Optimal Income Taxation with a Risky Asset –
  The Triple Income Tax

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Abstract

We show in a two-period world with endogenous savings and two assets, one of them exhibiting a stochastic return that an interest adjusted income tax is optimal. This tax leaves a safe component of interest income tax free and taxes the excess return with a special tax rate. There is no trade-off between risk allocation and efficiency in intertemporal consumption. Both goals are reached. As the resulting tax system divides income into three parts, the tax can also be called a Triple Income Tax. This distinction and a special tax rate on the excess return is necessary in order to have an optimal risk shifting effect.

JEL-Classification: H21

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1 Introduction

There is an old but still ongoing debate on the best tax system in most OECD countries. Especially in Germany and the USA there are proponents of a consumption orientated taxation on the one hand and defenders of the comprehensive income taxation on the other. In a major reform, the Nordic countries have implemented a Dual Income Tax, which apportions total income in segmented tax bases of labor and capital income (see, i.e., Sørensen, 1994). In all cases, however, this debate does not consider the effects of risk in personal income and therefore does not deal with risky tax revenue. Consequently, the questions we want to answer in this paper are: First, what is the optimal income taxation in case of aggregate risk? Second, which tax system should therefore be implemented?

In a world without uncertainty and distributional considerations the optimal tax structure for financing public expenditure is lump sum. In case of uncertainty about the individual wage rate, so called private risk which is unsystematic and can be insured, this statement does not hold. Eaton and Rosen (1980) showed in their seminal paper that for a one-period world with endogenous labor choice an income tax with a strictly positive marginal tax rate improves welfare. The government pools the private risk of all individuals and uses a lump sum transfer in order to return the deterministic tax revenue. The income tax takes the function of a social insurance scheme against private risk. Varian (1980) showed similar effects for a two-period world where the households work for a known wage rate and have to choose between consumption and savings. In his analysis, individuals face private risk because of uncertainty about the best investment portfolio.

The risk category however, we are interested in, is aggregate risk. This risk is systematic and hits all agents at the same time and in the same manner. Therefore, no insurance is possible, but incurring this risk pays out a risk premium. Amazingly, there are only few contributions to this topic. Richter and Wiegard (1991) examine a model with endogenous savings. In their two-period model the households have inelastic labor supply in period zero and divide their exogenous labor income between consumption and savings. Consumption in the following period is financed by savings and a stochastic interest income. Richter and Wie-
gard show that a tax on this risky interest income improves welfare under certain conditions – even in world with exogenous labor supply. The optimal tax rate is a trade-off between efficiency and insurance. Therefore, it depends on the elasticity of current consumption with respect to a compensated relative change of the tax rate. Further they show that a consumption tax cannot achieve this insurance function. Related studies have been done by Richter (1992). He examines the portfolio choice decision in a two asset world with one exhibiting a stochastic return and one safe asset. Richter develops an optimal elasticity rule for the taxation of asset returns and demonstrates that a cash flow tax is not optimal, if tax rates are not differentiated. Unfortunately, these papers assume risk neutrality in public consumption. Hence, they collapse to the case of private risk and are limited in their statements concerning risky tax bases. Christiansen (1993), instead, points out some optimal tax rules for portfolio choice in case of risk aversion in public consumption, but his approach cannot be linked to tax systems, mentioned above.

All these papers suggest that in case of uncertainty a consumption tax is always inferior compared to an income tax. This is because an income tax provides superior insurance by taxing capital income and the sacrifice of a distorted intertemporal consumption decision is more than compensated by the reduction of risk. However, looking at the capital taxation used, in all papers the return in each single asset is liable to one separated tax rate. Thus, they are not able to deal with risk effects separately, as we point out in the next section.

We show in our paper that the sacrifice of efficiency is not necessary. First, by using a adequately defined tax system, we can achieve both insurance and intertemporal efficiency. Second, we are able to show, that the resulting optimal tax scheme is a modern form of consumption taxation. Precisely, we will get a kind of consumption-orientated income tax with interest adjustment (see i.e., Rose 1999, pp. 35ff). Furthermore, we can state that a consumption tax is able to insure against risk in capital income in an optimal manner.

The remainder of the paper will be as follows. In section 2 we present the model and examine the household choice, whereas section 3 discusses the optimal tax structure for a welfare maximum. The paper closes with some conclusions.
2 The Model and Household Choice

We use a two-period model without any bequest motive. There is a homogenous individual, receiving exogenous labor income \( y \) in period one and dividing it on first period consumption \( c_0 \) and savings \( s_0 \). Savings can be invested in an asset \( A_0 \) with a certain return \( r > 0 \) and in a risky asset \( A_1 \), which has a stochastic return \( \tilde{x} \geq -1 \). We assume \( E[\tilde{x}] > r \). Savings are the only source of consumption in the second period.

The government can use both a proportional wage tax \( t^L \) in the first period\(^1\) and a tax on capital income. For capital income taxation we follow the approach of Hilgers and Schindler (2002) and use a two-part interest income tax. We tax the riskless return \( r \) in both assets with rate \( t_0 \) and the excess return \( (\tilde{x} - r) \) with rate \( t_1 \) and assume full loss offset. If the realization of the excess return is negative, this loss will lead to a tax refund of \( t_1 \cdot (\tilde{x} - r) \).

The motivation for the design of the capital tax system is as follows: The investor does not invest in assets in order to own the assets. On one hand, she does investments in order to shift resources into the future for financing future consumption. However, postponing consumption creates some kind of disutility as long as her marginal rate of time preference is positive and she has therefore to be compensated for giving up present consumption. The price of this compensation for resource shifting equals the riskless rate of return and is paid out by both the riskless asset and the risky one. On the other hand, the investor can expand her income for future consumption by buying asset \( A_1 \) and incurring risk. Risk creates disutility and for incurring this risk the investor receives a risk premium. After realization of risk, this price for risk equals the excess return \( \tilde{x} - r \), which can be negative in bad states of the world. Hence, the investor gets additional (risk-)income \( (\tilde{x} - r) \cdot A_1 \) according to her risk preferences which is positive in expected values. Taken together the prices for resource shifting and risk are liable to two separated tax rates under our tax system, whereas in most former papers the assets’ returns are liable to asset-specific tax rates. This means for the risky

\(^1\)This wage tax is equivalent to a lump sum tax.
asset that the prices mentioned are mixed and taxed at one rate. If so however, no separated treatment of risk is possible.\(^2\)

The savings can now be written as
\[
s_0 = A_0 + A_1 = (1 - t^L)y - c_0.\]
Consumption in period 1 is
\[
\bar{c}_1 = [(1 - t_1)(\bar{x} - r)A_1 + [1 + r(1 - t_0)])((1 - t^L)y - c_0).\]
We assume that the representative investor is risk averse in both private and public consumption and her von Neumann-Morgenstern utility function takes the form
\[
W = E[U(c_0, \bar{c}_1, \bar{g})] \text{ with } U_c > 0; U_{cc} < 0; U_{g} > 0; U_{gg} < 0.
\]
Furthermore, we assume that future consumption and the public good are complements, \(U_{c1g}, U_{gc1} \geq 0\).

In this model, the government chooses the tax rates and the tax revenue is used completely to finance the public good in period 1. Therefore, the probability distribution of \(\bar{g}\) is also an instrument variable of the government.

The household maximizes his expected utility \(W\) for given tax rates by choosing her optimal first period consumption \(c_0\) and her optimal savings \(A_0 + A_1 = (1 - t^L)y - c_0\) with respect to her budget constraint. She does not anticipate the effect of her saving behavior on the level of the public good. Inserting the budget constraint for \(\bar{c}_1\), the maximization problem can be written as
\[
\max_{c_0, A_1} W = E[U(c_0, (1 - t_1)(\bar{x} - r)A_1 + [1 + r(1 - t_0)]((1 - t^L)y - c_0), \bar{g})].
\]
The first order conditions of the household problem are:
\[
\begin{align*}
\frac{\partial W}{\partial c_0} &= E[U_{c0}] - E[U_{c1} \cdot [1 + r(1 - t_0)]] = 0 \\
\frac{\partial W}{\partial A_1} &= (1 - t_1)E[U_{c1} \cdot (\bar{x} - r)] = 0
\end{align*}
\]
Optimal values of \(c, A_1\) and \(s\) are denoted \(c_0 = c_0(t_0, t_1, t^L), A_1 = A_1(t_0, t_1, t^L)\) and \(s_0 = s_0(t_0, t_1, t^L)\). For the marginal rate of time preference we obtain:
\[
\rho = \frac{E[U_{c0}]}{E[U_{c1}]} - 1 = r(1 - t_0)
\]
\(^2\)In the Hilgers/Schindler model there is no labor income, but the tax system for capital income is equivalent to our approach. The idea is to have enough instruments for pursuing two goals, namely an optimal resource allocation as well as an efficient risk diversification.
Equation (3) indicates that our tax system does not distort portfolio choice, as the FOC is equal to the optimality condition in case of no taxation.

**Proposition 1** The tax rate \( t_1 \) on the excess return \((\bar{x} - r)\) does not affect overall savings \( s_0(t_0, t_1, t_1^L) \) as \( \frac{dc_0}{dt_1} = 0 \). Further, \( t_1 \) has only a Musgrave-substitution effect on \( A_1 \) and \( \frac{dA_1}{dt_1} = A_1 \).

**Proof:** Let \( p_r = 1 + r(1 - t_0) \) and \( p_A = (1 - t_1)(\bar{x} - r) \). Totally differentiating equations (2) and (3) with respect to \( c_0, A_1 \) and \( t_1 \) gives:

\[
\begin{pmatrix}
E \left[ U_{c_0c_0} - U_{c_0c_1}p_r - U_{c_1c_0}p_r + U_{c_1c_1}p_r^2 \right] & E \left[ U_{c_0c_1}p_A - U_{c_1c_1}p_rp_A \right] \\
E \left[ U_{c_1c_0}p_A - U_{c_1c_1}p_Ap_r \right] & E \left[ U_{c_1c_1}p_A^2 \right]
\end{pmatrix}
\cdot
\begin{pmatrix}
dc_0 \\
dA_1
\end{pmatrix}
= 
\begin{pmatrix}
E \left[ U_{c_0c_1}p_A - U_{c_1c_0}p_r + U_{c_1c_1}p_r^2 \right] \\
E \left[ U_{c_1c_1}p_Ap_r \right]
\end{pmatrix}
\cdot
A_1 \cdot dt_1
\tag{5}
\]

Using Cramer’s Rule, we get \( \frac{dc_0}{dt_1} = 0 \), as the modified determinant \( \det \alpha_{d t_1 A_1} \) in the nominator equals zero, and \( \frac{dA_1}{dt_1} = A_1 \cdot \frac{A_1}{1 - t_1} \) as \( \det \alpha_{c_0 d t_1} = \det \alpha_{c_0 A_1} \cdot \frac{A_1}{1 - t_1} \).

This result corresponds to the result for taxing capital gains in Sandmo (1969, Section 8) and is similar to the portfolio choice result for a net tax in case of several risky assets (Sandmo, 1977). By investing more in the risky asset according to \( \frac{dA_1}{dt_1} = \frac{A_1}{1 - t_1} \) and diminishing the investment in the riskless asset by the same amount and therefore keeping both first period and second period consumption constant, the tax rate change in \( t_1 \) does not change expected utility of the household.

### 3 Optimal Taxes on Interest Income

The government collects tax revenue not only from capital taxation in period 1 but also from the wage tax in the first period. However, all spending for the public good take place in period 1 only. Hence, we assume that the tax revenue of the wage tax in period 0 is invested entirely in the safe asset.\(^3\) Thus, in period 1, the

\(^3\)This may be interpreted as a short cut for an overlapping-generations model, where the government provides an old-generation specific public good and where riskless rate of return equals population growth in steady-state. See, e.g., Sandmo (1985), Section 7.
government’s budget restriction can be written:

\[
\tilde{g} = (1 + r)t^L \cdot y + t_1(\bar{x} - r) \cdot A_1(t_0, t_1, t^L) + t_0r \cdot ((1 - t^L)y - c_0(t_0, t_1, t^L))
\]

The government chooses now the tax rates and the public good \(g\) in order to maximize the social welfare function:

\[
\Omega = E\left[U(c_0(t_0, t_1, t^L), \tilde{c}_1(t_0, t_1, t^L), \tilde{g})\right]
\]
given optimal household choice and subject to its budget restriction. We get the following optimization problem:

\[
\max_{t_0, t_1, t^L} E\left[U(c_0(\cdot), \tilde{c}_1(\cdot), (1 + r)t^L \cdot y + t_1(\bar{x} - r) \cdot A_1(\cdot) + t_0r \cdot ((1 - t^L)y - c_0(\cdot))\right]
\] (6)

By using optimal household choice (2) and (3), we get as first order conditions:

\[
E\left[-U_{c_1}r \cdot s_0 + U_g \cdot t_1(\bar{x} - r) \cdot \frac{\partial A_1}{\partial t_0} + r \cdot s_0 + t_0r \cdot \frac{\partial c_0}{\partial t_0}\right] = 0
\] (7)

\[
E\left[U_g \cdot (\bar{x} - r) \cdot A_1 + t_1(\bar{x} - r) \cdot \frac{\partial A_1}{\partial t_1} + t_0r \cdot \frac{\partial c_0}{\partial t_1}\right] = 0
\] (8)

\[
E\left[-U_{c_1} (1 + r(1 - t_0)) + U_g \cdot (1 + r(1 - t_0) + t_1(\bar{x} - r) \cdot \frac{\partial A_1}{\partial t^L} + t_0r \cdot \frac{\partial c_0}{\partial t^L})\right] = 0
\] (9)

As \(\frac{\partial A_1}{\partial t^1} = \frac{A_1}{1 - t_1}\) and \(\frac{\partial c_0}{\partial t^1} = 0\) from Proposition 1, (8) can be rewritten as:

\[
E\left[U_g \cdot (\bar{x} - r)\right] \cdot \frac{A_1}{1 - t_1} = 0.
\] (10)

Then, we can conclude from using (10) and simplified versions of (7) and (9):

**Proposition 2** An optimal income tax system in case of exogenous labor income and risky returns to at least one asset does not tax the riskless rate of return \((t_0 = 0)\). The optimal tax rate on the excess return \((\bar{x} - r)\) is strictly positive and in the open interval \(t_1 \in (0; 1)\), if the households are risk averse in both private and public consumption. The wage tax is used to equate marginal utility of public and private consumption in period 1.
Proof: see Appendix.

If $t_1$ is set optimally, we have $\text{Cov}(U_{c_1}, \tilde{x}) = \text{Cov}(U_g, \tilde{x})$. As the households are risk averse in both private and public consumption, in an optimum, the risk must be diversified on both types of consumption. This diversification depends on the relative strength of the risk aversion in private consumption compared to the one in public consumption. Therefore, the tax rate $t_1$ depends on this relative strength: The higher the risk aversion in private consumption relative to the one in public consumption, the higher the tax rate on the excess return $(\tilde{x} - r)$.

As the government returns the risk to the households by providing a public good, our result is general and independent of any assumption whether the government can deal better with risk than the capital market. Furthermore, because of providing an additional asset (the public good), which cannot be provided by the capital market, the tax on the risk component is not neutral for the government and achieves positive tax revenue in expected values. Hence, Gordon’s (1985) result does not hold in our model.\footnote{See Kaplow (1994), p. 795.}

Bulow and Summers (1984) state that most risk is embedded in economic depreciations which are not liable to taxation. Thus, the government would not participate in the risk and there is no insurance effect of taxation. This is true for corporate taxation, using statutory depreciation rates. However, we tax ex-post capital income, where the risky rate of return depends on the economic value of the underlying firm.\footnote{Assume asset 1 as a stock. Then its return $\tilde{x}$ reflects the firm’s fluctuating value.} Thus, the government fully participates in all income risk and the critic of Bulow and Summers (1984) does not apply. But, if we assume risk neutrality in public consumption and marginal utility of future consumption is independent of stochastic fluctuations in the level of the public good, we get as special case $t_1 = 1$ and all risk is concentrated in public consumption. This would be in accordance with the Arrow-Lind Theorem, where the government can diversify aggregate risk perfectly. Finally, we can state:

**Proposition 3** If an interest adjusted income tax is implemented, taxing the excess return according to Proposition 2 ($t_1 \in (0; 1)$) and letting the
riskless component of interest yield tax free ($t_0 = 0$), an efficient risk allocation is achieved without disturbing the intertemporal consumption decision. There is no trade-off between risk and efficiency in allocation. The marginal rate of time preference equals the riskless rate of return ($\rho = r$).

**Proof:** From (4), the marginal rate of time preference is $\rho = r \cdot (1 - t_0)$. For $t_0 = 0$, $\rho = r$. In the optimum, the marginal rate of time preference is then independent of the tax rates and the intertemporal consumption decision is not distorted. Additionally, $\text{Cov}(U_{c1}, \tilde{x}) = \text{Cov}(U_g, \tilde{x})$ assures efficient risk allocation.

As mentioned above, the literature shows that a consumption-orientated taxation cannot achieve the insurance function of a traditional income tax in case of risky capital income. But is it true that an income tax always yields better results? Examining our analysis, this view must be handled with care.

We use a proportional wage tax on exogenous labor income in the first period. The safe rate of return on savings is tax-free, whereas the excess return or supernormal profits are taxed with a special tax rate. This tax scheme equals a modified consumption-orientated income tax with interest adjustment.

Therefore, we have a consumption tax, which optimally provides insurance against risky capital income and simultaneously avoids a distortion in the intertemporal consumption decision.

This tax scheme is an extension of the Dual Income Tax approach and can be named a *Triple Income Tax* as we divide the full income in three different parts. The excess return (or risk premium) is one of it. This distinction is necessary for achieving an optimal risk allocation by taxation.

The intuition behind these results is straightforward. On the one hand, it is optimal to diversify the aggregate risk between private and public consumption.

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6A consumption-orientated income tax with interest adjustment taxes the overall labor income and tax-exempts interest income. For excess returns in capital income a tax with the same tax rate as for labor income is possible. See i.e., Rose 1999, pp. 35ff.

7Calling a tax system Triple Income Tax, where one tax rate equals nil, seems not to be very intuitive on first sight. However, the tax system is also suitable in case of an extended model using endogenous labor choice. Then, all three tax rates are positive.
On the other hand, risk shifting has negative welfare effects by disturbing the intertemporal consumption decision, if we tax the risky asset with only one tax rate. In this case, there is a trade-off and the optimal tax rate depends on the strength of these effects. If we instead tax the excess return with a special tax rate, the tax system is well defined and the trade-off can be avoided. Thus, we reach both optimal risk allocation and efficiency in intertemporal consumption simultaneously.

4 Conclusions

We showed that an interest adjusted income tax can guarantee a welfare maximum in a two-period world with two assets, one of them exhibiting a stochastic return. The excess return must be taxed separately and possible losses in this tax base must be subsidized. In case of risk aversion in public consumption, we have an inner optimum with $t_1 \in (0; 1)$ because the risk must be diversified on both consumption types for having an optimal risk allocation.

As such a tax system is a kind of indirect consumption taxation, we showed that a consumption tax is able to insure against risky interest income. Furthermore, the tax system can be called Triple Income Tax, because it uses three separated tax bases. This extension of the Dual Income Tax approach is due to considering risk effects in personal income and tax revenue explicitly. Although not done in the paper, it is straightforward to show that these results also prevail in a multi-asset world.

The tax system has an interesting advantage from a political point of view. In public opinion there is usually major support for taxing supernormal profits in capital income, whereas, at least in Germany, such capital gains are mostly tax free at the moment. Now, this paper shows that taxing excess returns can be done in a welfare-enhancing manner without effecting utility of private consumption. Therefore, the risk tax creates no incentives for households to engage in tax planning. However, a disadvantage of our tax system may be that individuals have an incentive to declare labor income as preferred taxed riskless capital income in
order to avoid taxes. This problem is similar to the case of a Dual Income Tax with separate tax rates for labor and capital income (see, i.e., Sørensen, 1994).

Related work can be done in a multi-asset world with a fixed amount of savings. In such a world, Richter (1992) and Christiansen (1993) show that there is a trade-off between risk allocation and optimal portfolio choice. If the same tax system is introduced as in this paper, this trade-off should also be overcome. Further work could also examine a multi-asset world with endogenous savings and labor-supply.

Another interesting topic emerges if we look at further work concerning heterogeneous households. The taxation of the excess return effects utility solely by public consumption. Despite this, the risk tax can be used for redistribution, if individuals with lower income have higher preferences for the public good than high-income households. Here, redistribution is to be thought as enhancing utility of low-income households, which is possible by increasing the risk tax and creating higher tax revenue in expected values. Contrary to the traditional literature on redistribution we get an instrument which avoids any trade-off between efficiency and redistribution, but society has to pay for it with increased risk in consumption.

5 Appendix: Proof to Proposition 2

In proving Proposition 2 we use (10) in FOC (7) and (9) and obtain $E[U_g - U_{c_1}] \cdot r \cdot s_0 + E[U_g] \cdot t_0 r \cdot \frac{dc_0}{dt_0} = 0$ and $E[U_g - U_{c_1}] \cdot [1 + r(1 - t_0)] + E[U_g] \cdot t_0 r \cdot \frac{dc_0}{dt} = 0$. Combining these expressions results in

$$t_0 \cdot \left( [1 + r(1 - t_0)] \frac{dc_0}{dt_0} - r \cdot s_0 \frac{dc_0}{dt} \right) = 0. \quad (11)$$

Household’s budget constraint can be displayed as

$$\tilde{c}_1 + p_r \cdot c_0 = (1 - t_1)(\tilde{x} - r)A_1 + p_r \cdot (y - a), \quad (12)$$

where $p_r = [1 + r(1 - t_0)]$ is the price of present consumption, $p_r \cdot y$ is the value of endowment, $p_r \cdot a = p_r \cdot tL \cdot y$ is the lump-sum tax payment, induced by the wage tax, and $I = p_r(y - a)$ is full income.

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8See, for example, Hilgers and Schindler (2002) for a first analysis.
Thus, \( \frac{\partial c_0}{\partial t_0} = \frac{\partial c_0}{\partial p_r} (-r) \) and we define \( \frac{\partial c_0}{\partial t} = \frac{\partial c_0}{\partial a} \) because \( \frac{\partial c_0}{\partial t} \) is a pure income effect as long as labor supply is exogenous. Using the Slutsky equation for \( \frac{\partial c_0}{\partial p_r} \) and endowment effects in equation (11), the income effects cancel out and we are left with the substitution effect:

\[
t_0 \cdot (p_r \cdot (-r) \cdot S_{c_0c_0}) = 0
\]

(13)

and therefore unambiguously \( t_0 = 0 \), as \( S_{c_0c_0} < 0 \) as long as the tax on riskless return is not another lump-sum tax.

For \( t_0 = 0 \), from (7) and (9) then follows \( E[U_{c_1}] = E[U_g] \). Using FOC (3) of the household problem and (10), we can write

\[
E[U_{c_1} \cdot (\bar{x} - r)] = 0 = E[U_g \cdot (\bar{x} - r)].
\]

(14)

As \( E[Y \cdot Z] = E[Y] \cdot E[Z] + \text{Cov}(Y, Z) \) and \( E[U_{c_1}] = E[U_g] \), this expression can be simplified to \( \text{Cov}(U_{c_1}, \bar{x}) = \text{Cov}(U_g, \bar{x}) \). But, this is only possible for \( t_1 \in (0; 1) \).

\[\Box\]

**References**


