Corporate Tax Systems and Cross Country Profit Shifting: Formula Apportionment vs. Separate Accounting

September 13, 2004

Abstract

This paper shows in a symmetric tax competition model that a formula apportionment system can attain the first best welfare optimum without any political pre-agreed harmonization or coordination of tax bases and tax rates. The same result cannot be obtained under separating accounting even if countries agree on both tax rates and bases. The efficiency of company tax reform therefore requires more political cohesion under separate accounting than formula apportionment and yields lower welfare.

*JEL classification: H7, H73*
1 Introduction

The recent EU commission report on company taxation aims at leveling the playing field for company taxation in Europe by proposing four different blueprints for a single tax base for European multinationals. These blueprints eliminate the current system of separate accounting whereby profits of affiliates of multinationals are determined on the basis of the arm’s length principle as a corporate tax standard for the EU, and instead advocates the use of a formula apportionment rule. The basic idea underpinning each of the four proposals is that European multinationals should be able to calculate profits originating within the EU under a consolidated tax base. Hence, the total profit of a multinational originating within the EU should be allocated to EU countries based on activity weights that reflect the multinational’s relative activity in each of the EU countries.

The main argument in the literature in favor of using formula apportionment is that it is better suited at curbing profit shifting by multinationals.\(^1\) This comes at a cost, however, since any formula apportionment rule has the inherent structure that shifting of profit does not occur through the pricing of inter-firm transactions, but rather through the allocation of activity to low tax jurisdictions. Thus, the choice between separate accounting and formula apportionment is one where one has to trade off the different distortions under each system.\(^2\) Perhaps as a response to this insight, the political discussion on company tax reform in the EU has lately concentrated its efforts on the creation of a level playing field for corporate taxation by harmonizing or approximating tax bases.\(^3\) The implicit signal is that tax rates may be allowed to differ across countries, and that the issue of tax rate harmonization is a second step in the political process.

In the discussion of harmonization of tax bases, the emphasis has been on the activity weights under formula apportionment and how they should be constructed to display activities, minimize distortive taxation, and secure a fair share of revenue

\(^1\) See e.g. Musgrave (1973), Bird and Brean (1986), Shakelford and Slemrod (1998) and more recently Kind, Midelfart and Schjelderup (2004).

\(^2\) The possible distortions under formula apportionment are discussed in Gordon and Wilson (1986).

for each country that has some multinational activity. In doing so there has been a tendency to forget that one of the main problems in company taxation no matter what system, is the fact that one has to determine what taxable profit is and its relationship to the true economic profit of the firm. Any difference between what the firm sees as true costs or revenues, in comparison to taxable costs and revenues, creates a tax distortion that affects the allocation of profit even before profit is distributed by weights under formula apportionment.

The issue of tax neutrality under separate accounting has been examined both in a closed and open economy setting (see e.g. Sinn 1987), whereas the simultaneous choice of tax rate and tax base under separate accounting in a setting with multinationals and competition among countries to attract shifty profit has been studied by Haufler and Schjelderup (2000). They find that tax competition leads to a second best equilibrium where tax deductions are incomplete and thus distortive. A comparable analysis to Haufler and Schjelderup (2000) under formula apportionment has yet to be done, and the purpose of this paper is to do just that.

This paper sets up a model with multinationals and profit shifting were firm behavior has the potential of being distorted by tax deductible depreciation allowances, and where each country must decide on the optimal tax rate, tax base and apportionment weight. We then compare separate accounting to formula apportionment and find that formula apportionment has some qualities that so far has been neglected in the literature. In particular, formula apportionment can reproduce the famous Schanz-Haig-Simons tax in the sense that the tax does not affect the marginal investment decisions of the firm. This is a first best property that cannot be achieved under separate accounting. The implication is that a European leveling of the playing field can be agreed upon without political coordination under formula apportionment whilst political coordination and bargaining must be the outcome under separate accounting. Furthermore, the outcome of such a process can only reproduce a second best result due to the problem of profit shifting.

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4McLure (1980) shows that the use of activity weights implies that local corporate income taxes become taxes or subsidies to the factors entering the formula.

2 The model

We consider a model of two small countries each of which hosts a multinational enterprise (MNE). The multinationals are identical in structure and each has access to a market for internationally mobile capital and produces an output good using capital as an input. Subscripts denote the country where the MNE has its headquarter and superscripts the country where the economic activity takes place. The output of MNE\(_i\) in country \(i\) and \(j\) is accordingly given by the production functions \(f_i(k_i)\) and \(f_i^j(s_i k_i^j)\), where \(s_i\) is an essential service the headquarter delivers to the affiliate located in country \(j\), which is proportional to the amount of capital invested in the firm, and has the property of enhancing the productivity of the affiliate.\(^6\) The true cost of this service is \(p_i\) and it can be interpreted as the selling or lease of a patent or knowledge. We assume throughout the analysis that \(p_i\) is not observable for the tax authorities and that the multinational may choose to overinvoice or underinvoice the service and declare unit costs of the service equal to \(\gamma_i\) in order to reduce the total tax payments of the corporation. It is costly to make a false statement about the true cost of the service provided and we denote the concealment cost that each firm incurs as \(C_i(\gamma_i - p_i)\). The cost is assumed to be a convex function with a minimum at \(C_i(0) = 0\), and may be interpreted as the hiring of lawyers and accountants to hide the true nature of the transaction.

The rate of true economic depreciation is \(\delta_i\), and \(r\) is the cost per unit of capital. Before tax profits of MNE\(_i\) are

\[
\pi_i^T = f_i(k_i) - (r + \delta_i)k_i + [\gamma_i - p_i - C_i(\cdot)] s_i k_i^j,
\]

\[
\pi_i^T = f_i^j(s_i k_i^j) - (r + \delta_i)k_i^j - \gamma_i s_i k_i^j,
\]

where \([\gamma_i - p_i - C_i(\cdot)]\) is the net value of the transfer pricing transaction by the parent firm.

Each multinational can deduct a share \(\alpha_i \in [0, 1]\), \(i = 1, 2\) of its capital costs and true depreciation from the tax base. Taxable profits are

\[
\pi_i^{T_1} = f_i(k_i) - \alpha_i(r + \delta_i)k_i + [\gamma_i - p_i - C_i(\cdot)] s_i k_i^j,
\]

\[
\pi_i^{T_2} = f_i^j(s_i k_i^j) - \alpha_i'r + \delta_i)k_i^j - \gamma_i s_i k_i^j.
\]

\(^6\)This has been documented as an important mode of profit shifting (see e.g., Grubert, 2003).
In what follows we will use this framework to analyze the properties of formula apportionment and separate accounting. We start by examining formula apportionment.

2.1 Formula Apportionment

Under formula apportionment the income of the parent firm and its affiliate is combined into a single measure of global corporate income, which is then apportioned to each of the two countries based on some relative activity weight. In our model we use capital as a weight of activity, and the average tax rate on profits of MNE$_i$ then becomes

$$T_i = t^{l_i} \frac{k_i}{k_i + k_j} + t^{l_j} \frac{k_j}{k_i + k_j},$$

(1)

where $t^{l_i} \in [0, 1]$ and $\alpha_i \in [0, 1]$ are choice variables of country $i$, whilst $\alpha_j \in [0, 1]$, and $t^{l_j} \in [0, 1]$ are choice variables of country $j$. Using the above equations total after tax profit of MNE$_i$ is

$$\Pi_i = (\pi^T_i + \pi^T_j) - T_i \left( \pi^T_i + \pi^T_j \right).$$

(2)

MNE$_i$ maximizes $\Pi_i$ with respect to $\gamma_i$, $k^i_j$, and $k^{j}_i$. The first-order conditions are

$$k^j_i \gamma_i(1 - T_i)C_i' = 0 \quad (3a)$$

$$f^D_i' - r - \delta_i - k^j_i(t^{l_j} - t^{l_i}) \frac{\pi^T_i + \pi^T_j}{(k_i^j + k_j^i)^2} + T_i(\alpha_i^j(r + \delta_i) - f^D_i') = 0 \quad (3b)$$

$$s_i f^{D'}_i - r - \delta_i - p_is_i + k^j_i(t^{l_i} - t^{l_j}) \frac{\pi^T_i + \pi^T_j}{(k_i^j + k_j^i)^2} + T_i(p_i s_i + \alpha^j_i(r + \delta_i) - s_i f^{D'}_i' = 0 \quad (3c)$$

Inspection of (3a) shows that $C_i' = 0$ in the firm optimum, so $\gamma_i = p_i$ and there is no transfer pricing under formula apportionment. This result confirms the conventional wisdom pointed out and shown in several papers that under formula apportionment firms have no incentive to manipulate profit. Using the envelope theorem on (2) we get:

$$\frac{\partial \Pi_i}{\partial t^{l_i}} = - \frac{k^j_i}{k_i^j + k_j^i} \left( \pi^T_i + \pi^T_j \right), \quad \frac{\partial \Pi_i}{\partial t^{l_j}} = - \frac{k^j_i}{k_i^j + k_j^i} \left( \pi^T_i + \pi^T_j \right).$$

(4)

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7See e.g. Mintz (1999). This result, however, does not hold under imperfect competition as shown by Nielsen, S.B., P.Raimondos-Møller, and G. Schjelderup (2001a).
Differentiating (3) and then imposing symmetry \( t_i^{t_i} = t_j^{t_j}, \alpha_i^t = \alpha_j^t \) we get

\[
\begin{bmatrix}
(1 - T_i) f_i^{i''} & 0 \\
0 & (1 - T_i) s_i^t f_i^{i''}
\end{bmatrix}
\begin{bmatrix}
dk_i^t \\
dk_j^t
\end{bmatrix}
= \begin{bmatrix}
T_i(r + \delta_i) & 0 \\
0 & T_j(r + \delta_j)
\end{bmatrix}
\begin{bmatrix}
dt_i^t \\
dt_j^t \\
d\alpha_i^t \\
d\alpha_j^t
\end{bmatrix}
\] (5)

which allows us to derive the relevant responses of tax policy on \( k_i^t \) and \( k_j^t \).

**Welfare maximization.** Preferences of the representative household in country \( i \) are given by the utility function \( U_i(c_i) \), where the consumption level \( c_i \) is determined by the budget constraint \( c_i = rS_i + \frac{k_i^t}{k_i^t + k_j^t} \Pi_i + \frac{k_j^t}{k_i^t + k_j^t} \Pi_j \). Here, \( S_i \) is the capital that the household invests on the international capital market. We take this as exogenous, but this has no consequence for our results as will be clear later. Residents in country \( i \) own a share \( k_i^t/(k_i^t + k_j^t) \) of the capital used by MNE\(_i\) and a share \( k_j^t/(k_j^t + k_j^t) \) used by MNE\(_j\). In a later section we discuss the implications of relaxing this symmetry assumption.

Inserting the individual budget constraint into the direct utility function gives indirect utility \( v_i(t_i^t, t_j^t, t_i^t, \alpha_i^t, \alpha_j^t, r) \). For the main analysis, we assume that the objective of each country is to maximize this utility of residents subject to a public budget requirement. We consider a pre determined political equilibrium where the two countries do not cooperate over tax rates nor tax bases. It is clear from the political discussions surrounding EU company tax reform that there might be political will to harmonize or at least coordinate tax bases, but no commitment has been given by any country to such a strategy. It is therefore of interest to characterize the case when no restrictions apply to each country’s choice of tax rate and base. Initially, we assume that countries can agree to the apportionment weights, but this assumption will be relaxed later.

The objective of country \( i \) is to choose \( t_i^t, t_j^t, \alpha_i^t \) and \( \alpha_j^t \) to maximize indirect utility \( v_i(\cdot) \) subject to an exogenous revenue requirement \( R_i \). We do not model explicitly

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8 A similar matrix can easily be derived for MNE\(_j\) to obtain the changes in \( k_j^t \) and \( k_j^t \) that result from a change in \( t_j^t, t_j^t, \alpha_j^t, \alpha_j^t \).

9 The choice of weights do not seem to be a political issue within the EU, rather it is the choice among alternative formula apportionment schemes that is politicized, see Sorensen (2004).
why there is such a revenue requirement but the underlying assumption may be that there for political and redistributive reasons must be a certain amount of tax revenue collected from the corporate sector.

Using (4) the Lagrangian to the optimization problem is:

$$\mathcal{L}_i(\cdot) = v_i(\cdot) + \lambda_i \left(-t_i^i \frac{\partial \Pi_i}{\partial t_i^i} - t_j^j \frac{\partial \Pi_j}{\partial t_j^j} - R_i\right).$$

Using the envelope theorem on the indirect utility function where $\mu_i$ is the marginal utility of income, the first-order conditions for $t_i^i$ and $t_j^j$ are:

$$(\mu_i - \lambda_i) \frac{\partial \Pi_i}{\partial t_i^i} + \lambda_i t_i^i(1 - \alpha_i^i) \frac{r + \delta_i}{1 - T_i k_i^i + k_i^i} \left(\frac{\partial k_i^i}{\partial t_i^i} + \frac{\partial k_j^j}{\partial t_i^i}\right) = 0, \quad (6a)$$

$$(\mu_i - \lambda_i) \frac{\partial \Pi_j}{\partial t_j^j} + \lambda_j t_j^j(1 - \alpha_j^j) \frac{r + \delta_j}{1 - T_j k_j^j + k_j^j} \left(\frac{\partial k_j^j}{\partial t_j^j} + \frac{\partial k_j^j}{\partial t_j^j}\right) = 0. \quad (6b)$$

Inspection of (5) shows that $\partial k_i^i/\partial t_i^i + \partial k_j^j/\partial t_i^i \neq 0 \forall \alpha_i^i \neq 1$ and zero otherwise. Similarly, the term $(\partial k_j^j/\partial t_j^j + \partial k_j^j/\partial t_j^j)$ vanishes at $\alpha_j^j = 1$, but is non-zero otherwise. In (5) the second terms in both first-order conditions are multiplied with a factor $(1 - \alpha)$ where sub/superscripts have been supressed, which is determined by the depreciation allowance given to MNC_i and MNC_j. If $\alpha_i^i = 1 = \alpha_j^j$, inspection of (6) shows that the second term in both conditions vanishes and both first-order conditions imply $\mu_i = \lambda_i$. Hence, marginal utility of income equals the shadow price of public revenues. The implication of this is that we have a first-best welfare optimum if we can show that $\alpha_i^i = 1 = \alpha_j^j$ is also compatible with the first-order conditions for $\alpha_i^i$ and $\alpha_j^j$. From (5) we know that $\partial k_i^i/\partial \alpha_i^i = 0$. Similarly, $\partial k_j^j/\partial \alpha_j^j = 0$. Using this information, the first order conditions for $\alpha_i^i$ and $\alpha_j^j$ are:

$$(\lambda_i - \mu_i) t_i^i(r + \delta_i) k_i^i \frac{k_i^i}{k_i^i + k_i^i} + \lambda_i t_i^i(1 - \alpha_i^i) \frac{r + \delta_i}{1 - T_i k_i^i + k_i^i} \frac{\partial k_i^i}{\partial \alpha_i^i} = 0 \quad (7a)$$

$$(\lambda_i - \mu_i) t_j^j(r + \delta_j) k_j^j \frac{k_j^j}{k_j^j + k_j^j} + \lambda_j t_j^j(1 - \alpha_j^j) \frac{r + \delta_j}{1 - T_j k_j^j + k_j^j} \frac{\partial k_j^j}{\partial \alpha_j^j} = 0 \quad (7b)$$

It is seen from (5) that $\partial k_i^i/\partial \alpha_i^i \neq 0 \forall T_i \in (0, 1)$, and $\partial k_j^j/\partial \alpha_j^j \neq 0 \forall T_j \in (0, 1)$. We can then conclude that $\alpha_i^i = \alpha_j^j = 1$ is a solution to the maximization problem.

From the derivations above it follows that when foreign direct investments are introduced and multinationals can shift profit through transfer pricing, it is optimal for each country not to distort the investment decisions of multinationals. Since under
formula apportionment, the multinational cannot save tax payments by manipulating profit income in a single country, distorting investment behavior has no effect on the incentive to shift profit and would only serve to reduce the overall amount of profit to be shared. Thus, in a symmetric country setting a formula apportionment tax is a tax on pure profit and reproduces the well known Schanz-Haig-Simons property derived in a closed economy setting. In order to see this, note that under symmetry assumptions and with full deductions, the apportionment weights are identical in each country and the average tax rate is \( T_i = t_i = t_j \) from (1). One can then rewrite the firm’s first order conditions in (3) as

\[
\begin{align*}
k_i s_i (1 - t_i) C_i' &= 0, \\
(1 - t_i) \left[ f_i'' - r - \delta_i \right] &= 0, \\
(1 - t_i) \left[ s_i f_i'' - r - \delta_i - p_i s_i \right] &= 0,
\end{align*}
\]

where it is seen that the tax rate can be factored away and the firm’s decisions are unaffected by taxation. The implication of the neutrality result is that each country in the Nash equilibrium provides complete deductions \( \alpha_i = \alpha_j = 1 \), and no coordination among countries is needed neither on the tax base nor on the tax rate to achieve the first best welfare optimum.

In the analysis above we assumed that the apportionment weights were not choice variables. The analysis was also conducted with only one factor of production namely capital. Relaxing these assumptions we obtain;

**PROPOSITION 1:** In a non-cooperative tax equilibrium where each country maximizes welfare by choosing its tax rate, tax base and weight to apportion profit, and each firm employs two factors (labor and capital), the symmetric Nash equilibrium yields a first best outcome, with true costs equal to tax deductible costs.

**PROOF:** See http://www.nhh.no/sam/res-publ/supplements/appitax.pdf

Introducing apportionment weights as a choice variable to each country and allowing firms to employ several factors of production do not change the incentive of countries to allow depreciation allowances to equal true depreciation costs, since these additions to the model do not affect the definition of taxable profit.
2.2 Separate Accounting

How does the above results compare to what one would find under separate accounting? An answer to this question is given by Haufler and Schjelderup (2000), who shows that the problem of profit shifting under separate accounting does not lead to a first best outcome, and that in the Nash equilibrium, countries use incomplete deductions (in our setting $0 < \alpha^i_j = \alpha^j_i < 1$) in order to tax the profit of the multinationals. The intuition is that the tax rate and the tax base are equally effective tax instruments to tax multinationals. Thus, if profit shifting is triggered by differences in statutory tax rates (as suggested by the empirical literature, see Hines 1999), then the tax elasticity of the tax base is lowered by incomplete deductions. This comes as a cost, though, since it means that the tax at the margin distorts firm behavior and a first best solution can therefore not be obtained. The problem, of course, is that the delineation of the tax base by arm’s length prices give rise to profit shifting that can only partially be cured by the use of two tax instruments simultaneously.

3 Concluding remarks

Our analysis above have made several simplifying assumptions and we would like to relax in order to assess the robustness of our results. A first important feature of the analysis is that the two countries jointly own with equal shares the two multinationals. It is of course possible to perceive other ownership structures. One such structure would be to either let each country have an ownership share equal to $(1 - \theta)$ in both firms, or alternatively, to let each countries have a different ownership share in either firm. It is not clear to us if this would retain the presumption in favor of formula apportionment. A clue to this might be found in Nielsen, Raimondos-Moller and Schjelderup (2001b) who model profit shifting and tax competition, but without a choice variable for the tax base. They allow a share $\theta$ of the multinationals to be owned by residents in country $i$ and the residual $(1 - \theta)$ by residents in country $j$. In their welfare analysis they find that a switch from separate accounting to

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10 The paper by Haufler and Schjelderup (2000) is obtained in a framework where one firm exports a good to the other firms. It is at least in theory possible that their results do not carry over in our framework. However, we shown in the Appendix that using the model above replicates the findings in Haufler and Schjelderup (2000).
formula apportionment may reduce welfare depending on how costly it is to shift profit and the level of profit the multinationals generate.

A third possibility would be to let both multinationals be owned by residents of a third country. This case has been examined by Kind, Midelfart and Schjelderup (2004) in a setting with trade and barriers to trade, but where, again, there is no choice of profit base. They find that under separate accounting the transfer price is relatively tax sensitive for a high degree of economic integration. In contrast, under a formula apportionment the transfer price is not very tax sensitive for high levels of economic integration. Thus if economic integration and shifty profits are a concern to policymakers, their message is that for a high degree of economic integration, tax competition intensifies under separate accounting while the opposite is the case under formula apportionment. These findings are mirrored in their welfare analysis which show that for a low (high) level of economic integration a system of separate accounting (formula apportionment) is welfare dominating.

4 Appendix

In this appendix we show the outcome of the tax competition game under separate accounting.

**Firm behavior.** Under separate accounting the total net profits of MNE$_i$ are

$$\Pi_i = (\pi_i^i + \pi_i^j) - t_i^i \pi_i^{T_i} - t_i^j \pi_i^{T_j}$$

where $t_i^i \in [0, 1]$ is the rate of the corporation tax on MNE$_i$ in country $i$ and $t_i^j \in [0, 1]$ is the corporation tax on MNE$_i$ in country $j$. MNE$_i$ maximizes $\Pi_i$, and the first order conditions for chooses $\gamma_i$, $k_i^i$, and $k_i^j$, respectively are

$$k_i^i s_i (t_i^j - t_i^i) - (1 - t_i^j) C_i^j \leq 0$$

$$s_i (\gamma_i (t_i^j - t_i^i) - (p_i + C_i^j) (1 - t_i^j) + (1 - t_i^j) f_i^j) - (1 - \alpha_i t_i^j) (r + \delta_i) = 0 \quad (8a)$$

The first line in $(8a)$ shows that there will be no transfer pricing if $t_i^i = t_i^j \rightarrow C_i^j = 0$ or $C_i^j (\gamma_i = p_i) \neq 0$, i.e., concealment costs are prohibitive ($C_i^j > 0$). To

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11 In which case our maximization problem disappears.
concentrate on interior solutions we assume that concealment costs are such that the latter will never be the case. Given this, the first line holds with strict equality. The second and the third line have the usual interpretations of equating marginal revenues to marginal costs.

Using the envelope theorem on (8a) yields

\[
\frac{\partial \Pi_i}{\partial t_i^i} = -\pi_i^{T_i} \quad \frac{\partial \Pi_i}{\partial t_j^i} = -\pi_j^{T_i}
\]

We differentiate (8a) and then impose symmetry assumptions \( t_i^i = t_j^j, \alpha_i^i = \alpha_j^j \) to obtain

\[
\begin{bmatrix}
(1 - t_i^i)f_i^{in} & 0 & 0 \\
0 & (1 - t_j^j)s_i^2f_j^{in} & 0 \\
0 & 0 & -k_i^is_iC_i^{in}
\end{bmatrix}
\begin{bmatrix}
dk_i^i \\
dk_j^j \\
d\gamma_i
\end{bmatrix}
= \begin{bmatrix}
(1 - \alpha_i^i)r_i + \delta_i \\
0 & t_i^i(r + \delta_i) & 0 \\
0 & 0 & -t_i^i(r + \delta_i)
\end{bmatrix}
\begin{bmatrix}
dt_i^i \\
dt_j^j \\
d\alpha_i^i
\end{bmatrix}
\]

(9)

where \( \theta = s_i(C_i + p_i - \gamma_i)(r + \delta_i)\frac{1-\alpha_i^i}{1-\alpha_i^{j_i}} \). Using Cramer’s rule we can now get the changes in \( k_i^i, k_j^j, \gamma_i \) following changes in the policy instruments \( t_i^i, t_j^j, \alpha_i^i, \alpha_j^j \) for a given concealment costs function. A similar matrix can easily be derived for MNE\( j \) to obtain the changes in \( k_j^j, k_i^j, \gamma_j \) that follows when \( t_i^j, t_j^j, \alpha_i^j, \alpha_j^j \) changes.

**Maximizing welfare.** The objective of country \( i \) is to choose \( t_i^i, t_j^j, \alpha_i^i \) and \( \alpha_j^j \) to maximize indirect utility \( v_i(\cdot) \) subject to a budget constraint which amounts to raising an amount taxes \( R_i \). From the first order conditions of the firm the associated Lagrangian is:

\[
\mathcal{L}_i(\cdot) := v_i(\cdot) + \lambda_i \left( -t_i^i \frac{\partial \Pi_i}{\partial t_i^i} - t_j^j \frac{\partial \Pi_j}{\partial t_j^j} - R_i \right)
\]

We define \( \mu_i \) the marginal utility of income. Using the envelope theorem on the indirect utility function, the first-order conditions for and \( t_i \) and \( t_j^i \) respectively are:
\[(\mu_i - \lambda_i) \frac{\partial \Pi_i}{\partial t_i} + \lambda_i \left( (1 - \alpha_i^i) \frac{r + \delta_i}{1 - t_i} \frac{\partial k_i^i}{\partial t_i} - s_i (C_i + p_i - \gamma_i) \frac{\partial k_i^i}{\partial t_i} + k_i^i s_i \frac{\partial \gamma_i}{\partial t_i} \right) = 0 \]  
\[(10a)\]

\[(\mu_i - \lambda_i) \frac{\partial \Pi_j}{\partial t_j} + \lambda_i \left( (1 - \alpha_j^i) \frac{r + \delta_j}{1 - t_j} \frac{\partial k_j^j}{\partial t_j} - s_j (C_j + p_j - \gamma_j) \frac{\partial k_j^j}{\partial t_j} + k_j^j s_j \frac{\partial \gamma_j}{\partial t_j} \right) = 0 \]  
\[(10b)\]

Using (9) in (10) it is straightforward to see that the coefficient of the Lagrangian parameter in (10) is negative for all \(\alpha_i^i, \alpha_j^j \in [0, 1]\) and all \(t_i, t_j \in [0, 1]\) and therefore \(\mu_i \neq \lambda_i\). It then becomes immaterial to check the first-order conditions for the depreciation allowance in order to verify that the first-best cannot be obtained under separate accounting.■

### References


