost of opening a foreign plant. Accordingly, the multinational option is favored when plant-specific fixed costs are low relatively to trade costs (the so-called proximity-concentration trade-off).

As noted by Petit and Sanna-Randaccio (2000), besides the “scale” and the “firm size” effects, the mode of foreign entry is also affected by a “techno-
logical” effect (for example investment in R&D). Having this in mind, Petit and Sanna-Randaccio (2000) endogenize both the firms’ mode of foreign expansion and R&D investment

1. R&D investment is modeled as a cost reducing activity following Leahy and Neary (1997). Petit and Sanna-Randaccio (2000) then show that there is a positive relationship between multinational expansion and R&D investment. In other words, multinational firms invest more in R&D than domestic firms, because by having preferable access to foreign markets they have economies of scale in R&D. This allows multinationals to carry out more innovative activities than domestic firms. In this sense, Petit and Sanna-Randaccio (2000) endogenize the “technological” effect on FDI through exogenous “firm size” effects.

Underlying Petit and Sanna-Randaccio’s (2000) result, but not explicitly taken into account by them, is the fact that firms are by nature heterogeneous. This seems to be particularly important for multinationals. Multinational firms are invariably bigger in size and more productive than domestic firms (Markusen, 2002 and Helpman et al. 2004). In fact, Helpman et al. (2004) show that by introducing exogenous productivity differences between firms, only the more competitive firms become multinational, while the less competitive ones either do not enter the market or are relegated to the domestic strategy

2.

The model of this paper has three building blocks. First, we adopt Horstmann and Markusen’s (1992) horizontal FDI framework

3. Second, as in Petit and Sanna-Randaccio (2000), we introduce process R&D investment that reduces marginal costs but increases firm-specific fixed costs. Third, following Bagwell (1995), we assume that firms differ in the capacity to commit to R&D decisions, i.e.: some firms are leaders in R&D.

Given our modeling strategy, similar to Horstmann and Markusen (1992), Petit and Sanna-Randaccio (2000) and Helpman et al. (2004), FDI also arises as a result of a proximity-concentration trade-off. But this is only part of the story. We show that differences in R&D commitment power add a strategic

---

1Some other papers analyze only the innovative activities of multinational firms, i.e.: multinationality is exogenous (see de Bondt et al., 1988; Veugelers and Vanden Houte, 1990; and Wang and Blomstrom, 1992).

2Nocke and Yeaple (2007) and Lu (2007) also analyze the role of firm heterogeneity on FDI, the former on mergers and acquisitions and the latter on product life cycle dynamics.

3As in Helpman (1984) vertical FDI could be introduced by allowing for international differences in factor endowments. However, as Ihrig’s (2005) empirical study on US FDI shows, technology issues are more important for horizontal FDI than for vertical FDI.
technological dimension to the choice of foreign expansion. Accordingly, firms with higher commitment power, independently of being exporters or multinationals, tend to invest more in R&D and consequently to be more competitive than rivals that lack such capability. As a result, higher R&D commitment power firms have higher propensity to become multinational. In addition, firms with higher commitment power use the R&D leader advantage to compel rivals not to enter the market or, in case of entry, to force them to adopt the domestic strategy. Therefore, in addition to the proximity-concentration trade-off, we identify another FDI determinant: strategic international technological competitiveness.

The rest of the paper is organized as follows. In the next section we introduce the base model and define R&D commitment power. Then we derive the production equilibrium. In the fourth section we analyze how R&D and multinational activity affect competitiveness. In the fifth section we derive the entry equilibrium. We conclude by discussing results.

2 The Model

We consider an industry with two potential producer countries (home and foreign) and a third consumer country where all production is sold. Each country hosts one firm, the home firm and the foreign firm, which produce the same homogenous good. Each firm consists of two physical units: the headquarters and the production plant. The headquarters are always located in the country of origin of the firm. The production plant, in turn, can be located either close to the headquarters (domestic firm) or in the third market (multinational firm). Accordingly, when a firm chooses the domestic strategy it serves the third market through exports, while when a firm opts for the...
multinational strategy it serves the third market through local production.

Since the model is symmetric, in most of the following we concentrate
our attention on the home country. Equations for the foreign country (and
for the foreign firm) apply by symmetry. Foreign variables are indicated by
an asterisk.

The home and the foreign firm face the following indirect demand in the
third country:

\[ P_{i,j} = a - b \left( q_{i,j} + q^*_{i,j} \right) \] (1)

where \( q \) represents sales of the home firm in the third market and \( q^* \) is
the equivalent for the foreign firm. The sub-scripts \((i, j)\) represent market
structure. Accordingly, \( i = 0, E, M \) is the international strategy of the home
firm, where 0 stands for non-entry, \( E \) for the exporting strategy and \( M \) for the
multinational strategy. Similarly \( j = 0, E, M \) is the international strategy
of the foreign firm\(^8\). Also, \( a \) and \( b \) respectively stand for the intercept of
demand and for an inverse measure of market size.

Like in Leahy and Neary (1997), the home and the foreign firm invest in
process R&D that reduces marginal costs (\( C \)) but increases fixed costs (\( \Gamma \))\(^9\).

For the home firm this amounts to:

\[ C_{i,j} = (c - \theta k_{i,j}) \]
\[ \Gamma_{i,j} = \gamma \frac{k_{i,j}^2}{2} \] (2)

where \( k \) is R&D investment by the home firm, \( \theta \) is the cost-reducing effect of
R&D, \( \gamma \) is the cost of R&D and \( c \) is the initial marginal cost. The foreign firm
has a similar cost structure with \( c = c^*, \theta = \theta^* \) and \( \gamma = \gamma^* \). The symmetry
in technology is assumed so that competitiveness asymmetries between the
home and the foreign firm can only arise endogenously.

\(^8\)Then, for example, \( q_{M,E} \) is the home firm’s sales in the third country when the home
firm is a multinational and the foreign firm is an exporter (similarly \( q^*_{M,E} \) is the foreign
firm’s exports to the third country when the foreign firm is an exporter and the home firm
is a multinational).

\(^9\)See also Spence (1984) and Spencer and Brander (1983). Contrary to Horstmann
and Markusen (1992) we are then able to endogenize the firm-specific fixed costs. This is
particularly important, since firm-specific fixed costs intend to represent strategic assets,
such as R&D investment, which can hardly be seen as exogenous.
We model multinational activity similar to what is standard in the literature (see Horstmann and Markusen, 1992). In particular, since the multinational strategy demands firms to separate headquarters and production, we assume that is more costly to operate a plant in the third market than in the country of origin of the firm. Accordingly, we assume a plant specific fixed cost $\Delta$ that equals $G$ when a firm is an exporter and that equals $\rho G$, with $\rho > 1$, when a firm is a multinational, i.e.: $\Delta_E = G$ and $\Delta_M = \rho G$.

In this sense the home firm’s profits can be written as:

$$\Pi_{i,j} = (P_{i,j} - C_{i,j} - t_i) q_{i,j} - \Gamma_{i,j} - \Delta_i$$  \hspace{1cm} (3)$$

where $t = t^*$ represents trade costs, which are symmetric for both the home and the foreign firm. Like in Horstmann and Markusen (1992), only exporters face trade costs, i.e.: $t_E = t > 0$ and $t_M = 0$.

### 2.1 Commitment Power

The concept of commitment power, introduced by Stackelberg (1934), refers to the strategic advantages of moving before rivals. Bagwell (1995) gives a precise definition of the assumptions behind games where firms have differences in commitment power. First, moves in the game are sequential with some players committing to actions before other players select their respective actions. Second, late-moving players perfectly observe actions selected by the first movers.

In this paper we follow Bagwell’s (1995) definition and apply it to investment in R&D. Our objective is to analyze the effects of R&D commitment power differences on multinational activity.

In game terms, a firm has R&D commitment power when it can commit to the output stage, i.e.: R&D levels are chosen in a previous stage to outputs. The contrary happens when a firm has no commitment power: the firm sets outputs and R&D levels simultaneously. Thus, when a firm has commitment power, it can use R&D to improve productive efficiency and also to affect the rival’s strategic decisions. When a firm does not have R&D commitment power, it can only use R&D to improve efficiency but not to affect the rival’s strategic choices.

Like in a standard output-Stackelberg-leader set-up, R&D commitment power, then, gives leader advantages to a firm that competes with a firm that lacks such capability. However, differently from standard output-leader
models, and as we will show below, firms with different R&D commitment capabilities can become endogenously asymmetric in marginal costs, i.e.: our model endogenizes competitiveness. As we will see, this has important implications in terms of the mode of foreign expansion by firms.

In this sense we assume that only the home firm has R&D commitment power. The timing of the multinational game is then the following (see figure 1). In the first stage the home and the foreign firm decide the mode of entry in the third market (non-entry, export or multinational). In the second stage the home firm chooses R&D ($k$). In the third stage the home firm chooses outputs ($q$ and $x$) while the foreign firm chooses both outputs ($q^*$ and $x^*$) and R&D levels ($k^*$).

We are now ready to define the production equilibrium of our model.

### 3 Production Equilibrium

As usual the game is solved by backward induction. Output expressions are found by solving for the outputs’ first-order conditions (FOCs). However, since these FOCs depend on market structure, different output expressions apply for different market structure cases. Accordingly, when both firms are multinationals we have:

$$q_{M,M} = \frac{D+2\theta k_{M,M} - \theta k_{M,M}^2}{3\theta}$$

$$q_{M,M}^* = \frac{D+2\theta k_{M,M}^* - \theta k_{M,M}}{3\theta}$$

where $D = (a - c)$ is a measure of a firm “initial cost competitiveness” (i.e.: without R&D investment).

When both firms are exporters, we obtain:
\[
q_{E,E} = \frac{D - t + 2k_{E,E} - \theta k_{E,E}^*}{3b} \\
q_{E,E}^* = \frac{D - t + 2k_{E,E} - \theta k_{E,E}}{3b}
\]

If the home firm is an exporter and the foreign firm is a multinational:

\[
q_{E,M} = \frac{D - 2t + 2k_{E,M} - \theta k_{E,M}^*}{3b} \\
q_{E,M}^* = \frac{D + t + 2k_{E,M} - \theta k_{E,M}}{3b}
\]

If the home firm is a multinational and the foreign firm is an exporter:

\[
q_{M,E} = \frac{D + t + 2k_{M,E} - \theta k_{M,E}^*}{3b} \\
q_{M,E}^* = \frac{D - 2t + 2k_{M,E} - \theta k_{M,E}}{3b}
\]

In turn, if the home firm has a multinational monopoly we get:

\[
q_{M,0} = \frac{D - 2t + 2k_{M,0}}{2b}
\]

And if the home firm has an exporting monopoly it results in:

\[
q_{E,0} = \frac{D - t + 2k_{E,0}}{2b}
\]

Obviously, the expressions \(q_{0,M}^*\) and \(q_{0,E}^*\) are exactly the same as \(q_{M,0}\) and \(q_{E,0}\), with \(k_{M,0}\) substituted for \(k_{0,M}^*\) and \(k_{E,0}\) substituted for \(k_{0,E}^*\), respectively.

To derive the R&D expressions we can proceed in the same way as for outputs by working with the FOCs for R&D investment. These FOCs however depend on market structure (in particular on whether we are in the presence of a monopoly or a duopoly) and on commitment power (i.e.: whether a firm has commitment power or not). In this sense, it can be helpful to write down the general R&D’s FOC for the home firm:

\[
\frac{d\Pi_{i,j}}{dk_{i,j}} = \frac{\partial \Pi_{i,j}}{\partial k_{i,j}} + \frac{\partial \Pi_{i,j}}{\partial q_{i,j}} \frac{dq_{i,j}}{dk_{i,j}}
\]

A similar expression holds for the foreign firm. The first and second terms on the right hand side of equation 10 are usually called the non strategic and
the strategic motive for R&D, respectively (see Leahy and Neary, 1997). Two situations can be identified related with these two terms: first, when a firm has commitment power and has a rival firm (i.e.: the home firm in the duopoly cases); and second, when a firm has either no R&D commitment power or is a monopolist (i.e.: the foreign firm in all duopoly cases and the home and the foreign firm in the monopoly cases, respectively). When a firm has no commitment power or is a monopolist, the second term in equation 10 vanishes, i.e.: R&D investment is non strategic. If, on the contrary, a firm has commitment power and has a rival, the second term is non-zero, i.e.: R&D investment is strategic.

We then have that the foreign firm’s R&D expressions, independently of market structure, are always equal to:

$$k^*_{i,j} = \frac{\theta}{\gamma} q^*_{i,j}, \text{ for } j \neq 0 \text{ and } i = 0, E, M$$

(11)

In turn, R&D expressions for the home firm can take two forms. The first holds in the monopoly market structures where the home firm has a similar R&D expression to that of the foreign firm:

$$k_{i,0} = \frac{\theta}{\gamma} q_{i,0}, \text{ for } i = E, M$$

(12)

The second holds in the duopoly cases, where R&D by the home firm is:

$$k_{i,j} = \frac{4\theta}{3\gamma} q_{i,j}, \text{ for } i, j \neq 0$$

(13)

We can then see that in the duopoly cases, the home and the foreign firm have asymmetric incentives to invest in R&D (see equations 11 and 13). This results from the home and the foreign firm having asymmetric R&D commitment power levels. Accordingly, differences in R&D commitment power create endogenous competitiveness asymmetries between the home and the foreign firm. In particular, the home firm (the firm with higher commitment power) over-invests by a proportion of $4/3$ relatively to the foreign firm (the firm with no commitment power)\(^{11}\).

\(^{10}\)Note that the whole R&D’s FOC for the home firm is: \(\frac{\partial \Pi}{\partial k} = \frac{\partial \Pi}{\partial k} + \frac{\partial \Pi}{\partial q} \frac{dq}{dk} + \frac{\partial \Pi}{\partial q} \frac{dq^*}{dk}\). However, from the output’s FOC we have that \(\frac{\partial \Pi}{\partial q} = 0\).

\(^{11}\)This is so because in Cournot competition outputs are strategic substitutes (see Bulow et al. 1985), i.e.: if \(q^*\) increases, \(q\) decreases (and in consequence also the home firm’s profits). But since when \(k\) increases, \(q^*\) decreases, then \(\frac{\partial \Pi}{\partial q} \frac{dq^*}{dk} = \frac{\theta}{3q} > 0\), i.e.: the strategic effect of R&D is positive for the home firm.
The important thing to note is that over-investment in R&D by the home firm intends not only to discourage entry by the foreign firm (as in Fudenberg and Tirole, 1984), but also in case of entry to affect the foreign firm’s mode of entry (export versus FDI). In other words, in case the foreign firm enters the market, the home firm prefers that the foreign firm enters as a domestic firm and not as a multinational.

We can now solve for the explicit output and R&D expressions for the different market structure cases to obtain:

\[
q_{M,M} = \frac{3(D(1-\eta))}{b(9-2\eta(7-2\eta))}
\]

\[
q^*_{M,M} = \frac{D(3-4\eta)}{b(9-2\eta(7-2\eta))}
\]

\[
k_{M,M} = \frac{4\theta(1-\eta)}{b\gamma(9-2\eta(7-2\eta))}
\]

\[
k^*_{M,M} = \frac{6(D(3-4\eta))}{b\gamma(9-2\eta(7-2\eta))}
\]

\[
q_{E,E} = \frac{3((D-t)(1-\eta))}{b(9-2\eta(7-2\eta))}
\]

\[
q^*_{E,E} = \frac{(D-t)(3-4\eta)}{b(9-2\eta(7-2\eta))}
\]

\[
k_{E,E} = \frac{4\theta(D-t)(1-\eta)}{b\gamma(9-2\eta(7-2\eta))}
\]

\[
k^*_{E,E} = \frac{\theta(D(3-4\eta)+3t)}{b\gamma(9-2\eta(7-2\eta))}
\]

\[
q_{E,M} = \frac{3((D-t)(1-\eta)-t)}{b(9-2\eta(7-2\eta))}
\]

\[
q^*_{E,M} = \frac{(D-t)(3-4\eta)+3t}{b(9-2\eta(7-2\eta))}
\]

\[
k_{E,M} = \frac{4\theta(D-t)(1-\eta)-t}{b\gamma(9-2\eta(7-2\eta))}
\]

\[
k^*_{E,M} = \frac{\theta(D(3-4\eta)+3t)}{b\gamma(9-2\eta(7-2\eta))}
\]
\[ q_{M,0} = \frac{D}{b(2-\eta)} \]
\[ k_{M,0} = \frac{\theta D}{b\gamma(2-\eta)} \]
\[ q_{E,0} = \frac{D-t}{b(2-\eta)} \]
\[ k_{E,0} = \frac{\theta(D-t)}{b\gamma(2-\eta)} \] (14)

We restrict the parameter space so that trade costs do not forbid exports. If we assume otherwise, our model would be biased for the multinational strategy. It can easily be checked that in order to have trade in all market structure configurations we just need to guarantee that \( q_{M,E}^* > 0 \), or:

\[ \hat{t} < \frac{(3-4\eta)D}{2(3-2\eta)} \]
\[ 0 < \hat{\eta} < \frac{3}{4} \] (15)

The second equation above also assures that all second-order conditions (SOCs) are always satisfied (see appendix).

4 R&D, Multinationals and Competitiveness

The model in this paper can then predict asymmetric R&D patterns even with initially symmetric firms. As we mentioned above, what drives this result is differences in R&D commitment power, that endogenize competitiveness asymmetries between the home and the foreign firm.

In this section we are going to show that the firm with higher commitment power (the home firm) tends to invest more in R&D and to produce more than the firm with lower commitment power (the foreign firm). As a result the home firm also tends to be more competitive and bigger in size than the foreign firm.

To see this, note first that the home firm always has a competitiveness advantage over the foreign firm when the two firms have symmetric entry strategies (multinational and exporting duopolies) or the home firm is a multinational and the foreign firm is an exporter (see appendix):
Furthermore, even when the home firm is in disadvantage in the entry strategy (i.e.: \((E, M)\) market structure) it is possible for the home firm to be more competitive than the foreign firm if trade costs are not very high:

\[
\begin{align*}
    k_{E,M} > k_{E,M}^* \quad \text{and} \quad q_{E,M} > q_{E,M}^*, \quad \text{if} \quad t < \frac{D}{11-4\eta} \\
    k_{M,E} > k_{M,E}^* \quad \text{and} \quad q_{M,E} > q_{M,E}^*, \quad \text{if} \quad t < \frac{\eta D}{3(3-\eta)}
\end{align*}
\] (16)

Not surprising then, that a multinational firm with commitment power that faces a domestic firm with no commitment power \(((M, E)\) market structure) is more competitive and bigger in size than a multinational firm with no commitment power that faces a domestic firm with commitment power \(((E, M)\) market structure):

\[
\begin{align*}
    k_{M,E} > k_{E,M}^* \quad \text{and} \quad q_{M,E} > q_{E,M}^*
\end{align*}
\] (17)

**Proposition 1** In an international duopoly, a firm with higher commitment power tends to be more competitive than a firm with lower commitment power.

As we will see in the next section this exogenous competitiveness asymmetry property has important consequences in the entry decisions of both the home and the foreign firm.

## 5 Equilibrium Market Structure

To study the entry decision of the home and the foreign firm we have first to define firms’ profits under the different market structure configurations. We start with the monopoly cases since a monopolist always has the same profits independently of R&D commitment power:

\[
\begin{align*}
    \Pi_{M,0} &= \Pi_{0,M}^* = \frac{(2-\eta)D^2}{2b(2-\eta)^2} - \rho G \\
    \Pi_{E,0} &= \Pi_{0,E}^* = \frac{(2-\eta)(D-t)^2}{2b(2-\eta)^2} - G
\end{align*}
\] (19)
In the duopoly cases, in turn, the home and the foreign firm will have different profit levels, since differences in R&D commitment power make them endogenously asymmetric:

\[
\begin{align*}
\Pi_{M,M} &= \frac{(9-8\eta)D^2(1-\eta)^2}{b(9-2(7-2\eta)^2)} - \rho G \\
\Pi_{E,E} &= \frac{(9-8\eta)(D-t)^2(1-\eta)^2}{b(9-2(7-2\eta)^2)} - G \\
\Pi_{E,M} &= \frac{(9-8\eta)(D-t)^2(1-\eta)-t}{b(9-2(7-2\eta)^2)} - G \\
\Pi_{M,E} &= \frac{(9-8\eta)(D(1-\eta)+t)^2}{b(9-2(7-2\eta)^2)} - \rho G
\end{align*}
\]

\[
\begin{align*}
\Pi^*_{M,M} &= \frac{(2-\eta)D^2(3-4\eta)^2}{2b(9-2\eta(7-2\eta)^2)} - \rho G \\
\Pi^*_{E,E} &= \frac{(2-\eta)(D-t)^2(3-4\eta)^2}{2b(9-2\eta(7-2\eta)^2)} - G \\
\Pi^*_{E,M} &= \frac{(2-\eta)((3-4\eta)(D-t)+3t)^2}{2b(9-2\eta(7-2\eta)^2)} - \rho G \\
\Pi^*_{M,E} &= \frac{(2-\eta)((3-4\eta)(D-t)-3t)^2}{2b(9-2(7-2\eta)^2)} - G
\end{align*}
\]

To derive the entry equilibrium, however, it is not sufficient to just have the profit equations. We also need to compute firms’ preferences over different market structure configurations. In fact, as shown in figure 2, the Nash solution of the entry stage is a three-by-three matrix with three strategic choices (non-entry, export and multinational) and two players (the home and the foreign firm). Then, we also have to compare for the home firm \(\Pi_{M,M}\) with \(\Pi_{E,M}\), \(\Pi_{M,E}\) with \(\Pi_{E,E}\), and \(\Pi_{M,0}\) with \(\Pi_{E,0}\); and for the foreign firm \(\Pi^*_{M,M}\) with \(\Pi^*_{E,M}\), \(\Pi^*_{M,E}\) with \(\Pi^*_{E,E}\), and \(\Pi^*_{0,M}\) with \(\Pi^*_{0,E}\). In other words, in addition to equations 19 to 20 we also need to compute:

\[
\begin{align*}
\Pi_{M,M} - \Pi_{E,M} &= (9-8\eta) t \frac{2D(2-\eta(3-\eta)-t(4-\eta(4-\eta))}{b(9-2\eta(7-2\eta)^2)} - G (\rho - 1) \\
\Pi_{M,E} - \Pi_{E,E} &= (9-8\eta) t \frac{2D(2-\eta(3-\eta)+t(4-\eta(4-\eta))}{b(9-2\eta(7-2\eta)^2)} - G (\rho - 1) \\
\Pi_{M,0} - \Pi_{E,0} &= \Pi^*_{0,M} - \Pi^*_{0,E} = t \frac{2D-\eta b}{2(2-\eta)^2} - G (\rho - 1) \\
\Pi^*_{M,M} - \Pi^*_{M,E} &= 2 (2-\eta) t \frac{(D-t)(9-4\eta(3-\eta)-2D\eta(3-2\eta))}{b(9-2\eta(7-2\eta)^2)} - G (\rho - 1) \\
\Pi^*_{E,M} - \Pi^*_{E,E} &= 2 (2-\eta) t \frac{D(9-2\eta(9-4\eta)+2\eta(3-2\eta))}{b(9-2\eta(7-2\eta)^2)} - G (\rho - 1)
\end{align*}
\]
We are now able to derive the entry equilibrium. Before that, however, we perform two comparative static exercises. First, we analyze the role of transport costs ($t$), plant-specific fixed costs ($G$) and the return on R&D ($\eta$) on the entry decision of firms. Second, we look at a hypothetical scenario where firms cannot choose between export versus multinational but only between export versus non-entry or between multinational versus non-entry. This exercise can give us some insights into the role of competitiveness asymmetries on the mode of entry chosen by firms.

5.1 Transport Costs, Plant-Specific Fixed Costs and Return on R&D

We study the role of $t$, $G$ and $\eta$ on the entry decisions of the home and the foreign firm by looking at equations 19 to 21. Four patterns arise.

First, for both the home and the foreign firm, the multinational strategy is penalized for high $G$, and the reverse for the export strategy.

Second, as shown in appendix, also for both the home and the foreign firm, the exporting strategy is penalized for high $t$, and the contrary for the multinational strategy:

$$\frac{d\Pi_{E,j}}{dt} < 0 \quad \text{and} \quad \frac{d\Pi^*_i, E}{dt} < 0 \quad \text{with} \quad i, j = 0, E, M$$

$$\frac{d\Pi_{M,j}}{dt} > 0 \quad \text{and} \quad \frac{d\Pi^*_i, M}{dt} > 0 \quad \text{with} \quad i, j = 0, E, M$$

$$\frac{d\Pi_{M,j-E,j}}{dt} > 0 \quad \text{and} \quad \frac{d\Pi^*_i, M-E_i, E}{dt} > 0 \quad \text{with} \quad i, j = 0, E, M$$

(22)

Third, in the duopoly cases, increases in $\eta$ in general penalize the foreign firm. Only in the $(E, M)$ market structure might this not be the case if $\eta$ is
sufficiently low. The contrary happens with the home firm. In general, the home firm benefits from increases in $\eta$. The only exception is the $(E, M)$ market structure, where for sufficiently low $\eta$, profits by the home firm may decrease with $\eta$ (see appendix):

$$
\frac{d\Pi_{M,M}}{d\eta} > 0, \frac{d\Pi_{E,E}}{d\eta} > 0 \text{ and } \frac{d\Pi_{M,E}}{d\eta} > 0
$$

$$
\frac{d\Pi_{E,M}}{d\eta} > 0 \text{ for high } \eta, \frac{d\Pi_{E,M}}{d\eta} < 0 \text{ for low } \eta
$$

$$
\frac{d\Pi_{M,M}^*}{d\eta} < 0, \frac{d\Pi_{E,E}^*}{d\eta} < 0 \text{ and } \frac{d\Pi_{M,E}^*}{d\eta} < 0
$$

$$
\frac{d\Pi_{E,M}^*}{d\eta} < 0 \text{ for high } \eta, \frac{d\Pi_{E,M}^*}{d\eta} > 0 \text{ for low } \eta
$$

(23)

In turn, for the monopoly cases, profits always increase with $\eta$ for both the home and the foreign firm (see appendix).

Fourth, in the duopoly cases, for the home firm increases in $\eta$ promote the multinational strategy relatively to the exporting one. For the foreign firm, in general the opposite occurs except when $\eta$ is sufficiently low (see appendix):

$$
\frac{d\Pi_{M,j-E,j}}{d\eta} > 0 \text{ with } i, j = E, M
$$

$$
\frac{d\Pi_{M-E,i,E}^*}{d\eta} < 0 \text{ for high } \eta, \frac{d\Pi_{M-E,i,E}^*}{d\eta} > 0 \text{ for low } \eta, \text{ with } i, j = E, M
$$

(24)

In the monopoly cases, an increase in $\eta$ always promotes the multinational strategy relatively to the exporting one.

**Proposition 2** In an international duopoly, higher $t$ and lower $G$ promote the multinational strategy over the export one. In turn, for the firm with higher commitment power, higher $\eta$ tends to promote: international activity (export and multinational) and favour the multinational strategy over the export one. The reverse happens for the firm with lower commitment power.

We can then see that while $G$ and $t$ work symmetrically for the home and the foreign firm, the opposite happens with the parameter $\eta$. The reason for this asymmetry in behavior is commitment power asymmetries in R&D. For high $\eta$, the firm with higher commitment power (the home firm) can more easily impose the leader advantages on the firm with lower commitment power (the foreign firm).
5.2 Export, Multinational and Non-Entry

From equations 19 to 21 it is also possible to derive the threshold levels of $G$ that make a market structure profitable. We denominate these plant-specific threshold levels as respectively: $\hat{G}_{M,0} = \hat{G}_{0,M}^*, \hat{G}_{E,0} = \hat{G}_{0,E}^*$, $\hat{G}_{M,M}^*$, $\hat{G}_{M,M}^*$, $\hat{G}_{E,E}^*$, $\hat{G}_{E,M}^*$, $\hat{G}_{E,M}^*$, $\hat{G}_{E,M}^*$, $\hat{G}_{M,M-E,M}^*$, $\hat{G}_{M,M-E,M}^*$, $\hat{G}_{M,0-E,0}^*$, $\hat{G}_{M,M-M,E}^*$ and $\hat{G}_{E,M-E,E}^*$.

If firms can only choose between exporting and non-entry, the relation between $\hat{G}_{E,0}$, $\hat{G}_{E,E}$ and $\hat{G}_{E,E}^*$ is going to be (see appendix):

$$\hat{G}_{E,0} > \hat{G}_{E,E} > \hat{G}_{E,E}^*$$  \hspace{1cm} (25)

In other words, as shown in figure 3, for $G > \hat{G}_{E,0}$ there will be no entry ($(0,0)$); if $\hat{G}_{E,E} < G < \hat{G}_{E,0}$ either the home or the foreign firm will have an export monopoly ($(0,E)$ or $(E,0)$); if $\hat{G}_{E,E}^* < G < \hat{G}_{E,E}$ the home firm will have an export monopoly ($(E,0)$); and if $G < \hat{G}_{E,E}^*$ there will be an export duopoly ($(E,E)$).
Similarly, if firms can only choose between the multinational strategy and non-entry, the relation between $\hat{G}_{M,0}$, $\hat{G}_{M,M}$, and $\hat{G}_{M,M}^*$ is (see appendix):

$$\hat{G}_{M,0} > \hat{G}_{M,M} > \hat{G}_{M,M}^*$$ (26)

Accordingly, as shown in figure 4, for $G > \hat{G}_{M,0}$ there will be no entry $((0,0))$; if $\hat{G}_{M,M} < G < \hat{G}_{M,0}$ either the home or the foreign firm will have a multinational monopoly $((0, M)$ or $(M, 0))$; if $\hat{G}_{M,M}^* < G < \hat{G}_{M,M}$ the home firm will have a multinational monopoly $((M, 0))$; and if $G < \hat{G}_{M,M}^*$ there will be a multinational duopoly $((M, M))$.

**Proposition 3** In an international duopoly, the decision between export versus non-entry and multinational versus non-entry depends on R&D commitment power. Sufficiently low $G$ promotes symmetric duopoly equilibriums, medium-low $G$ promotes multinational or exporting monopolies by the firm with higher commitment power (home firm), medium-high $G$ promotes multinational or exporting monopolies by either the home or the foreign firm, while very high $G$ promotes non-entry.
In the previous exercise, there are two equilibriums that make our model more directly comparable with the one by Helpman et al. (2004): \((E, 0)\) and \((M, 0)\). Accordingly, in these two equilibriums only the more competitive firm (the home firm) enters the market, while the less competitive one (the foreign firm) is not able to face competition. As such, like in Helpman et al. (2004) we can also show that depending on market conditions (i.e.: \(G\)) the more competitive firms have more chances to become exporters or multinationals.

5.3 Entry Equilibrium

The entry equilibrium in our model is, however, more complicated than merely the decision between export versus non-entry and multinational versus non-entry. We also need to consider the multinational versus the exporting strategy. Using the relations in the entry profit matrix (figure 2) it is possible to construct the entry equilibrium in the \((G, t)\) space as shown in figures 5, 6 and 7. Figure 5 arises for high values of \(\eta\), while figures 6 and 7 arise for low \(\eta\) (figure 7 shows a detail of figure 6)\(^\text{12}\).

As can be seen from figures 5 to 7, the solution of the entry stage shares some similarities with other FDI models, such as Horstmann and Markusen (1992) and Petit and Sanna-Randaccio (2000). In the first place, due to the modeling strategy adopted, our model also displays the proximity concentration trade-off: for high trade costs and low plant specific fixed costs the multinational strategy is preferred; while for high plant specific fixed costs and low trade costs the exporting strategy is favored.

There are also some differences, however. First, our model predicts three market structure equilibriums not possible in either Horstmann and Markusen (1992) or Petit and Sanna-Randaccio (2000): \((E, 0)\), \((M, 0)\) and \((M, E)\). Contrary to what happens with all market structure equilibriums in Horstmann and Markusen (1992) and Petit and Sanna-Randaccio (2000), the previously mentioned equilibriums are single Nash equilibriums\(^\text{13}\). As such,

\(^{12}\)Figures 5, 6 and 7 depict the following profit curves (colors indicated in parenthesis): \(\Pi_{M,M}\) and \(\Pi^*_M\) (black), \(\Pi_{E,E}\) and \(\Pi^*_E\) (blue), \(\Pi_{E,M}\) and \(\Pi^*_{E,M}\) (brown), \(\Pi_{M,E}\) and \(\Pi^*_{M,E}\) (cyan), \(\Pi_{M,0}\) (orange), \(\Pi_{E,0}\) (dark green), \(\Pi_{M,M} - \Pi_{E,M}\) and \(\Pi^*_{M,M} - \Pi^*_{E,M}\) (magenta), \(\Pi_{M,E} - \Pi_{E,E}\) and \(\Pi^*_{E,M} - \Pi^*_{E,E}\) (light green), \(\Pi_{M,0} - \Pi_{E,0}\) (red). The home firm’s profits curves are represented by solid lines while the foreign ones by dash lines. Figure 5 is constructed with \(D = 20\), \(b = 1\), \(\rho = 2\) and \(\eta = 0.2\); and figures 6 and 7 with \(D = 20\), \(b = 1\), \(\rho = 2\) and \(\eta = 0.1\).

\(^{13}\)Horstmann and Markusen (1992) predict the following market structures in equilib-
asymmetries on commitment power reduce the indeterminacy in the solution of the market structure equilibrium.

In addition, the market structure $(E, M)$, which only arises in Petit and Sanna-Randaccio (2000) together with the $(M, E)$ market structure, only emerges in our model for low values of $\eta$ (see figures 6 and 7). Then, a multinational firm that has no commitment power finds it difficult to compete successfully with a domestic firm with commitment power. This shows, as already mentioned in a previous section, that the firm with higher commitment power (the home firm) uses R&D strategically to affect the entry choices of the firm with lower commitment power (the foreign firm). In particular, the home firm over-invests in R&D to restrict entry or, in case of entry, to force the foreign firm to adopt the domestic strategy.

In this sense, like in Helpman et al. (2004), here competitiveness asymmetries between firms also plays an important role in connection with firms’ entry-FDI decisions. In the previous subsection we discussed the impor-

---

19
Figure 6: Entry Equilibrium: Low $\eta$

tance of competitiveness in explaining the emergence of the market structures $(E, 0)$ and $(M, 0)$. We can also see the same effects at work in the $(M, E)$ market structure. Depending on market conditions $(G$ and $t)$, it is possible that only the more competitive firm can become a multinational while the less competitive one can only aspire to be an exporter. Then, similarly to Helpman et al. (2004), also in our paper competitiveness asymmetries between firms can separate multinationals from exporters.

The difference relative to Helpman et al. (2004) is that in the present paper asymmetries between firms are endogenous. Accordingly, for high return on R&D (figure 5), the firm with higher commitment power can more easily impose the R&D leader advantage. In turn, for low return on R&D (figures 6 and 7) the firm with lower commitment power can more easily face the firm with higher commitment power. As a result, when the return on R&D $(\eta)$ is very low, new mixed equilibriums arise: $(E, E)(M, 0)(0, M)$, $(E, E)(M, 0)$ and $(E, M)(M, 0)$. Of these three equilibriums only the first one has been uncovered previously by Horstmann and Markusen (1992) and Petit and Sanna-Randaccio (2000).
In our view, however, the important thing to notice is that while the proximity-concentration trade-off works symmetrically for both the home and the foreign firm, the “technological” effect (that runs through $\eta$) affects the two firms asymmetrically: high return on R&D favours the firm with higher commitment power firm (the home firm) more than the firm with lower commitment power (the foreign firm), and the reverse when the return on R&D is very low. In this sense, this paper then introduces a new FDI determinant: strategic international technological competitiveness.

6 Conclusion

In this paper we have studied the interrelation between FDI, R&D and endogenous asymmetries between firms. We have introduced endogenous competitiveness asymmetries by assuming that firms differ in R&D commitment power. Accordingly, we show that a firm with higher commitment power tends to invest more in R&D, and therefore, tends to be more competitive, than a firm with lower commitment power. The competitiveness advantage
of the firm with higher commitment power, in turn, increases the chances of becoming a multinational. In other words, the “technological” effect can be more important than “firm size” effects for the multinational choice.

In addition, a firm with higher commitment power plays strategically against a firm with lower commitment power in order to affect the rival’s entry decision. In particular, the former over-invest in R&D in order to force the latter to not enter, or at least to relegate the competitor to the domestic strategy.

As a result, while high trade costs promote the multinational strategy and higher plant-specific fixed costs promote the exporting strategy, the return on R&D has opposite effects on the higher and on the lower R&D commitment power firms. Higher return on R&D favors the firm with higher commitment power over the firm with lower commitment power. Low return on R&D in turn mitigates the R&D leader advantages of the firm with higher commitment power. In addition, higher return on R&D promotes the firm with higher commitment power to become multinational while it promotes the firm with lower commitment power to become domestic. What this means is that although the proximity-concentration trade-off affects all firms symmetrically, the “technological” effect affects firms asymmetrically.

There are several issues that are disregarded in our paper. First we do not consider the question of the location of R&D (see for example Ekholm and Hakkala, 2007). Second, we do not take into account the welfare policies directed either to attract FDI or to promote R&D (see Sanna-Randaccio, 2002). Also, our framework should be extended to a more general context (as in Helpman, 2004). Future work should aim at incorporating these issues in the model introduced here.

A Appendix

R&D Second-Order Condition For the home firm in all the duopoly cases we obtain:

\[
\frac{d^2\Pi_{i,j}}{dk_{i,j}^2} = -\frac{\gamma(9-8\eta)}{9} < 0 \quad \text{for } i, j \neq 0
\]  

(27)

For the foreign firm in all the duopoly cases we obtain:

\[
\frac{d^2\Pi_{i,j}^*}{d(k_{i,j}^*)^2} = -\frac{(3-2\eta)}{3} < 0 \quad \text{for } i, j \neq 0
\]  

(28)
In turn, in all the monopoly cases we have:

\[
\frac{d^2\Pi_{i,j}}{dk_{i,j}^2} = \frac{d^2\Pi_{i,j}^*}{dk_{i,j}^*} = -\frac{\gamma(2-\eta)}{2} < 0 \quad \text{for } i \text{ or } j = 0
\]  

(29)

Then, the most restricted SOC is \(0 < \eta < \frac{9}{8}\).

**Proof of Proposition 1**  
The following relations hold as long as \(0 < \eta < \frac{3}{4}\):

\[
egin{align*}
k_{M,M}^* - k_{M,M}^* &= \frac{\theta D}{b(9-2\eta(7-2\eta))} > 0 \\
k_{E,E}^* - k_{E,E}^* &= \frac{\theta(D-t)}{b(9-2\eta(7-2\eta))} > 0 \\
k_{M,E}^* - k_{M,E}^* &= \frac{\theta(D+t)(10-4\eta)}{b(9-2\eta(7-2\eta))} > 0
\end{align*}
\]

(30)

Then, the following relations also hold if and only if:

\[
\begin{align*}
k_{E,M}^* - k_{E,M}^* &= \frac{\theta(D-t)(11-4\eta)}{b(9-2\eta(7-2\eta))} > 0 \text{ iff } t < \frac{D}{11-4\eta} \\
q_{E,M}^* - q_{E,M}^* &= \frac{D\eta-t(9-3\eta)}{b(9-2\eta(7-2\eta))} > 0 \text{ iff } t < \frac{\eta D}{3(3-\eta)}
\end{align*}
\]  

(31)

**Proof of Proposition 2**

**Profits versus trade costs**  
For multinationals:
\[
\frac{d\Pi_{M,0}}{dt} = \frac{d\Pi_{M,M}}{dt} = \frac{d\Pi_{M,E}}{dt} = 0
\]
\[
\frac{d\Pi_{M,E}}{dt} = \frac{2(9-8\eta)(D(1-\eta)+t)}{b(9-2\eta(7-2\eta))^2} > 0
\]
\[
\frac{d\Pi_{E,M}}{dt} = \frac{3(2-\eta)(D(3-4\eta)+3t)}{b(9-2\eta(7-2\eta))^2} > 0
\]

(32)

For exporters:

\[
\frac{d\Pi_{E,0}}{dt} = -\frac{D-t}{b(2-\eta)} < 0
\]
\[
\frac{d\Pi_{E,E}}{dt} = -\frac{2(9-8\eta)(D-t)(1-\eta)^2}{b(9-2\eta(7-2\eta))^2} < 0
\]
\[
\frac{d\Pi_{E,M}}{dt} = -\frac{2(9-8\eta)(D-t)(1-\eta-t)(2-\eta)}{b(9-2\eta(7-2\eta))^2} < 0
\]
\[
\frac{d\Pi_{M,E}}{dt} = -\frac{(2-\eta)(D-t)(3-4\eta)^2}{b(9-2\eta(7-2\eta))^2} < 0
\]

(33)

For multinational versus exporting strategy:

\[
\frac{d\Pi_{M,M}-\Pi_{E,M}}{dt} = \frac{2(9-8\eta)(D(2-\eta(3-\eta))-t(4-\eta(4-\eta)))}{b(9-2\eta(7-2\eta))^2} > 0
\]
\[
\frac{d\Pi_{M,E}-\Pi_{E,E}}{dt} = \frac{2(9-8\eta)(D(2-\eta(3-\eta))+(\eta(2-\eta))}{b(9-2\eta(7-2\eta))^2} > 0
\]
\[
\frac{d\Pi_{M,0}-\Pi_{E,0}}{dt} = \frac{D-t}{b(2-\eta)} > 0
\]
\[
\frac{d\Pi_{M,M}-\Pi_{M,E}}{dt} = \frac{2(2-\eta)(D(9-2\eta(9-4\eta))-2t(9-4\eta(3-\eta)))}{b(9-2\eta(7-2\eta))^2} > 0
\]
\[
\frac{d\Pi_{E,E}-\Pi_{E,0}}{dt} = \frac{2(2-\eta)(D(9-2\eta(9-4\eta))+4t(3-2\eta))}{b(9-14\eta+4\eta^2)^2} > 0
\]

(34)

**Profits versus Return on R&D**  For the monopoly cases:

\[
\frac{d\Pi_{M,0}}{d\eta} = \frac{D^2}{(2-\eta)b^2} > 0
\]
\[
\frac{d\Pi_{E,0}}{d\eta} = \frac{(D-t)^2}{2(2-\eta)^2b} > 0
\]

(35)

For the home firm:
For the foreign firm:

\[
\frac{d\Pi_{M,M}}{d\eta} = \frac{D^2(1-\eta)(9-4\eta(5-\eta(7-4\eta)))}{b(9-2\eta(7-2\eta))^3} > 0
\]

\[
\frac{d\Pi_{E,E}}{d\eta} = \frac{2(D-t)^2(1-\eta)(9-4\eta(5-\eta(7-4\eta)))}{b(9-2\eta(7-2\eta))^3} > 0
\]

\[
\frac{d\Pi_{M,E}}{d\eta} = \frac{(D(1-\eta)+t)(D(9-\eta(20-4\eta(7-4\eta)))+t(90-16\eta(8-3\eta)))}{b(9-2\eta(7-2\eta))^3} > 0
\]

\[
\frac{d\Pi_{E,M}}{d\eta} = \frac{2((D-t)(1-\eta)-t)(D(9-4\eta(5-\eta(7-4\eta)))-(99-4\eta(37-\eta(19-4\eta))))}{b(9-2\eta(7-2\eta))^3} \geq 0
\]

\[
< 0 \text{ for low } \eta \text{ and } > 0 \text{ for high } \eta \tag{36}
\]

For the home firm, multinational versus exporting strategy:

\[
\frac{d(\Pi_{M,M}-\Pi_{E,M})}{d\eta} = 2t \frac{D(117-4\eta(74-\eta(75-2\eta(19-4\eta))))-t(19-\eta(395-4\eta(75-\eta(27-4\eta))))}{b(9-2\eta(7-2\eta))^3} > 0
\]

\[
\frac{d(\Pi_{E,M}-\Pi_{E,E})}{d\eta} = 2t \frac{D(117-4\eta(74-\eta(75-2\eta(19-4\eta))))+t(81-\eta(99-4\eta^2(11-4\eta)))}{b(9-2\eta(7-2\eta))^3} > 0
\]

\[
\frac{d(\Pi_{M,E}-\Pi_{E,0})}{d\eta} = t \frac{2D-t}{2(2-\eta)^2}b > 0 \tag{38}
\]

For the foreign firm, multinational versus exporting strategy:

\[
\frac{d(\Pi_{M,M}-\Pi_{M,E})}{d\eta} = 2t \frac{D(99-2\eta(153-2\eta(81-8\eta(5-\eta))))-t(207-2\eta(195-4\eta(36-\eta(13-2\eta))))}{b(9-2\eta(7-2\eta))^3} \leq 0
\]

\[
\frac{d(\Pi_{E,M}-\Pi_{E,E})}{d\eta} = 2t \frac{D(99-2\eta(153-2\eta(81-8\eta(5-\eta))))+4t(27-\eta(21+\eta(9-2\eta(7-2\eta))))}{b(9-2\eta(7-2\eta))^3} \leq 0
\]

\[
> 0 \text{ for low } \eta \text{ and } < 0 \text{ for high } \eta \tag{39}
\]
Proof of Proposition 3  Export versus non-entry:

\[
\hat{G}_{E,0} - \hat{G}_{E,E} = \frac{(D-t)^2(45-2\eta)(58-15\eta)}{2(2-\eta)b(9-2\eta(7-2\eta))^2} > 0
\]

\[
\hat{G}_{E,E} - \hat{G}_{E,E}^* = \frac{(5-6\eta)(D-t)^2\eta}{2b(9-2\eta(7-2\eta))^2} > 0
\] (40)

Multinational versus non-entry:

\[
\hat{G}_{M,0} - \hat{G}_{M,M} = \frac{D^2(45-2\eta)(58-15\eta)}{2\rho b(2-\eta)(9-2\eta(7-2\eta))^2} > 0
\]

\[
\hat{G}_{M,M} - \hat{G}_{M,M}^* = \frac{(5-6\eta)D^2\eta}{2\rho b(9-2\eta(7-2\eta))^2} > 0
\] (41)

References


