On the profitability of cross-border mergers

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Abstract
A horizontal merger is unlikely to be profitable unless it involves the large majority of firms in an industry. This well established result was developed by Salant et al (1983) in a closed economy setting. The present paper studies the profitability of mergers in an open economy. A cross-border merger provides the acquiring firm with market access. If alternative modes of market entry are sufficiently costly, a merger may indeed be profitable. The relationship between entry costs and the profitability of merger is, however, not a monotonic one. An increase in entry costs may cause a change in the optimal entry mode of rival firms such that a merger may be unprofitable even for higher entry costs. The paper also derives results regarding the nationality of the acquiring firm.

JEL classification: F15, F21, F23, L12, L13
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1 Introduction
It is well known from the literature on mergers that it is generally more profitable to be outside a merger than to participate in it, see Stiegler (1950).

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In a Cournot model with symmetric firms, Salant et al. (1983) show that a merger involving less than 80 percent of the industry will not be profitable. The reason is that the outside firms will “steal business” from the merging parties. Due to the business stealing effect, the profits of the merged firm may be less than the combined pre-merger profits of the merging parties, thus violating the traditional theoretical criterion for a merger to take place.

In light of these results, the fact that a large number of mergers actually take place is rather surprising. There are two main explanations to this puzzle. First, mergers may be guided by other objectives than profits. Second, mergers may give rise to cost synergies that are not captured by the standard models referred to above.

The present paper maintains the assumption of profit maximization, and studies the cost synergies that arise from cross-border mergers. Acquiring a firm in a foreign market may function as a mode of entry, thus bringing additional gains to the merging parties relative to a single-market context. If alternative modes of market entry, which here means exports or greenfield investment, are sufficiently costly, a merger may indeed be profitable to the merging parties. The relationship between entry costs and the profitability of merger is, however, not a monotonic one. An increase in entry costs that changes the optimal entry mode of rival firms may turn the profitability of a merger from positive to negative.

Cross-border mergers and acquisitions have increased sharply in recent years. In the second half of the 1990s the yearly growth was close to 50 percent, reaching USD 720 billion in over 6000 deals at the end of that decade, see UNCTAD (2000). The economic literature on cross-border mergers is however relatively small. Most of it is normative, analysing the welfare effects of mergers and policy implications, see for instance Barros and Cabral (1994) and Horn and Levinsohn (1997). The present paper addresses the positive issue of equilibrium market structure, and is related to Horn and Persson (2001), Norbäck and Persson (2001 a,b), Görg (1999).

The distinguishing feature of the present paper relative to the existing lit-
erature on cross-border mergers is the focus on the profitability of a merger. Horn and Persson (2001) analyse equilibrium market structure, focussing on whether mergers cross borders or not. The main result of their paper is that an increase in trade costs may increase the profitability of domestic mergers relative to cross-border mergers. The intuition is basically that when trade costs are high, a domestic merger results in national monopolies, which is more profitable than a cross border merger resulting in an international duopoly. They do not analyse the possibility of mergers not being profitable to the involved parties. Moreover, they do not consider greenfield as alternative entry mode.

Norbäck and Persson (2001a) consider the case of privatization. Given that a firm is for sale, they ask who will purchase the firm. As in Horn and Persson, they show that when trade costs are high, the firm located in the same market as the firm for sale will be the buyer. Allowing for greenfield investment, they also demonstrate that when greenfield costs are low, the price of the privatized firm will be low and the foreign firm will end up buying it. The intuition is basically the following: The only reason why the local firm should buy the privatized firm is to avoid entry through acquisition by the foreign firm. But if greenfield costs are low, the most profitable alternative to acquisition for the foreign firm is greenfield. Hence, the local firm has nothing to gain from buying the firm for sale. Low greenfield costs therefore lead to foreign acquisition. Again, in their paper the possibility of an acquisition not being profitable is not an issue. The reason is basically that the reservation price of the privatized firm is assumed to be zero, so that there will always be a buyer.4

Finally, Görg (2000) analyses the choice between acquisition and greenfield. He abstracts from exports as possible entry mode and, by considering only a two-firm case, does not address the issue of possible losses from the acquisition due to business stealing effects from outside firms.

The paper is organized as follows. Section 2 presents the model and section 3 the analysis. Section 4 concludes.

4In a related paper, Norbäck and Persson (2001b) focus on the welfare effects of foreign acquisitions.
2 The model

Consider a market where demand for the homogenous good $Q$ is given by

$$Q = 1 - p,$$  \hspace{1cm} (1)

where $p$ is the price. The market can be supplied by local firms and by firms located elsewhere. We shall refer to the latter as foreign firms. Operating profits for a firm $i$ are given by

$$\pi_i = (p - s_i) Q_i,$$  \hspace{1cm} (2)

where $s_i$ is marginal sales costs for firm $i$. Assuming Cournot competition between firms, equilibrium operating profits can be found as$^5$

$$\pi_i = \frac{(1 - ns_i + \sum s_k)^2}{(n + 1)^2}, \ k \neq i,$$  \hspace{1cm} (3)

where $n$ is the number of firms competing in this market. A foreign firm has three modes of servicing the market. First, it may acquire a firm already located there. This is the acquisition strategy $A$. Second, it may enter by investing in a new production plant at a fixed cost $f$. This is the greenfield strategy $G$. Third, it may choose exports at a per unit trade cost $t$, which we shall call strategy $X$. Finally, the firm may choose not to serve the market at all, which we call strategy $0$.

The sequence of moves is as follows. At stage one, firms simultaneously decide on whether or not to invest, and in case of investment, whether to choose strategy $A$ or $G$. At stage two, there is production and sales, with Cournot-competition between the firms.

To study the issue at hand, it suffices to consider three firms, call them $a, b$ and $c$. Let $c$ be the target firm, located in the market in question. Marginal production costs are assumed to be identical across firms, and are normalized to zero. Note that in the closed economy context studied by Salant et al (1983), a merger between a pair of firms in the symmetric triopoly case would not be profitable. In line with the literature, we ignore the trivial case of the three firms merging to form a monopoly. We can think of such monopolization as being prohibited by competition policy.

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$^5$See Appendix A for a derivation.
The standard criterion for a merger to take place is that there is an increase in the joint profits to the insiders of a merger. Let $\pi^*_j$ define the post-merger operating profits of the merged entity when firm $j = a, b$ is the merging partner with $c$, and let $\Pi_j$ define net profits, i.e., net of any fixed investment cost $f$, in the absence of a merger. The change in joint profits to the merging parties caused by the merger is thus given by

$$
\mu_j \equiv \pi^*_j - \Pi_j - \pi_c.
$$

(4)

A merger will be assumed to take place between $j$ and $c$ only if $\mu_j > 0$. In terms of an acquisition, $\mu_j > 0$ can be interpreted as saying that the acquiring firm $j$ will place a bid on the target firm $c$ only if $j$’s reservation price, $(\pi^*_j - \Pi_j)$, exceeds the reservation price of $c$, $\pi_c$. Note that the present paper focuses on the incentive to merge. I shall therefore limit myself to answering the question whether we can expect an acquisition to take place or not (i.e., whether $\mu_j$ is positive or not) and not address the question of what the acquisition price will be if indeed $\mu_j$ is positive.

Operating profits for the possible combinations of entry modes for two foreign firms are derived from (3) and are summarized in Table 1.

Table 1. Matrix of operating profits

<table>
<thead>
<tr>
<th></th>
<th>$G$</th>
<th>$X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0, 0</td>
<td>0, $\left[\frac{1}{2}\right]$</td>
</tr>
<tr>
<td>$G$</td>
<td>$\left[\frac{1}{8}\right]$, 0</td>
<td>$\left[\frac{1}{16}\right]$, $\left[\frac{1}{16}\right]$</td>
</tr>
<tr>
<td>$X$</td>
<td>$\left[\frac{(1-2t)^2}{9}\right]$, 0</td>
<td>$\left[\frac{(1-3t)^2}{16}\right]$, $\left[\frac{(1+t)^2}{16}\right]$</td>
</tr>
<tr>
<td>$A$</td>
<td>$\left[\frac{1}{4}\right]$, 0</td>
<td>$\left[\frac{1}{7}\right]$, $\left[\frac{1}{7}\right]$</td>
</tr>
</tbody>
</table>

Entry costs affect profits, and thereby $\mu_j$, through two channels. First, a change in entry costs affects profits for a given set of entry modes. For instance, higher trade costs lowers profits for an exporting firm and raises profits for a local firm. Second, a change in entry costs may alter firms’ choice of entry mode. For instance, higher trade costs may induce a firm to

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6To save space, we have not included the payoffs for the situation where the vertical player is the acquiring firm. These payoffs would, however, be entirely similar to the horizontal player’s acquisition payoffs.
choose greenfield rather than exports, leading to a reduction in the profits of a local firm.

In order to determine the sign of $\mu_j$, we have to derive the equilibrium entry strategies. For the relevant entry strategies we can then derive critical values of entry costs at which the sign of $\mu_j$ changes. The discussion of the two scenarios will be based on two figures, one for each scenario, with the calculations underlying the figures reported in Appendices B and C. The axes of the figures measure entry costs: On the vertical axis is the fixed greenfield cost $f$, and on the horizontal axis is the per unit trade cost $t$.

In terms of notation, each area in the figures is assigned a roman number and letters describing the equilibrium entry mode. The letters in parenthesis give the equilibrium entry mode in the no-merger case. For instance, $(GG)$ means that both foreign firms in the absence of merger would choose greenfield investment, and $(XG)$ that one firm chooses exports, the other greenfield. Letters preceding a parenthesis refers to entry mode when a merger takes place. Hence, $AX (XX)$ means that the equilibrium outcome is one in which one firm acquires $c$, the optimal response of the outside firm being exports. Moreover, the parenthesis indicates that in the absence of a merger, both foreign firms would have chosen exports.

When studying cross-border mergers in our three-firm model, there are two relevant initial market structures to analyse. In scenario 1, both $a$ and $b$ are located abroad, facing the same trade and greenfield costs. We shall call this the symmetric case. This scenario allows us inter alia to analyse how the profitability of a merger is affected by the response of the outside firm, i.e., the firm not involved in the merger. In the second scenario, only one firm is located abroad, with the other two already being established in the market in question. We shall refer to this as the asymmetric case. An interesting issue in this second scenario is the question of who will buy firm $c$, i.e., the foreign or the local firm.

3 Scenario 1. The symmetric case

It is instructive to start the analysis by asking what the alternative to a merger is. The equilibrium market structure in the no-merger case brings no great surprises. Basically, when trade costs are low relative to greenfield costs (areas $I$, $IV$, $VI$), the foreign firms choose exports, and when trade costs are high (areas $II$, $VII$), they choose greenfield. In the central areas

6
III, VIII, and V, there is room for only one greenfield investor; in III and VIII the optimal response of the rival is to choose exports, and in V to stay out of the market. The MM-curve shows combinations of investment and trade costs above which an entrant, in case he meets a single local producer in the foreign market, chooses exports and below which he chooses greenfield. This information is relevant if the rival chooses to stay out of the market or acquires firm c. The horizontal axis is bounded above by \( t = 0.5 \), at which point profits for two foreign exporters are zero.

Figure 1: Scenario 1. The symmetric case

We now have the information we need on no-merger market structure and can turn to the main issue, namely the profitability of merger. In the figure, the shaded areas I – V are combinations of greenfield costs and trade costs for which \( \mu_j \leq 0 \) and hence where a merger will not take place. Not surprisingly,

\footnote{We limit ourselves to studying equilibria in pure strategies.}
with zero entry costs there will be no merger since we are effectively in the single market setting studied by Salant et al (1983). The same is true for regions $I - III$ characterized by low entry costs. However, starting in the low-cost areas $I - III$, we find that:

**Proposition 1** An increase in entry costs, in the form of higher greenfield investment costs or trade costs, may make a cross-border merger profitable.

**Proof.** From Figure 1, with derivations in Appendix B2, we see that an increase in entry costs, moving the economy from $I$ to $VI$, from $II$ to $VII$, from $III$ to $VII$ or $VIII$, results in the profitability of an acquisition turning from negative to positive.

The intuition for this result is straightforward: An increase in entry costs related to trade and investment increases the attractiveness of acquisition as a cost effective way of entering foreign markets. While an increase in trade costs and greenfield costs may induce a cross-border merger, this is not necessarily the case. From Figure 1 we see that:

**Proposition 2** An increase in entry costs that causes a change in the entry mode of the rival firm may turn the profitability of a merger from positive to negative.

**Proof.** From Figure 1, with derivations in Appendix B2, we see that an increase in entry costs, moving the economy to $V$, or from $VI$ or $VIII$ to $IV$, results in the profitability of an acquisition turning from positive to negative.

This proposition is less intuitive, and requires some elaboration. Consider first an increase in greenfield costs such that the economy moves from $VII$ to $V$. This brings about a change in the no-merger market structure from $(GG)$ to $(0G)$. This means that in $V$ the equilibrium market structure would be the same with or without a merger, in both cases characterized by two producers located in the market. There is therefore no potential gain from a merger. Since no firm would spend money on acquiring firm $c$, the outcome is characterized by no-merger equilibrium $(0G)$. An increase in trade costs causing a shift from $VI$ or $VIII$ to $V$ has an entirely similar effect.

Consider next an increase in greenfield costs that takes us from $VIII$ to $IV$, with the associated change in no-merger entry strategies from $(XG)$ to $(XX)$. This change makes the merger unprofitable by making the alternative
to merger more profitable: Without a merger, the response of the rival would be the fairly ”soft” choice of exports, while a merger would trigger the more ”aggressive” response of greenfield from the rival.

Similarly, an increase in trade costs that brings us from VI to IV also results in negative profits from merger. Here, the no-merger market structure remains the same, namely (XX). What changes is the optimal response to a merger by the outside firm, from exports to greenfield. By intensifying post-merger competition, this makes a merger less attractive to the merging parties.

Finally, a few words on the profitability of being an insider versus an outsider to the merger in this symmetric case. Appendix B3 shows that it is generally more profitable to be outside the merger. The exception to this rule is for $t > \frac{1}{3}$ in region VI and in region IX. Here, the cost saving effect is sufficiently strong, and the business stealing effect sufficiently weak (due to the optimal choice of the outside firm being exports) to make the insider better off than the outsider. In areas VII and VIII, a first mover would choose $G$, leaving the acquisition to the second mover. Trivially, a first mover would also choose $G$ in area V, thus forming a duopoly with firm $c$.

4 Scenario 2: The asymmetric case

In this version of the model we explore the consequences of an asymmetry between $a$ and $b$ in terms of their initial location. For concreteness, let $b$ and $c$ be located in the same market with $a$ being the only foreign firm. In this case, market entry is of course an issue only for firm $a$. This allows us to deal with the issue of the nationality of the acquiring firm. When should we expect to see cross-border mergers and when are mergers between two firms located in the same market more profitable?

Figure 2 illustrates the outcome of this scenario, with the algebra presented in Appendix D. The no-merger equilibrium market structure is relatively straightforward. Again, low trade costs relative to greenfield costs leads to exports (areas I, V and VI), whereas high trade costs relative to greenfield costs leads to investment (areas II and VII). When the cost of both kinds of entry are high, the potential entrant stays out (areas III, IV, and VIII).

Turning to mergers, we can derive the same kind of results as in Scenario 1: Increased entry costs may make a merger profitable, but not necessarily so.
Figure 2: Scenario 2. The asymmetric case

Hence, propositions 1 and 2 apply also to this scenario. The most important insight derived from the present scenario relates to the buyer of $c$. The general insight can be stated as:

**Proposition 3** A cross-border merger brings the added gain of market access and is therefore generally more profitable than a merger between two firms located in the same market.

**Proof.** See Appendix C

The reason why the foreign firm $a$ is more likely to merge with $c$ than is the local firm, $b$, is of course that for $a$ an acquisition brings the additional gain of market entry. There is, however, an exception to this general insight, which can be stated as:
Proposition 4 When entry costs are sufficiently high, a merger is profitable only between two firms located in the same market.

Proof. See Appendix C ■

In area $VIII$ the local firm is the only one with an incentive to merge. The reason is as follows. As noted above, in regions $III$, $IV$, and $VIII$, the optimal no-merger strategy of $a$ is not to service the market. Since an acquisition by $a$ would essentially replace one local duopoly with another, there is no gain from such a move. The question is therefore whether firm $b$ will choose acquisition or not. For sufficiently high trade costs, namely in region $VIII$, firm $b$ finds it profitable to spend money on acquiring $c$, the advantage for $b$ being that it thus replaces a local competitor with a foreign based one, a fact which softens the competitive pressure on $b$.

5 Concluding remarks

The present paper analyses the incentives to undertake cross-border mergers. Relative to the analysis of mergers in a single market, the geographical dimension adds the issue of market entry to the analysis. An increase in trade costs and/or greenfield costs increases the profitability of acquisition as a mode of market entry and may thus increase the gains from horizontal mergers.

There is, however, no monotone relationship between entry costs and the profitability of cross-border mergers. The reason is that a change in entry costs may induce a change in the entry mode of the rival firm. For instance, an increase in trade costs that changes the optimal entry mode of the outside firm from exports to greenfield, increases the business stealing effect and may render an acquisition unprofitable.

The analysis also demonstrates that mergers are more likely to take place between firms located in different markets than between firms located in the same market. The reason is, as noted above, that a cross-border merger brings with it the additional gain of market access to the foreign firm. Hence, the willingness to pay for a firm in a specific location is generally higher for

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8This result is essentially the same as that found in Horn and Persson (2001) referred to in the introduction: Abstracting from greenfield investment (which in region $VIII$ is true because of high greenfield costs), an increase in trade costs results in a domestic merger.
a foreign than a local firm. However, when entry costs are sufficiently high, a merger may be profitable only between two local firms.

The analytical framework in which the issue of cross-border mergers has been analysed here is certainly not a general one. However, the mechanisms presented in this relatively simple model are fairly intuitive, and should be present also in a more complex model. Indeed, Salant et al (1983) report that the loss-from-merger result can also arise in more complex Nash equilibrium models, with differentiated products, increasing marginal costs, and so on. Naturally, allowing for new entry following a merger would make the merger less profitable. But if we believe that increased market concentration is an important motivation for mergers, the Cournot model without additional entry may be realistic.

One suggestion for future research is to analyse welfare implications of the model. Another is to analyse the consequences of the outcome when there are asymmetries in firms’ technologies.

Appendix A Derivation of equation (3).

Combining (1) and (2), firm i profits can be expressed as

\[ \pi_i = \left(1 - \sum Q_k - s_i \right) Q_i. \]  
(A1)

Maximizing this expression with respect to \( Q_i \), taking as given all \( Q_k \), \( k \neq i \), we get the first order condition

\[ Q_i = 1 - s_i - \sum Q_k. \]  
(A2)

Since this condition must hold for all \( n \) producers, we have

\[ \sum Q_k = n - \sum s_k - n \sum Q_k, \]  
(A3)

which can be written as:

\[ (n + 1) \sum Q_k = n - \sum s_k. \]  
(A4)

Using (A2), we can express (A4) as
\[ 1 + \sum s_k = (n + 1)(Q_i + s_i), \quad \text{(A5)} \]

which in turn can be expressed as

\[ Q_i = \frac{1 - (n + 1)s_i + \sum s_k}{n + 1}, \quad \text{(A6)} \]

The equilibrium price can be found by using (A4) into the demand function (1), resulting in

\[ (1 - p)(1 + n) = n - \sum s_k, \]

which in turn can be expressed as

\[ p = \frac{1 + \sum s_k}{n + 1}. \quad \text{(A7)} \]

Equilibrium profits can be found by using (A6) and (A7) in the profit function (2). This yields

\[ \pi_i = \left( \frac{1 + \sum s_k}{n + 1} - s_i \right) \left( \frac{1 - (n + 1)s_i + \sum s_k}{n + 1} \right), \]

which simplifies to

\[ \pi_i = \frac{(1 - (n + 1)s_i + \sum s_k)^2}{(n + 1)^2}, \quad \text{(A8)} \]

which can be written as

\[ \pi_i = \frac{(1 - ns_i + \sum s_k)^2}{(n + 1)^2}, \quad k \neq i. \quad \text{(A9)} \]
Appendix B: Scenario 1 (The symmetric case)

B1. The no-merger case

From Table 1 we can derive the critical values of entry costs at which equilibrium market structures change. A word on notation: \( \Pi X | G \) means net profits from exports, given that the rival foreign firm has chosen greenfield. Similarly for the other entry modes. For instance, from \( \Pi X | G = \Pi G | G \) we can find the border between the \((XG)\) and the \((GG)\) market structures.

\[
\begin{align*}
\Pi X | G = \Pi X | G : 0 &= \left( 1 - 2 t \right)^2 \Rightarrow t = \frac{1}{16} \\
\Pi G | G: 0 &= \left( 1 - 2 t \right)^2 - \left( 1 + 2 t \right)^2 < 0 \text{ for } t < \frac{1}{16} \text{ (Area I)} \\
\Pi G | G: 0 &= \left( 1 - 2 t \right)^2 - \left( 1 + 2 t \right)^2 < 0 \text{ for } f < \frac{1}{16} \text{ (Area II)} \\
\Pi G | G: 0 &= \left( 1 - 2 t \right)^2 - \left( 1 + 2 t \right)^2 < 0 \text{ for } t < \frac{1}{15} \text{ (Area III)} \\
\Pi G | G: 0 &= \left( 1 - 2 t \right)^2 - \left( 1 + 2 t \right)^2 < 0 \text{ for all } t \text{ (Area IV)} \\
\Pi G | G: 0 &= \left( 1 - 2 t \right)^2 - \left( 1 + 2 t \right)^2 < 0 \text{ for all relevant } t \text{ (Area V)}
\end{align*}
\]

B2. The profitability of merger

Notation: Let the profitability of acquisition, given that the optimal response of the outside firm is exports, be given by \( \mu [AX (XX)] \equiv \pi A | X - \Pi X | X - \pi_c | XX \), where \( \pi_c | XX \) is the payoff to \( \pi \) given that both foreign firms choose exports. Similarly for other other entry modes.

\[
\begin{align*}
\mu [AX (XX)] &= \frac{1}{9} \left( 1 + t \right)^2 - \frac{1}{16} \left( 1 - 2 t \right)^2 - \frac{1}{16} \left( 1 + 2 t \right)^2 < 0 \text{ for } t < \frac{1}{10} \text{ (Area VI)} \\
\mu [AG (GG)] &= \frac{1}{9} - \frac{1}{16} \left( 1 - 2 t \right)^2 - \frac{1}{16} \left( 1 + 2 t \right)^2 < 0 \text{ for all } t \text{ (Area VII)} \\
\mu [AG (XX)] &= \frac{1}{9} - \frac{1}{16} \left( 1 - 2 t \right)^2 - \frac{1}{16} \left( 1 + 2 t \right)^2 < 0 \text{ for all } t \text{ (Area VIII)} \\
\mu [AG (0G)] &= \frac{1}{9} - \frac{1}{16} \left( 1 - 2 t \right)^2 - \frac{1}{16} \left( 1 + 2 t \right)^2 < 0 \text{ for all } t \text{ (Area IX)}
\end{align*}
\]
\[ \mu [AG(XG)] < \Pi G|A : \frac{1}{9} - \frac{(1-3t)^2}{16} - \frac{(1+t)^2}{16} < \frac{1}{9} \quad \text{for all relevant } t \quad (\text{Area VIII}) \]

\[ \mu [AG(GG)] < \Pi G|A : \frac{1}{9} - \left( \frac{1}{16} - f \right) - \frac{1}{16} < \frac{1}{9} \quad \text{for all relevant } f \quad (\text{Area VII}) \]

Appendix C: Scenario 2 (The asymmetric case)

All critical values and functions can be found from Scenario 1, just keeping in mind the fact the asymmetry of initial location. Hence, the definition of the border between \((X)\) and \((G)\) is identical to the one defining the border between \((XG)\) and \((GG)\) in Scenario 1; the condition \(\mu [A(X)] < 0\) is identical to \(\mu [AG(XG)] < 0\) in Scenario 1, and so on.

Generally, it is not profitable for the local firm to acquire \(c\). The exception is area \text{VIII}, as shown below.

\[ \mu [AX(X)] = \frac{(1+t)^2}{9} - \frac{(1+t)^2}{16} - \frac{(1+t)^2}{16} < 0 \quad \text{for all relevant } t \quad (\text{Area V}) \]

\[ \mu [AG(X)] = \frac{1}{9} - \frac{(1+t)^2}{16} - \frac{(1+t)^2}{16} < 0 \quad \text{for all relevant } t \quad (\text{Area VI}) \]

\[ \mu [AG(G)] = \frac{1}{9} - \frac{1}{16} - \frac{1}{16} < 0 \quad (\text{Area VII}) \]

\[ \mu [AX(0)] = \frac{(1+t)^2}{9} - \frac{1}{9} - \frac{1}{9} < 0 \quad \text{for } t < .41421 \quad (\text{Area IV}) \]

\[ \mu [AX(0)] = \frac{(1+t)^2}{9} - \frac{1}{9} - \frac{1}{9} > 0 \quad \text{for } t > .41421 \quad (\text{Area VIII}) \]

References


