Private Labels, Price Rivalry, and Public Policy

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Abstract
The article examines how the existence of a retailer owned brand, private label, affects the price setting of a national brand. We find that the potential for a private label introduction may lead to price concessions from the national brand producer, but that actual private label introduction as such may very well lead to higher retail prices on national brands. We argue that this may have important implications for the interpretation of empirical results and the public policy towards national brands.

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1 Introduction

Retailer owned brands in the grocery sector, often denoted private labels (or simply store brands), have had an enormous growth in the last decades in many countries and many product categories (Dobson, 1998; Connor et al., 1996). However, only recently the academic literature has begun to examine the impact of private labels. Most of the literature, though, are primarily empirical studies trying to explain the variation in private labels penetration across product categories (e.g. Sethuraman, 1992; Hoch and Banerji, 1993). The purpose of this paper is to add to the existing theory of private labels. In particular, we focus on the intra-category rivalry between a national brand and a - potential or active - private label.

Results from empirical studies indicate that some national brands use either brand proliferation (Putsis, 1997) or advertising (Cotterill et al, 2000; Parker and Kim, 1995) as a strategy to meet the challenge from private labels, and that a private label’s quality can be crucial for its success in terms of market share (Hoch and Banerji, 1993). Strategies towards brand proliferation, advertising and quality is not an issue in this paper. Instead we are interested in how the threat from private label introduction, and its actual introduction, will affect prices on national and private labels. The results from the empirical literature are mixed. Putsis (1997) finds that private label introduction lowers the average price of national brands, while Parker and Kim (1995) find that private label introduction can increase prices on national brands. Cotterill et al (2000) find that in some product categories an increase in private label distribution has the effect of increasing the prices of national brands, while the opposite is true in other product categories.

A natural response to the observed ambiguity concerning rivalry on prices, is to step back and start by examining what theory predicts. In a theoretical model, Mills (1995) predicts that the introduction of a private label results in lower prices. The reason is that a private label by definition eliminates the double marginalization problem in a distribution channel. While the price of the private label is set only once, by the retailer, the price of a national brand is set first by the brand manufacturer and then by the retailer. Narasimhan and Wilcox (1998) find that the introduction of a private label triggers a battle over market shares which results in lower wholesale price from manufacturers of national brands.\footnote{Raju et al (1995) are also investigating the effect of the introduction of a private label. However, their main focus is on how the introduction of a private label affects a retailer’s profits and how different factors affect the private label’s market share. Comparative statics concerning how the introduction of a private label affects prices on national brands is not reported.} However, in their model private
label introduction has no effect on the retail price of the national brand.

Our theoretical model is much in the same spirit as Narasimhan and Wilcox (1998).\textsuperscript{2} In line with them we distinguish between what we call loyal and switching consumers. Only the latter group of consumers considers to switch from buying a national brand to buying a private label. Contrary to Narasimhan and Wilcox (1998), in our model the switching consumers have price elastic demand. It turns out that this has important implications for the retail price of the national brand. In addition, we allow the manufacturer of the national brand to condition his wholesale price on whether a private brand is introduced or not.\textsuperscript{3} In Narasimhan and Wilcox (1998) the national brand producer has not the option to offer an exclusivity contract.\textsuperscript{4} Therefore, our focus is distinctly different from theirs. We examine whether the retailer will introduce a private label, and, if it does, how the wholesale price and thereby the retail price of the national brand is affected. Since the retail price of the national brand is unaffected by the private label in Narasimhan and Wilcox (1998), their main concern is how a private label affects the national brand producer’s wholesale price. Moreover, they are not focusing on the question whether a private label will be introduced or not, but rather on the rivalry that takes place given that a private label is introduced.

We examine how the mere existence of a private label affects the equilibrium outcome. If private label entry is blockaded the national brand has a monopoly. This situation is contrasted with the possibility of introducing a private label. When private label entry is feasible, then one of three situations may emerge. First, national brand exclusivity may still arise as an equilibrium outcome and the price of the national brand will go down

\textsuperscript{2}Frank and Salkever (1992) is also a study in much the same spirit, although applied on a different industry. They analyse the effects of entry of generic drugs in the pharmaceutical industry, and show that it may lead to a price increase for brand name drugs. In contrast to our setting, they do not model the vertical relationship within the industry, and therefore they do not raise the issue whether the national brand producer can offer an exclusivity contract and thereby deter entry by the generic brand. Moreover, while they assume a Stackelberg game with sequential price setting we assume that the firms set prices simultaneously.

\textsuperscript{3}Although the producer can offer an exclusivity contract, it is common in the existing literature on vertical restraints to assume that the retailer decides which products it should carry, see e.g. Bernheim and Whinston (1998), O’Brien and Shapiro (1993,1997) and Gabrielsen and Sørgard (1999b). We follow that approach here. For more specific studies of retailer power, see e.g. Dobson and Waterson (1997, 1999) and Gabrielsen and Sørgard (1999a).

\textsuperscript{4}Note, though, that even then the retailer cannot decide to not introduce the private label. This is the case if the national producer’s wholesale price is so low that it is not profitable for the retailer to introduce the private label, even though it is allowed to do so.
compared to the monopoly outcome. The reason is that the producer of the national brand may lower its exclusive dealing wholesale price so as to make it unattractive for the retailer to introduce a private label. Second, the private label is introduced and the producer of the national brand may increase its wholesale price compared to monopoly and thereby induce an increase in the retail price of the national brand as well. The reason is that the competition for the switching consumers is harsh after a private label is introduced, and he chooses to concentrate on his loyal consumers and increases his price. Third, the private label is introduced and the price of the national brand remains unchanged. This is the case if the national producer serves only the loyal consumers under monopoly, and thus sets its wholesale price at its maximum both before and after the introduction of a private label.

We show that the number of loyal consumers relative to the number of switching consumers is crucial for the equilibrium outcome. For a relatively low number of loyal consumers, the switching consumers are of relative large importance to the manufacturer of the national brand. In that case the threat of private label introduction will induce the producer of the national brand to reduce its wholesale price and offer the retailer exclusivity. For an intermediate number of loyal consumers the national brand producer serves also switching consumers in the absence of a private label. Under the threat of private label the producer of the national brand is better at serving only its loyal consumers. It therefore increases its wholesale price and stops serving the switching consumers allowing the retailer to introduce the private brand and thereby serve the switching consumers. When the number of consumers that are loyal to the national brand is large enough, the price of the national brand will be high absent the private label and it will remain so even if a private label is introduced.

We also examine how the existence of a private label affects consumer surplus and welfare. Compared to a benchmark situation where private labels are non-existing (national brand monopoly), we show that their mere existence is beneficial for both consumers and society. However, the relevant comparison from a public policy perspective should be between potential and actual private label introduction. The relevant public policy question is whether the manufacturer of the national brand imposes national brand exclusivity in cases where the consumers and society as a whole would prefer exclusivity. The answer to this is that we may have too much exclusivity from the viewpoint of the consumers and society, but the opposite may also be true. Whether or not we will have too much or too little national brand exclusivity depends on the productions costs of private labels and whether or not consumers incur costs when switching from a national brand to a private
brand. If private label production is inefficient there is a tendency towards too much exclusivity from the consumers’ and society’s point of view, and if consumers have switching costs the opposite may be true.

The article is organized as follows. In Section 2 we formulate our model, and present the benchmark with national brand monopoly. In Section 3 we report results for the case of cost differences between the national brand and the private label, while we in Section 4 report results for the case of consumer switching costs. Our results are summarized in Section 5, where we also discuss some implications for empirical testing and for public policy towards the manufacturer of the national brand.

2 Some preliminaries

We consider a situation where a producer of a national brand sells its brand through a single retailer. The retailer may distribute the national brand exclusively but may also introduce its own private label. Initially we consider the equilibrium outcome when there is no threat from a private label. If so, the national brand manufacturer has a monopoly, and we denote this case with subscripts \( m \). Thereafter we allow for the introduction of a private label. The potential introduction of a private label may affect the pricing policy of the national brand manufacturer. If the private label is not introduced we denote this by subscripts \( e \) (for national brand exclusivity). Finally, if the private label is actually introduced and distributed alongside with the national brand, we denote this case with subscripts \( c \) (for common distribution). Let \( r \) be the retail price of the private label, and \( p_i; w_i; S_i \) and \( W_i \), \( i \in \{m,e,c\} \) denote the retail price of the national brand, the wholesale price of the national brand, consumers’ surplus and welfare in the three cases.

The production costs for the national label is normalized to zero, and we assume that the private label can be procured at a constant marginal cost \( c \geq 0 \). The demand for the national brand consists of two types of consumers.

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\(^5\)This assumption deserves some attention. The normal case would be that private labels are procured by the retailers at lower costs than national brands. This is a feature that is captured in our model even if the private brand has higher marginal production cost than the national brand. The reason for this is that the price-cost margin charged by national brand producers often exceeds the marginal production cost of the private label. In fact, in equilibrium we ... that this is true. Furthermore, when allowing the private brand to have higher marginal production costs than the national brand, we ...nd this realistic for two reasons. First, private brands often are imported goods and for that reason they incur trade costs, for instance transportation costs. Second, national brands may be able to exploit economies of scale as they per definition has larger sales as they are sold in more retail outlets.
A subset of the consumers are loyal consumers. These consumers purchase a fixed quantity of the national brand provided that the price is below a choke price $p_1$; and will never consider to purchase the private label. A second set of consumers are potential switchers - denoted by switching consumers. These consumers have a price elastic demand $q = (1 - p)^\gamma$. The present model makes two different assumptions regarding the switching consumers. First, in Section 3 we consider the case where the switching consumers regard the national and private label as perfect substitutes. In this case the introduction of a private label at a price lower than the price of the national brand will attract all switching consumers that have willingness to pay higher than or equal to the price of the private label. Second, in Section 4 we consider the case when the switching consumers incur costs switching from the national brand to the private label (switching costs). If so, only the switching consumers with high enough willingness to pay and low enough switching costs will buy the private label.

Before this, we consider our benchmark case when the national brand producer is an unthreatened monopolist. Aggregate demand for the national brand then is

$$q_m = \frac{1}{2} \cdot \gamma + (1 - p_m) \cdot \gamma$$

where $\gamma$, 0 is the number of loyal consumers and the parameter $\gamma$, 0 scales up and down the number of switching consumers. Only the relative size between $\gamma$ and $\gamma$ are going to be of importance in the following. We therefore normalize the demand system above by defining $1 = \gamma$ and setting $\gamma = 1$: The interpretation of a large 1 is that there are many loyal consumers relative to switching consumers and vice versa when 1 is small. The profit of the retailer (r) is written:

$$r_m = (p_m \cdot w_m) \cdot (1 + (1 - p_m))$$

and the profit of the national brand producer (n) is given by:

$$n_m = w_m \cdot (1 + (1 - p_m))$$

The following proposition depicts the equilibrium outcomes, consumers' surplus (S) and welfare (W) for different $\gamma$ assuming national brand monopoly:

**Proposition 1 (National brand monopoly).** There exists a number $1^M$ such that if $1^2 < 0; 1^M, w_m = \frac{1 + 1}{2} < 1; p_m = \frac{3(1 + 1)}{4} < 1; S_m = \frac{1}{2}(5^1 + 1)(1 - 3^1) \cdot W_m = \frac{1}{2}(49 \cdot 3^1 + 14^1)$. Otherwise, $w_m = 1; p_m = 1; S_m = 0$ and $W_m = 1$.
Proof. See the appendix. ■

For the national brand monopolist there is a trade-off between exploiting loyal consumers by charging a high price and selling to switching consumers at a lower price. When the number of loyal relative to switching consumers is high, the monopolist tends towards exploitation of loyal consumers. It then sets its wholesale price at its maximum, and serves the loyal consumers exclusively. When the number of switching consumers relative to loyal consumers is high, it may be worthwhile to sell to both types of consumers. It then sets a lower wholesale price and serves both groups. This explains why the manufacturer sells to both groups of consumers when \( \frac{1}{3} < \frac{1}{2} \).

Now we can contrast our model with Narasimhan and Wilcox (1998). In their model, where demand is unaffected by price (rectangular demand curve), a national brand monopolist has no reason to lower its wholesale price below the consumers' reservation price. In contrast, in our setting it can be profitable for the manufacturer to attract switching consumers by setting its wholesale price below the loyal consumers' reservation price and thereby encourage the retailer to set a price below the loyal consumers' reservation price.

3 Private label and cost asymmetries

We now allow for the possibility that the retailer can introduce a private label at marginal cost \( c < 0 \). Even if the retailer has this possibility it may still choose to grant exclusivity to the national brand. Alternatively, it introduces a private label and distributes it alongside with the national brand. In this section we assume that if a private label is introduced, the switching consumers are indifferent between the two products. Furthermore, we allow the manufacturer of the national brand to condition its wholesale price on whether a private label is introduced or not. Let \( w_e \) denote the producer's wholesale price given that the retailer does not introduce the private label, while \( w_c \) denotes the wholesale price under private label introduction. We study the following simple game:

Stage 1: The national brand producer offers wholesale prices \( w_e \) and \( w_c \).
Stage 2: The retailer introduces a private brand or not, and sets retail price(s). If the retailer sells the national brand exclusively, for the given \( w_e \):

\[6\text{Formally, we may also have that the retailer excludes the national brand when introducing a private label. However, in the present model the loyal consumers would never consider buying the private label anyway, so we need not consider this option in our model.}\]

\[7\text{This may seem as a strong assumption in many circumstances. The next subsection relaxes this assumption.}\]
It sets $p_e$. If it introduces a private label, for given $w_c$ and $c$, it sets $p_c$ and $r$:

If the retailer does not introduce a private label its profit is written:

$$\pi_c = (p_e \cdot w_e)(1 + (1 - p_e));$$

(3)

and if it introduces the private label its profit is:

$$\pi_r = (p_c \cdot w_c)q_c + (r \cdot c)q_r;$$

(4)

where $q_c$ and $q_r$ are the quantities sold of the national and private brand, respectively. Then we have the following result:

**Proposition 2** (Equilibrium outcome). For $c \in [0; 1)$ there exist a number $\xi N(c)$ such that if $\xi > 0$; $\xi N(c)$; $q_r = 0$ and $w_e = 1 + c < 1$ and $p_e = \frac{2^\xi + (1+c)}{2} < 1$. Otherwise, $q_r > 0$ and $w_c = 1; p_c = 1$ and $r = \frac{1+c}{2}$. Furthermore, $\xi N(c) < \xi M$.

**Proof.** See the appendix.

As was the case under national brand monopoly, the producer of the national brand serves only its loyal consumers if the number of loyal consumers is relatively high ($\xi$ is sufficiently large). In addition, we see that the private label’s unit cost matters. When the production cost of the private label increases, the attractiveness for the retailer of introducing a private label is reduced. Therefore, for a given number of loyal versus switching consumers the national brand producer can increase its wholesale price and still enjoy exclusivity of his brand in the retail store. This is demonstrated in Proposition 2 by the fact that both $w_e$ and $p_e$ increase in $c$.

We also see that the critical $\xi$ to induce a change in pricing strategy from the part of the national brand producer is lower under the threat of private labels than without such a threat. This implies that under the threat of a private label a lower number of loyal consumers is needed for the manufacturer of the national brand to give up exclusivity by setting a high price and only sell to its loyal consumers. The reason for this is that under the threat of private label introduction the national brand producer is forced to price concessions. Consequently, the benefits from including the switching consumers are faster eroded for a threatened national brand producer as $\xi$ increases.

**Corollary 1** (Blocked entry) If $c > \frac{1 - \xi}{2}$, the introduction of a private label will never occur and its existence will not affect the pricing policy of the national brand producer.
Proof. See the appendix. ■

Now we are ready to investigate the effect of the potential for a private label. From now on we will restrict attention to the case when entry is not blockaded, i.e., $c < c$. By comparing Propositions 1 and 2, we have the following result:

**Proposition 3 (Outcome comparison).**

1. If $\xi_2 \leq 0$, $\eta N(c) \leq q$, $p_e < p_m < 1$ and $w_e < w_m < 1$. If $\xi_2 > 0$, $p_m < p_c = 1$ and $w_m < w_c = 1$. If $\xi_2 > 0$, $p_c = p_m = 1$ and $w_c = w_m = 1$.

2. The existence of a private label, even if it is not introduced by the retailer, will always increase the consumers’ surplus and welfare compared to national brand monopoly.

Proof. See the appendix. ■

The essence of Proposition 3 is illustrated in Figure 1, where we have set $c = 1.5$.

![Graph illustrating the retail price of the national brand with and without private label introduction.](image)

The solid line in Figure 1 illustrates the retail price of the national brand when the producer is a monopolist, while the dotted line illustrates the retail price of the national brand under the threat of a private label.

We see from Figure 1 that there are three regimes. For low $\xi$, the national brand has exclusivity. In this case the private label threat results in a lower price of the national brand compared to the monopoly case. The reason is that the number of loyal relative to switching consumers is low, and the producer is willing to lower its wholesale price to prevent the introduction of
a private label. He will continue to serve the switching consumers despite the loss it causes on the existing sale to the loyal consumers. However, the larger the number of loyal relative to switching consumers is, the larger the loss from such a strategy. Therefore, for intermediate values of $^1$ the producer decides not to serve the switching consumers, but instead concentrate on the loyal ones, and the private label is introduced. In this case the wholesale price, as well as the final price, of the national brand increases to unity as a response to the existence of a private label. This will hurt the loyal consumers as the price will be above the price resulting from national brand monopoly. For sufficiently high $^1$, the producer would choose to serve the loyal consumers exclusively even without the threat of a private label. Then the existence of a private label has no effect on neither the wholesale nor the retail price of the national brand, despite the fact that the retailer introduces the private label.

Compared with national brand monopoly, both the threat and the actual introduction of private labels always improve both consumers' surplus and welfare in this model. In relation to Figure 1 this is easy to understand for low and high values of $^1$: For low values the threat of a private label lowers wholesale and retail price while preserving exclusivity of the national brand, which must increase welfare and consumers' surplus. For high levels of $^1$ a private label is introduced without affecting the price of the national brand, and the gain in consumers' surplus and welfare stems from the sale of the private label to the switching consumers. In the intermediate case there are two effects. First, the introduction of a private label induces an increase in the price of a national brand. The price increase for the national brand is negative for welfare, but the introduction of the private label is positive. As it turns out, the latter effect dominates and both the consumers and the society at large are better off with private label introduction.

The comparison with national brand monopoly is valuable as a benchmark, but it is not particularly relevant when it comes to public policy issues. A much more interesting and relevant question for policy is the following: Given that private labels can be introduced, will they be introduced when they should? Are they sometimes introduced when they should not? In other words: are the private incentives to introduce private labels in line with the social ones? To answer these kinds of questions we must compare consumers' surplus and welfare under the threat of introduction and actual introduction of private labels. To this purpose define $^1^{W(c)}$ as the critical $^1$ above which welfare under private label introduction is higher than under national brand exclusivity. In the same manner define $^1^{S(c)}$ as the critical $^1$ above which consumers' surplus under private label introduction is higher than under national brand exclusivity.
Then we can show:

**Proposition 4** (Public policy). \(0 = 1^S(c) < 1^W(c) < 1^N(c)\) for \(c \in [0; \tau]\).

**Proof.** See the appendix. ■

We see from the proposition there is too much exclusivity seen from both the consumers' and the society's point of view. For low enough \(\tau\)'s the retailer accepts national brand exclusivity without taking into account that the switching consumers would have been better \(\propto\) with the introduction of a private label, supplied at a lower price than the price of the national brand.

Will consumers always prefer that private labels actually are introduced? On the one hand, private label introduction leads to higher or unchanged price of the national brand. On the other hand, the switching consumers would be better \(\propto\) with a private label at a lower price than an exclusive national brand. The statement \(1^S(c) = 0\) in Proposition 4 says that the latter effect dominates. Consumers will always be better \(\propto\) with actual introduction of a private label rather than exclusivity.

From a welfare point of view, the introduction of a private label is cost inefficient. This explains why welfare is higher under exclusivity for small values of \(\tau\). When \(\tau\) is low, the wholesale and retail price of the national brand is relatively low under exclusivity. Introducing a private label would benefit consumers on aggregate, but we would incur inefficient production enough to reduce welfare. The larger the relative number of loyal consumers \(1^L\); the higher the price of the national brand under exclusivity, and the larger is the gain to consumers from the introduction of a private label. Therefore, for sufficiently high \(\tau\) the gain to the consumers is enough to dominate the welfare loss from inefficient production.

4 **Private label and switching costs**

When consumers have switching costs and a private label is introduced, only the share of the switching consumers with low switching costs will buy the private label. Let \(i(s)\) denote the share of the switching consumers that has switching costs lower than or equal to \(s\):° If a private label is introduced, the switching consumers will choose whether to buy the private or the national label. Let \(c = p_c - r\) denote the price difference between the national and private label. In equilibrium we must have:

\[q_c = 1^L + (1^L i(c))(1^L p_c):\]

°See Klemperer (1987) for a similar modelling approach to switching costs.
The national label sells to its loyal consumers\(^1\); and the share of the switching segment whose consumers have reservation price above \(p_c\) and switching costs that are higher than the price difference between the two labels. The private label faces demand from two types of consumers. First, the private label will sell to switching consumers with reservation price above \(p_c\) and switching costs lower than the price difference. Second, the private label will sell to consumers with reservation price between \(r\) and \(p_c\), and who have reservation price minus switching costs above \(r\). Hence, demand for the private label is written:

\[
q_p = i \left( (x - r) \right) \left( 1 - \frac{p_c}{x - r} \right) + \frac{Z_{p_c}}{\mu} \int_{x=r}^{\infty} i \left( x \right) \left[ d(\left( x \right)) \right] \tag{5}
\]

To simplify, we normalize marginal costs of the private label to zero \((c = 0)\) and assume that \(s \approx U[0; L]\).

Consider the case when the retailer introduces the private label. The profit function of the retailer is written:

\[
\Pi_r = \left( p_c - w_c \right) \mu + \mu \left( 1 - \frac{p_c}{L} \right) \int_{r}^{\infty} \frac{x}{L} \mu + \mu \left( r - \frac{p_c}{L} \right) \int_{r}^{\infty} \frac{dx}{L}
\]

In a similar setting, Narasimhan and Wilcox (1998) found that different equilibria will arise for different parameter values. In some cases the national brand manufacturer lowers its wholesale price to induce the retailer to increase the retail price of the private label and thereby reduce the market share of the private label. However, for other parameter values the national brand manufacturer decides to set a high wholesale price so that it serves only the loyal consumers after the private label is introduced.\(^9\) This latter case can be an equilibrium outcome in our model as well:

\(^9\)In Table 1 in Narasimhan and Wilcox (1998), type 3 equilibrium is the one where the manufacturer of the national brand decides to set a high wholesale price and only serve the loyal consumers. We see from the parameter values defining type 3 equilibrium, that this equilibrium can be present if one of the following is true: (1) the reservation price is sufficiently high, (2) the cost of the private label is sufficiently high, or (3) the switching cost is sufficiently low.
Proposition 5 (Equilibrium with private label) If the retailer sells the private label and \[ \frac{1}{6L} + \frac{2p_3}{3L} \leq \frac{1}{3} K(L), \] then \( w_c = 1 \) and \( p_c = 1 \):

Proof. See the appendix. ■

The Proposition shows that if the number of loyal consumers are sufficiently large, then the introduction of a private label would in equilibrium imply that the retail price of the national brand is set at its maximum. This can be seen by considering the national producer's decision problem. If there are many loyal consumers, the national producer would respond to the introduction of a private label by setting his wholesale price as high as possible (\( w_c = 1 \)) and only serve the loyal consumers. Then, obviously, the retailer sets a high retail price (\( p_c = 1 \)).

We have chosen to focus on the case shown in Proposition 5, where \( 1 - \frac{1}{L} \leq K(L) \). In that case the profit function of the retailer reduces to

\[ r_c = r \left( \frac{1}{2} - \frac{1}{2L} \right). \]  \hfill (6)

When the retailer carries the national brand exclusively his profit is

\[ r_e = (p_e - w_e) (1 + (1 - p_e)). \]

Then we have the following result:

Proposition 6 (Equilibrium outcomes) Let us assume that \( 1 - \frac{1}{L} \leq K(L) \):

i) When \( L \geq \frac{4}{9} p_3 \); \( q_f > 0 \). ii) When \( L > \frac{4}{9} p_3 \); there exists a positive number \( N(L) \) such that for \( L > \frac{4}{9} p_3 \); \( N(L) < \frac{1}{3} L \); \( q_f = 0 \);

\[ w_e = \frac{1}{3} + \frac{2}{3} \frac{q_f}{L} < 1 \text{ and } p_e = \frac{1}{3} + \frac{2}{3} \frac{q_f}{L} < 1; \] Otherwise, \( q_f > 0 \) and \( w_e = 1 \); \( p_e = 1 \) and \( r = \frac{2}{3} \); Furthermore, \( K(L) < N(L) < M \) for \( L > \frac{4}{9} p_3 \):

Proof. See the appendix. ■

When switching costs are low enough there exist no equilibria with national brand exclusivity. With low switching costs the national brand producer must set a very low wholesale price to prevent the introduction of a private label. In this case it might be better for the manufacturer to allow the introduction of a private label and concentrate on his loyal consumers. When switching costs are high enough we have an equilibrium with national brand exclusivity.
As in the previous section the relative number between loyal and switching consumers is of importance for the equilibrium outcome. If $^1$ is low enough, exclusivity will arise and the intuition is as in the previous section. When $^1$ is high enough, the national brand producer decides not to deter the private label and concentrates on exploiting his large group of loyal consumers.

Now we are ready to investigate the effect of the potential for a private label. By comparing Propositions 1 and 6, we have the following result:

**Proposition 7** (Outcome comparison with monopoly). Assume that $L > \frac{4}{9} P$: i) If $^1 \leq \frac{2}{\omega} K (L); ^1 N (L); q = 0$, $p_e < p_m$ and $w_e < w_m$. If $^1 \geq \frac{2}{\omega} N (L); ^1 M q > 0$, $p_m < p_c = 1$ and $w_m < w_c = 1$. Otherwise, $q > 0$, $p_c = p_m = 1$ and $w_c = w_m = 1$. ii) The existence of a private label, even if it is not introduced by the retailer, will always increase the consumers' surplus and welfare compared to national brand monopoly.

**Proof.** See the appendix.

Again, we see that the national brand producer threatened to price concessions by a private label gives in earlier and starts charging high prices from loyal consumers than a pure monopolist would do. When $^1$ is small the retailer carries the national brand exclusively. Due to price concessions from the producer, both wholesale and retail prices are lower than under monopoly. When $^1$ becomes larger the national brand producer gives in and starts to exploit his loyal customers and the retailer introduces a private label. In this case both wholesale and retail prices are higher than those charged by an unthreatened monopolist. The intuition is as in the previous section. The monopolist serves both groups, and have to set a low wholesale price to attract switching consumers. When $^1$ increases further, even the monopolist reaches a point where he exploits only his loyal customers and does not sell to the switching consumers. In this area retail and wholesale prices for the national brand are the same in the two cases. However, as in the previous section the existence of a private label is always better for consumers and welfare compared to a pure national brand monopoly.

As in the previous section define $^1 W (L)$ and $^1 S (L)$ as the critical $^1$ above which welfare and consumers' surplus are higher under private label introduction than under national brand exclusivity. Then we have:

**Proposition 8** (Public policy) When $L > \frac{4}{9} P$: then $^1 K (L) < ^1 N (L) < ^1 W (L) < ^1 S (L)$.

**Proof.** See the appendix.

Note that compared to Proposition 4 the ranking of $^1 N$ and $^1 S$ have changed. When some consumers have switching costs, the private incentives
to introduce private labels are too strong both from a welfare and the consumers’ point of view. This implies that in equilibrium there can be too little exclusivity seen from the society’s and the consumers’ point of view.

As before the parameter \( \alpha \); measuring the relative number of loyal and switching consumers in the market, is crucial for the outcome. When \( \alpha \) is very low, below \( \alpha^N(L) \); the national brand is carried exclusively by the retailer and this is to the benefit of both the consumers and the society as a whole. A relatively low number of loyal consumers make the national brand producer offer a low wholesale price which translates into a relatively low retail price. As the number of loyal relative to switching consumers increases, the national brand producer gives up selling to the switching consumers, and increases its price to unity and the private label is introduced. Once \( \alpha \) is large enough to make it privately profitable for the retailer to introduce a private label, the loyal consumers have zero consumers’ surplus. The switching consumers now buy the private label, but must bear their switching costs. The switching costs alone explain why the consumers on aggregate are much less keen on private label introduction now than in the model without switching costs. When considering to increase its price to exploit the loyal consumers and thereby trigger the introduction of a private label, the national brand producer does not take into account the switching costs that have to be born by the switching consumers. That is why the national brand producer tend to give in too early (\( \alpha \) too low) from the consumers’ and a society’s point of view.

5 Discussion and concluding remarks

The received theoretical literature predicts that private labels will have a price-reducing effect or no effect at all on the prices on national brands. Elimination of double marginalization within a distribution system may result in lower retail prices on national brands. However, as shown by Narasimhan and Wilcox (1998), private label introduction may affect the rent distribution between a manufacturer and a retailer without affecting the retail price on the national brand to consumers. Although we do not disagree that these effects are of importance, we have pointed out that there are other mechanisms that in some product categories may reverse the competitive effect of private labels. Our argument is that the existence of a private label as such may force the manufacturer of the national brand to give price concessions so that the retailer decides to carry the national brand exclusively. If so, private label introduction may actually increase retail prices on national brands. The reason is that the manufacturer of the national brand then decides to give
up serving the price elastic consumers and rather sells to its loyal consumers at a high price.

The contrast between our results and the results in the received theoretical literature suggests that the competitive effect of private labels can be distinctly different from one product category to another. A natural response to this is to examine the results from empirical studies, to find out more about the competitive (or anticompetitive) effect of private labels. However, our study indicates that one should be careful with the interpretation of empirical results. In principle, one should distinguish between three different situations: (1) no threat of private label introduction, (2) threat of private label introduction and (3) actual introduction of private labels. Our theory predicts that the price of national brands are lower in case (2) than in case (1) and (3), if any difference at all.\footnote{Note that Narasimhan and Wilcox (1998) predicts that the price of the national brand is unchanged in all situations.} To examine the effect of actual introduction of private labels, one should compare (2) and (3). However, in data it is difficult to distinguish between (1) and (2). A natural way to test would then be to compare the combination of (1) and (2) with (3). For example, this is what Putsis (1997) has done, and he finds that the higher the market share of private labels the lower the price of the national brands. However, such a finding is not inconsistent with our theory, that the actual introduction of private labels may result in higher prices on national brands. We simply do not know whether a low market share of private labels is due to the fact that there is no serious threat from potential private labels [case (1)] or is due to aggressive price setting by the manufacturer of the national brand [case (2)]. A more natural empirical test would be to disaggregate data - if possible - and examine how the manufacturer of a national brand responds to an actual introduction of a private label. Parker and Kim (1995) explore this issue, and they find that actual introduction results in higher prices on national brands. However, their explanation is that this is due to heavy advertising and/or tacit collusion.

Any possible price-increasing effect of private labels raises questions whether consumers and society as a whole is better off by the introduction of private labels. The received theoretical literature compare a national brand monopoly with a situation where a private label is introduced. However, we have argued that the relevant comparison would be between the case where a manufacturer of a national brand offers the retailer exclusivity and the case where it does not offer exclusivity and allow the retailer to introduce a private label. We find that in some cases there is too little exclusivity. The manufacturer decides not to offer exclusivity when both consumers and so-
ciety as a whole would prefer such an outcome. The driving force is that the consumers incur switching costs when a private label is introduced, a cost not taken into account by the manufacturer of the national brand. Note that our model may even underestimate the potential welfare loss from the introduction of private labels, because we have not taken into account any possible dead weight loss associated with a higher price on the national brand. Our results therefore suggest that one should be careful in implementing any restrictions on national brand manufacturers exclusivity clauses with retailers. The consumers and the society as a whole may be better off with price concessions from the national brand producer than with an actual introduction of a private label.

6 Appendix

Proof of Proposition 1:

Maximizing (1) with respect to \( p_m \) yields the first-order condition:

\[
\frac{1}{2} + 1 + 2p_m + w_m = 0
\]

and the price

\[
p_m = \frac{1}{2} (\frac{1}{2} + 1 + w_m)
\]  \( \text{(7)} \)

The producer maximizes (2) with respect to \( w_m \) yielding the first-order condition

\[
\frac{1}{2} \frac{1}{2} + \frac{1}{2} w_m = 0
\]

and the equilibrium wholesale price

\[
w_m = \frac{1}{2} \left( \frac{1}{2} + 1 \right) \text{ if } 0 \leq \frac{1}{2} \leq 1
\]

\[
w_m = \frac{1}{2} \left( \frac{1}{2} + 1 \right) \text{ if } \frac{1}{2} > 1
\]

The equilibrium retail price is

\[
p_m = \frac{3}{4} \left( \frac{1}{2} + 1 \right) \text{ if } 0 \leq \frac{1}{2} \leq 1
\]

\[
p_m = \frac{3}{4} \left( \frac{1}{2} + 1 \right) \text{ if } \frac{1}{2} > 1
\]

Hence, depending on the parameters we have two different equilibria. Type I: \( \frac{1}{2} \leq \frac{1}{2} \leq 1 \): In this case retail and wholesale prices are set at

\[
w_m = \frac{1}{2} \left( \frac{1}{2} + 1 \right)
\]

\[
p_m = \frac{3}{4} \left( \frac{1}{2} + 1 \right)
\]
The retailer’s profit is written
\[
\pi^r = (p_m \times w_m)(1 + (1 \times p_m)) = \frac{1}{16}(1 + 1)^2;
\]
and the producer’s profit is
\[
\pi^n = w_m(1 + (1 \times p_m)) = \frac{1}{8}(1 + 1)^2;
\]
The consumers’ surplus is written:
\[
S_m = \frac{2^1 + (1 \times p_m)}{2}(1 \times p_m) = \frac{1}{32}(5^1 + 1)(1 \times 3^1)
\]
Welfare is written
\[
W_m = p_m(1 + (1 \times p_m)) + \frac{1 + 1 + (1 \times p_m)}{2}(1 \times p_m)
\]
\[
= \frac{1}{32} 49 \times 9^{1^2} + 14^{1^4}
\]
Type II: \(1 > ^1 M\): In this case retail and wholesale prices are set at
\[
w_m = 1
\]
\[
p_m = 1
\]
and the retailer’s profit is written
\[
\pi^r = (p_m \times w_m)^1 = 0;
\]
The producer’s profit is
\[
\pi^n = w_m^1 = 1
\]
Welfare is written
\[
W_m = p_m^1 = 1
\]
hence the consumers’ surplus is zero. QED.

Proof of Proposition 2:
We solve the game backwards, and start with the retailer’s price setting. If it carries only the national brand, we have from (7) that it sets the following retail price:
\[
p_e = \frac{1 + (1 + w_e)}{2};
\]
and earns:

\[ r_e = \frac{(1 + (1 \cdot w_e))^2}{4} \]

If the retailer carries both brands, it charges the loyal consumers \( p_c = 1 \) and maximizes profit on the falling demand curve by setting a lower price for the private label. The producer of the national brand realizes this and therefore offers \( w_e = 1 \), and the retailer earns zero profits selling the national brand.\(^{11}\)

If so, the retailer sets the following price of the private label:

\[ r = \frac{1 + c}{2} \]

and the retailer’s profit is the following:

\[ r \cdot r_c = \frac{(1 \cdot c)^2}{4} \]

At stage 1, the producer sets wholesale prices contingent on whether the retailer carries a national brand or not. First, it can choose to serve only its loyal customers. Then it has the following profit:

\[ r_e = 1 \]

Alternatively, it can set \( w_e \) such that the retailer is better off with only carrying his product than with carrying both products. This is true if:

\[ r_e - r_c > 0 \]

Solving with respect to \( w_e \), we have that the retailer is better off with only the national brand if

\[ w_e < 1 + c - w_e^n \]

If \( w = w_e^n \), the producer’s profit is the following:

\[ r_e = \frac{(1 \cdot c)(1 + c)}{2} \]

By comparison, we find that \( r_e > r_c \) if:

\[ \frac{c(1 \cdot c) \cdot (1 + c)^1}{2} > 0 \]

\(^{11}\)This would make the retailer indifferent between carrying the national label or not. As a tie-break assumption we assume that when indifferent, he chooses to sell the national brand.
We see that this is true if
\[ 1 < \frac{c(1 - c)}{1 + c}, \quad ^1N(c) : \]

Then we have that
\[ ^1N(c) \quad \frac{c(1 - c)}{1 + c} \quad \frac{1}{3}, \quad ^1M \]
\[ m \quad c < 1 \]
which is always true. QED.

Proof of Corollary 1:
Follows from comparing \( w_m \) and \( w_e \) from Proposition 1 and 2. When \( w_m \neq w_e, \quad c \neq \frac{1}{2} \) the national brand producer always sets the monopoly wholesale price and the retailer will never introduce a private label. QED.

Proof of Proposition 3:
Part i) follows directly from a comparison between Proposition 1 and 2. To prove part ii) note ..rst that the mere threat of a private label is bene.cial for the consumers and welfare if \( ^1 \geq 2 \quad ^1N(c) \). The producer of the national brand lowers the wholesale price to stop the retailer from introducing the private label. Second, if \( ^1 \geq 1M \), the monopolist serves only loyal consumers, and the introduction of a private label is bene.cial for the switching consumers who initially were not served by the producer of the national brand which also enhances welfare. However, for \( ^1 \geq 1N(c), ^1M \), the exact is ambiguous because loyal consumers get higher prices whereas switching consumers get lower prices. The consumers’ surplus in this case is written:
\[ S_c = \frac{(1 - r)^2}{2} = \frac{1}{8} (\frac{1}{4} + \frac{1}{2} c + \frac{1}{8} c^2) \]
whereas \( S_m \) is given by Proposition 1.
\[ S_c > S_m \]
\[ \frac{1}{4} \quad \frac{1}{2} \quad c + \frac{1}{2} c^2 \quad m \]
\[ \frac{1}{32} (5^1 + 1)(1 \quad 3^1) \]
which is always positive for \( c \geq 0 \) and \( ^1 \geq 0 \): Welfare when the private label is introduced is given by
\[ W_c = ^1 + S_c + (1 \quad r)(r \quad c) = \quad \frac{3}{8} (\frac{3}{4} c + \frac{3}{8} c^2) \]
and $W_m$ is given in Proposition 1.

\[
\begin{align*}
W_c + W_m & = m \\
& = 1 + \frac{3}{8} i + \frac{3}{4} c + \frac{3}{8} c^2 + \frac{1}{32} i + 49 i + 9 i^2 + 14 i^3
\end{align*}
\]

which is always positive for $c \in [0; 1]$ and $i \geq 0$: QED.

Proof of Proposition 4: \(\xi_1^M \leq \frac{1}{3}; 1 \leq i\) it is better for both consumers and welfare that the private label is introduced. The reason is that the private label includes the switching consumers without affecting the pricing of the national brand. Second, look at the interval \(1 \leq 0; 1; M \leq 0\): Consumers' surplus when national brand has exclusivity is

\[
S_e = \frac{(1 + (1 i p_e) + 1}{2} \mu \frac{1}{2} i \frac{1}{2} c \frac{1}{2} i \frac{1}{2} c
\]

whereas if the private label is introduced consumers' surplus is

\[
S_c = \frac{(1 i r)^2}{2} = \frac{1}{8} (c i 1)^2
\]

Then we have that consumers are better off under private label introduction if:

\[
\begin{align*}
W_e & = S_e + p_e(1 + (1 i p_e)) \\
& = \frac{1}{2} i \frac{1}{2} c + \frac{1}{2} i \frac{1}{2} c + \frac{1}{2} i \frac{1}{2} c + 1 + \frac{1}{2} c \frac{1}{2} i \frac{1}{2} c
\end{align*}
\]

i.e. always. Then look at welfare under national brand exclusivity:

\[
\begin{align*}
W_e & = S_e + p_e(1 + (1 i p_e)) \\
& = \frac{1}{2} i \frac{1}{2} c + \frac{1}{2} i \frac{1}{2} c + \frac{1}{2} i \frac{1}{2} c + \frac{1}{2} c \frac{1}{2} i \frac{1}{2} c
\end{align*}
\]
and welfare under private label introduction

\[ W_c = 1 + S_c + (1 + r)(r + c) \]

\[ = 1 + \frac{3}{8}c^2 + \frac{3}{4}c + \frac{3}{8} \]

Then we have that welfare is higher under private label introduction if:

\[ W_c > W_e \]

\[ \left( \frac{1}{2} + \frac{1}{2}c^2 \right) \left( \frac{1}{2}c + \frac{1}{2}r + \frac{1}{2}c \right) > 0 \]

\[ \left( \frac{1}{2}c + \frac{1}{2}r + \frac{1}{2}c \right) \left( \frac{1}{2}c + \frac{1}{2}r + \frac{1}{2}c \right) \left( 1 + \frac{6c}{1 + 3c^2} \right) \frac{1}{2} \left( 1 + c \right) < W(c) \]

Moreover, we have that

\[ W(c) < N(c) \]

\[ \frac{p}{1 + 6c} \frac{1 + 3c^2}{2} \frac{1}{1 + c} \frac{6}{1 + c} c(1 + c) \]

Solving this with equality yields two solutions: \( f_c = 1 \) and \( f_c = 0 \); hence for \( c \in (0; 1) \) the inequality either holds for all values or it does not hold. Inserting for \( c = \frac{1}{2} \) we have that \( 0.15139 < 0.16667 \) which holds, hence \( W(c) < N(c) \) for \( c \in (0; 1) \): QED.

**Proof of Proposition 5:**

We have that if

\[ \frac{\partial r}{\partial c} = L(1 + 1) i \left( 2p_c(L + 1) + 3p_c^2 + r(2 + r) \right) i \left( 3r + w_c(1 + 2pc + L + r) \right) = 0; \]

then the retailer would set \( p_c = 1 \): If \( p_c = 1 \); then we know that \( r = \frac{p_c}{3} \) (see the proof of Proposition 6 that follows). Evaluated at \( p_c = 1 \); we thus have that \( \frac{\partial r}{\partial c} = 0 \) if

\[ 1 \left( 3L i + 4 + p \frac{3(1 + w_c) + 3w_c(1 + L)}{3L} \right) > 1 K(L; w_c) \]

It can easily be seen that if \( w_c = 1 \); the condition is met for all relevant values of \( 1 \); that is \( 1 > 0 \): For lower values of \( w_c \); though, \( 1 K \) can be positive. It
is obvious that the national producer will never set \( w_c < w_e \) the wholesale price that deters the retailer from introducing the private label. If we set \( w_c = w_e \) (where \( w_e \) is found in Proposition 6 to follow) then we have the expression shown in the Proposition. This is a sufficient condition for \( p_c = 1 \) in equilibrium. QED.

Proof of Proposition 6:
Given that \( p_c = 1 \) and \( w_e = 1 \); maximizing (6) with respect to the price of the private label yields the first-order condition:

\[
\frac{1}{2L} \frac{3r^2}{r} = 0;
\]

and the optimal price for the private label is

\[
r = \frac{p_3}{3};
\]

Inserting this and \( w_c = 1 \) in (6) yields:

\[
\frac{1}{r_c} = \frac{\bar{A} p_3!}{3} \frac{\mu}{3L} = \frac{1}{3L};
\]

(8)

Suppose now that the retailer only carries the national brand. From (7) we have that for a given \( w_e \) it sets the price

\[
p_e = \frac{1}{2} \left( 1^i + 1 + w_e \right)
\]

(9)

and earns

\[
\frac{1}{r_e} = \frac{(1 + 1) w_e^2}{4}
\]

(10)

Comparing (10) and (8) reveals that the retailer will not introduce the private label if

\[
\frac{(1 + 1) w_e^2}{4} \geq \frac{1}{9L} p_3
\]

which defines the highest wholesale price that the producer can charge to prevent the introduction of a private label. We have that \( w_e^* < 1 \) if \( 1^i < \)
Inserting $w_e^r$ for $w_e$ in (9) yields the retailer’s optimal price given that the private label is going to be deterred, $p_e$: Given this, quantity sold is
\[ q_e = 1 + (1 - p_e) \]
Doing this yields:
\[ p_e = \begin{cases} \frac{8}{3L^2 + 3L_i} & \text{if } 1 < \frac{q}{3L} \\ 1 & \text{if } 1 \leq \frac{q}{3L} \end{cases} \]
\[ q_e = \begin{cases} \frac{q}{3L} & \text{if } 1 < \frac{q}{3L} \\ 1 & \text{if } 1 \leq \frac{q}{3L} \end{cases} \]

For $1 < \frac{q}{3L}$ we will have exclusivity and the national brand producer will earn
\[ \bar{A} = \frac{r - p_e - q}{L^3} \]
\[ \bar{A} = \frac{w_e q_e}{3L^2 + 3L_i} \]
Hence, we have that
\[ \frac{1}{n_e} = w_e q_e = \frac{3L^1 + 3L_i}{2L^3} \frac{1}{9L^2} \]
By simple computation we have that $1^{N(L)} < 1^{M(L)}$ if $L > \frac{4p_3}{3}$ proving part i). We must now check that $1^{N(L)} < \frac{q}{3L^2 + 3L_i}$ so that in fact $p_e < 1$ for $L > \frac{4p_3}{3}$: This amounts to the condition $i \frac{1}{3} \frac{q}{3L_i} - \frac{2p_3}{L^3} < 0$ which is always true.
We have that $1^{N(L)} < 1^{M(L)}$ if $L > \frac{4p_3}{3}$: Finally, by comparison we have that $1^{K(L)} < 1^{N(L)}$ if:
\[ \frac{3L_i p_3 p_3}{(3L_i + L^3)(6L_i + 3p_3)} > 0 \]
It can be shown that this condition is met if \( L = \frac{49}{3} p \): Then, obviously, it also holds for \( L > \frac{49}{3} p \). QED.

Proof of Proposition 7:

Part i) follows directly from comparing Propositions 1 and 5. To prove part ii) first note that for \( 1 \leq 1 < 0; 1 \in N(L) \) no private label is introduced, but due to threat of introduction the national brand producer sets a lower price than the monopolist, hence both consumers' surplus and welfare is increased by the existence of a private label. Second, when \( 1 \leq \frac{1}{2} < 1 \in M; 1 \in N \) the monopolist serves loyal consumers exclusively, whereas private label introduction also includes some switching consumers and welfare and consumers' surplus also increase in this case. For \( 1 \leq 1 < 1 \in N(L); 1 \in M \) private label introduction will increase the price to loyal consumers whereas switching consumers will get a lower price compared to the monopoly case. Under private label introduction the consumers' surplus is

\[
S_c = \frac{1}{2} (1 - r) q = \frac{1}{18} 3 1 \frac{p}{3} \frac{1}{L}:
\]

Then we have that

\[
S_c > S_m
\]

\[
\frac{1}{18} 3 1 \frac{p}{3} \frac{1}{L} > \frac{1}{32} (5^1 + 1)(1 - 3^1)
\]

which holds for any \( 1; L > 0 \). Welfare under private label introduction is

\[
W_c = 1 + r q + \frac{1}{2} (1 - r) q = \frac{1}{18} \frac{18L^1 + p 3 + 3}{L}:
\]

Then we have that

\[
W_c > W_m
\]

\[
\frac{1}{18} \frac{18L^1 + p 3 + 3}{L} > \frac{1}{32} 49 1 \frac{9^2 + 14^1}{4}
\]

which is easily verified to hold for any \( 1; L > 0 \). QED.

Proof of Proposition 8:
The consumers' surplus and welfare under private label introduction is derived in the proof of Proposition 6. When the private label is not introduced the consumers' surplus is

\[ S_e = \frac{1}{2} (q_e + 1) (1 - p_e) = \frac{1}{18} \overline{A} + \frac{r}{3} \left( \overline{p} \right)^{3} + \frac{i}{3} \overline{L} + \frac{q}{L} p^{\frac{2}{3}} \]

and welfare

\[ W_e = p_e q_e + \frac{1}{2} (q_e + 1) (1 - p_e) \]
\[ = \frac{1}{18} \left( 6 L^{2} 3^{1} + 6 p - \frac{P_{e}}{3} \right) 9 L^{1} 2 \]

Comparing these yields:

\[
\begin{align*}
S_e & \leq S_c \\
& \leq \frac{1}{18} \overline{A} + \frac{r}{3} \left( \overline{p} \right)^{3} + \frac{i}{3} \overline{L} + \frac{q}{L} p^{\frac{2}{3}} + \frac{1}{18} \overline{L} + \frac{p}{3} + \frac{2}{3} i + \frac{1}{18} \overline{L} \overline{W}(L) \\
W_e & \leq W_c \\
& \leq \frac{1}{18} \left( 6 L^{2} 3^{1} + 6 p - \frac{P_{e}}{3} \right) 9 L^{1} 2 + \frac{1}{18} \overline{L} + \frac{p}{3} + \frac{2}{3} i + \frac{1}{18} \overline{L} \overline{W}(L)
\end{align*}
\]

and for welfare
The following figure plots $N^L$ (lower line), $W^L$ (middle) and $S^L$ (upper line) for $L = \frac{1}{3}, \frac{5}{9}, 1$.

QED.

References


