Do Consumers Buy Less of a Taxed Good?∗

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Abstract

This paper shows that consumers may buy more of a taxed good if it is sold by a two-sided platform firm. Two-sided platform industries serve distinct customer groups that are connected through interdependent demand, and include major businesses such as the media industry (newspapers/magazines and advertisers), banking (cardholder and merchant), and the software industry (users and application developers). The paper compares ad valorem and specific taxes and shows that they may have opposite effects on quantities sold, and that the ad valorem tax - the most commonly used tax throughout the OECD - has effects on prices and quantities not previously recognized.

Keywords: Two-sided markets, ad-valorem taxes, specific taxes

JEL Codes: D4; D43; H21; H22; L13

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1 Introduction

A benchmark result in economics is that consumers buy less of a taxed good.\(^1\) In this paper we show a new result. Consumers may actually buy more of a taxed good if it is sold by a two-sided platform firm. In particular, a higher ad-valorem tax may lower the end-user price and increase sales.

By definition, a two-sided platform firm serves two different groups of customers that are connected through interdependent demand.\(^2\) Our analysis shows that in such markets, an increase in the ad valorem tax in one side of the market affects the relative profitability between the two markets, making the firm want to shift its earnings to the market where the tax rate is unchanged. To see the logic involved consider a firm that sells good A in market A and good B in market B. Suppose there is a positive externality from market A to market B, say, in the sense that sales in market B are positively correlated with sales in market A. In such a case the firm may generate more income in market B if it reduces the price and increases output in market A.

An example of the incentive mechanism at work above can be illustrated by a media firm, which derives income from selling a newspaper and advertisements, and where the income from advertisements depends positively on newspaper sales. An increase in the ad valorem tax rate on the newspaper may induce the media firm to rely more on income from advertisements because it can reduce the burden of the tax by lowering the price on the newspaper and attract more readers. This will increase the profit margin of the media firm in the market for advertisements. In the extreme, a very high tax on newspapers can lead the media firm to provide the newspaper free of charge and rely on income from advertising only.

In order to bring forth our results we set up a general model of a two-sided market, and show the exact conditions for when a tax increase causes the end-user price to fall and demand to rise. For the sake of convenience, we use the media

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\(^1\)An overview of the tax incidence literature is given by Fullerton and Metcalf (2002).

\(^2\)Evans (2003a,b) provides examples and classifications of two-sided markets.
example above to explain our results, but our model is general in nature.

Two-sided platform firms are found in major businesses such as the media industry, the financial sector, real-estate brokerage, and the computing industry. Many two-sided platform firms receive favorable tax treatment. Newspapers, for example, are taxed at a reduced rate or completely exempted from value-added taxation in most countries, since governments consider such publications to be an essential channel for disseminating vital information about e.g. culture, politics, and international affairs. Similarly, many countries exempt credit card services from value added taxation, partly in order to disseminate the use of these services among all income groups.

An important feature of a two-sided platform firm is that its pricing strategies must account for interactions between the demands of multiple customer groups and the externalities that arise in these relationships (Rochet and Tirole, 2003). In the media industry, advertising may be perceived as a nuisance (a negative externality) or a benefit (a positive externality) by readers.\(^3\) A media firm can internalize this externality by charging advertisers and readers/viewers appropriately. We show that the sign and size of such externalities are decisive for the effects of changes in ad valorem tax rates.

Our analysis is related to a growing literature on Industrial Organization that analyzes the price-setting behavior of firms in two-sided markets. However, this literature does not consider taxation issues.\(^4\) The literature on indirect taxation, on the other hand, does not consider two-sided markets.\(^5\)

Closest to the spirit of our analysis are an early paper by Edgeworth (1925) and follow-up contributions by Hotelling (1932), Wicksell (1934), and Bailey (1954). Edgeworth argued that higher commodity taxation under certain conditions may

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\(^3\)The nature of these externalities is further discussed in the formal model below.


\(^5\)E.g., Keen and Delipalla (1992), Dierickx, Matutes and Neven (1998) and Anderson et. al. (2001a, 2001b). For a survey, see Fullerton and Metcalf (2002).
reduce end-user prices if demand for two different goods is directly interrelated. This possibility has later been labelled Edgeworth’s Taxation Paradox. As an illustration, Edgeworth considered demand for first-class and third-class railway tickets. His assessment was that a tax imposed on first-class tickets may give the railway company an incentive to reduce the price of the untaxed good - third-class tickets - in order to sell more of it. Indeed, under certain conditions the price of both types of tickets will fall subsequent to the tax increase.⁶

There are probably many reasons why no link has been made between Edgeworth’s Taxation Paradox and indirect taxation in two-sided markets. First, the specific example used by Edgeworth is admittedly peculiar, and may explain why it has been almost forgotten in the literature.⁷ Second, the example given by Edgeworth relates to a one-sided market with substitutes.⁸ Third, Edgeworth focused solely on specific taxes, and not on the more widely used ad valorem tax. As shown in our analysis, the most interesting policy recommendations arise when we compare the effects of specific and ad valorem taxes, as indeed more recent contributions have emphasized (e.g., Delipalla and Keen 1992).

The rest of the paper is organized as follows: Section 2 sets up a model of a two-sided platform and analyzes the effects of an ad valorem tax on quantities and prices. Section 3 discusses policy implications and carries out an analysis with respect to specific taxes, while Section 4 presents conclusions and discusses some extensions to the model.

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⁶See Creedy (1988) for a good overview and discussion of this literature.
⁷For example, it is not mentioned in the Handbook of Public Economics (see Fullerton and Metcalf 2002).
⁸Bailey (1954) shows the precise conditions under which the tax paradox may arise, and points out a mistake in Hotelling’s analysis.
2 The Model

Consider a two-sided monopoly platform firm selling good $C$ to consumers, say, at price $q$, and good $X$ to producers, say, at price $p$. We assume that both the consumers and the producers are price takers. Let $c$ and $x$ denote the respective quantities of the two goods. The inverse demand function for each good is downward-sloping, so that the own-price effects are negative; $q_c \equiv \partial q/\partial c < 0$ and $p_x \equiv \partial p/\partial x < 0$ (subscripts henceforth denote partial derivatives). The willingness to pay for each good, however, may also depend on how much is sold of the other good. The sale of good $X$ imposes a positive externality on the willingness to pay for good $C$ if $q_x > 0$ and a negative externality if $q_x < 0$. In the same manner, good $C$ may impose a positive or negative externality on demand for good $X$. The inverse demand functions can thus be written as $q = q(c, x, \omega)$ and $p = p(c, x, \omega)$, where $\omega$ is a vector of other factors that may affect demand, including the general VAT rate ($T$) in society.

Examples that fit the model structure above can be found in many sectors of the economy, such as the media industry (or banking) where $C$ is a newspaper/broadcasting (banking; credit cards) and good $X$ is advertising space (banking; shops that accept cards). For the sake of convenience and to emphasize the economic intuition and policy relevance of our results, we shall in what follows relate our model to a media firm (the platform) selling a newspaper to readers and advertising space to firms.

An ad valorem tax ($t$) is levied on good $C$, which implies that the platform receives the price $q/(1 + t)$ from the consumers. The tax rate $t$ may deviate from the general VAT rate $T$. Our focal point here is to examine the effects of a change in the tax rate $t$, holding $T$ and other elements in $\omega$ fixed. For this reason we do not model ad valorem taxes on ads, and in what follows we shall suppress $\omega$ in the

\footnote{This is an externality since producers and consumers are price takers. Thus, they do not take into account the effect of their actions on the demand in either side of the market.}
inverse demand functions.\footnote{We hold $T$ fixed, reflecting the view that the general VAT level is determined by more overriding concerns than targeted tax policy in one single market. Note also that interfirm value-added taxes (e.g. on ad revenues) are fully rebated by the government under VAT.}

The platform has the following profit level:

$$\pi = \max_{c,x} \left[ xp(c, x) + \frac{cq(c, x)}{1 + t} - k(c, x) \right],$$

where $k(c, x)$ is the cost function, with $k_i \geq 0$ ($i = c, x$).

The first-order condition for good X ($\pi_x = 0$) implies

$$[p + xp_x] + [cq_x (1 + t)^{-1}] = k_x.$$  \hfill (2)

The first squared bracket in equation (2) is marginal revenue on the advertising side of the market of selling more ads. In optimum, this term would be equal to marginal cost ($k_x$) in a standard one-sided market. The second squared bracket captures the fact that sales of advertising (good X) may influence sales of newspapers (good C). This term is negative if demand for newspapers is decreasing in the level of advertising (that is, $q_x < 0$), while it is greater than zero if advertising imposes a positive externality on demand for newspapers. In the former case, the level of advertising should be set lower than the level that maximizes profit in the advertising market in isolation, while the opposite is true if a larger advertising volume increases demand for newspapers.

Setting $\pi_c = 0$ we further find that

$$[(q + cq_c) (1 + t)^{-1}] + xp_c = k_c.$$ \hfill (3)

The first squared bracket is marginal revenue from selling the newspaper (good C) to consumers, and should be equal to $k_c$ if $p_c = 0$. However, if demand for ads is higher the larger the number of readers ($p_c > 0$), profit is maximized by raising the sale of newspapers beyond the volume that maximizes profit in the consumer market (and vice versa for $p_c < 0$).
From the first-order conditions we see that equilibrium prices and quantities on both sides of the market depend on the tax rate. Since \( p = p(c, x) \) and \( q = q(c, x) \), we therefore find that the price changes subsequent to a tax increase are given by

\[
\frac{dp}{dt} = p_x \frac{dx}{dt} + p_c \frac{dc}{dt}, \quad \text{and} \quad \frac{dq}{dt} = q_c \frac{dc}{dt} + q_x \frac{dx}{dt}.
\]  

The second-order conditions for profit maximum require that \( \pi_{xx} < 0 \), \( \pi_{cc} < 0 \), and \( H \equiv \pi_{xx} \pi_{cc} - \pi_{xc}^2 > 0 \).

In order to have a two-sided market, there must be positive externalities from at least one side of the market to the other.\(^{11}\) The implication is that \( p_c > 0 \) and/or \( q_x > 0 \), but whether both terms are positive depends on the particularities of the industry in question. Related to our media example, we cannot predetermine the sign of \( q_x \), since empirical evidence does not give a clear answer as to whether consumers consider advertising to be a good or a bad. However, other things equal, it is reasonable to assume that the willingness to pay for advertising (\( p \)) is increasing in the number of readers. We shall consequently assume that \( p_c > 0 \):

**Assumption 1:** \( p_c > 0 \).

It should be emphasized that the model is applicable to two-sided markets in general and that our mathematical derivations and results also hold for \( p_c \leq 0 \) (in which case two-sidedness requires \( q_x > 0 \)).

For the analysis to follow, the sign of \( \pi_{xc} \) is of particular relevance. Differentiating equation (2) or (3) we find

\[
\pi_{xc} = p_c [1 + \varepsilon_p] + q_x [1 + \varepsilon_q] - k_{xc},
\]

where \( \varepsilon_p \equiv \frac{x \partial p}{p_c \partial x} \) and \( \varepsilon_q \equiv \frac{x \partial q}{q_x \partial x} \).

\(^{11}\) Evans (2003b) defines a two-sided market as one where we have (a) two distinct groups of customers, (b) positive network externalities (at least from one of the customer groups to the other), and (c) an intermediary that internalizes the externalities between the groups. See Rochet and Tirole (2004) for a more formal definition.
The variable \( \pi_{xc} \) measures how the marginal profitability of selling advertising space, \( \pi_x \), changes if the number of readers increases. One might think that \( \pi_{xc} \) is positive, given the assumption that the willingness to pay for advertising is increasing in the number of readers; \( p_c > 0 \). However, this is not necessarily true. To see why, note that \( \partial p_c / \partial x < 0 \) if the marginal value of a larger readership for the advertisers is decreasing in the advertising volume. Thus, the first term in (5) may be negative; this is the case when the elasticity of \( p_c \) with respect to \( x \) is smaller than minus one \( (\varepsilon_p < -1) \). The interpretation of the second term in (5) is similar; this term is negative if consumers are ad-lovers \( (q_x > 0) \) and \( \varepsilon_q < -1 \), or if consumers dislike ads \( (q_x < 0) \) and \( \varepsilon_q > -1 \). Summing up, it is thus clear that the sign of \( \pi_{xc} \) is ambiguous.\(^{12}\) In order to simplify the discussion in the main text, we shall assume that \( \pi_{xc} > 0 \):

**Assumption 2:** \( \pi_{xc} > 0 \).

In the Appendix we discuss how to interpret our results if \( \pi_{xc} < 0 \).

### 2.1 Non-positive externalities from the producer side

In what follows, we examine in detail how different assumptions regarding the externalities between the two groups affect output and prices.

#### 2.1.1 Zero externalities from the producer side \( (q_x = 0) \)

If readers are indifferent to the advertising level, we have \( q_x \equiv 0 \). To find how a higher value-added tax affects sales in the two sides of the market, we totally differentiate first order conditions (2) and (3) to find\(^{13}\)

\[
\left. \frac{dc}{dt} \right|_{q_x=0} = \frac{-\pi_{xx} (xp_c - k_c)}{H (1 + t)} > 0 \text{ if } (xp_c - k_c) > 0.
\]

\(^{12}\)Note also that with a sufficiently high value of \( k_{xc} \), \( \pi_{xc} \) may be negative even if the first two terms in (5) are positive.

\(^{13}\)The full derivation is stated in the Appendix.
With $q_x = 0$ we further see from equation (4) that the effect on prices is

$$\left. \frac{dq}{dt} \right|_{q_x=0} = q_c \left. \frac{dc}{dt} \right|_{q_x=0} < 0 \text{ if } (x p_c - k_c) > 0. \quad (7)$$

The sign of $dc/dt$ and $dq/dt$ is determined by the sign of $(x p_c - k_c)$, which has a straightforward economic explanation: The willingness to pay for advertising increases by $p_c$ units if the newspaper attracts one more reader. With a total advertising volume equal to $x$, the value for the newspaper of attracting one extra reader is thus equal to $x p_c$. If this value is greater than the marginal cost $k_c$ of an extra copy of the newspaper (i.e., $x p_c - k_c > 0$), we see from equations (6) and (7) that $dc/dt > 0$ and $dq/dt < 0$. From equations (2) and (3) we further find that a larger number of readers allows the newspaper to sell more advertising:

$$\left. \frac{dx}{dt} \right|_{q_x=0} = \frac{\pi_{xc} (x p_c - k_c)}{H (1 + t)} > 0 \text{ if } x p_c - k_c > 0. \quad (8)$$

To put the result in eqs. (6) and (7) into perspective, we show that, in sharp contrast to results obtained in one-sided markets (see e.g., Delipalla and Keen, 1992), a higher VAT may increase sales ($dc/dt > 0$) and reduce the price ($dq/dt < 0$) of the good subject to higher taxes.

The reason for this rather paradoxical result is that in the market where the tax is increased (newspaper) the profitability falls relative to the profitability in the other market (advertising). The firm therefore wants to shift income from the consumer side of the market to the producer side. In order to do so, it must increase the sale of newspapers, since this leads to a higher demand for ads (as is evident from (8)). To obtain higher sales of the newspaper ($dc/dt > 0$), the price of the newspaper must be reduced ($dq/dt < 0$).\textsuperscript{14}

Since $p(c, x)$ is downward-sloping in own quantity, an increase in the advertising volume tends to reduce $p$ ($p_x < 0$). At the same time, the firm can charge a higher

\textsuperscript{14}To see the intuition for this result as clearly as possible, assume that $t$ approaches infinity. Obviously, the newspaper would then have no reason to charge a positive consumer price. However, it can still raise revenue through the advertising market and give the newspaper away for free.
advertising price if the size of the readership increases (since \( p_c > 0 \)). Consequently, it is uncertain whether the price of advertising will go up or down:

\[
\frac{dp}{dt}_{q_x=0} = p_x \frac{dx}{dt} + p_c \frac{dc}{dt} \geq 0 \text{ if } \pi_{xc} > 0.
\]

### 2.1.2 Negative externalities from the producer side (\( q_x < 0 \))

When demand for newspapers depends negatively on the advertising level, we have \( q_x < 0 \). One might think that higher value-added taxes are more likely to reduce sales of newspapers the more consumers dislike ads (as an increase in \( x \), motivated by profit shifting to the ad market, now lowers \( c \)). However, total differentiation of equations (2) and (3) makes it clear that the opposite is true:

\[
\frac{dx}{dt}_{q_x<0} = \frac{dx}{dt}_{q_x=0} + \left( \frac{1}{1+t} \right)^2 \frac{\pi_{xc}q_x}{H}, \tag{9}
\]

\[
\frac{dc}{dt}_{q_x<0} = \frac{dc}{dt}_{q_x=0} + \left( \frac{1}{1+t} \right)^2 \left( -\pi_{xc}q_x \right) \frac{1}{H}. \tag{10}
\]

The first term in (9) and (10) shows how advertising and newspaper sales respond to a tax increase if consumers are indifferent about ads (\( q_x = 0 \)). As argued above, this term may be positive or negative. The second term, though, is unambiguously positive and increasing in the consumers’ disutility of ads. The reason is that if sales in the newspaper market are adversely affected by advertising (\( q_x < 0 \)) the media firm has a smaller advertising level than the volume which maximizes profit in the advertising market (c.f. equation (2)). With a heavier taxation of newspaper sales, this effect becomes less important for the media firm. Other things equal, it is optimal to increase the volume of ads, but in order to facilitate a rise in demand for advertising the size of the readership must increase. The latter requires a reduction in the price charged by the media firm, and more so the stronger the consumers’ distaste for advertising. In particular, this implies that the tendency for the consumer
price to fall subsequent to a tax increase is even more pronounced when \( q_x < 0 \) than when \( q_x = 0 \).\(^{15}\) It should be noted, though, that we still cannot sign the change in the price of advertising if both the advertising level and the size of the readership increase.

We can now state:

**Proposition 1:** Suppose that \( q_x \leq 0 \). A sufficient condition for a higher value-added tax on good \( C \) to increase equilibrium quantities of both goods is that \( xp_C > k_c \). The price of good \( C \) (inclusive of VAT) is lowered, while the sign of the change in the price of the untaxed good \( (X) \) is ambiguous.

### 2.2 Positive externalities from the producer side \( (q_x > 0) \)

If demand for good \( C \) depends positively on output of good \( X \), we have \( q_x > 0 \). Such a positive externality is characteristic for the banking industry, where consumers presumably have a higher willingness to pay for holding a credit card the larger the number of merchants that accept it. It may also be characteristic for specialized magazines, where \( q_x > 0 \) reflects a taste for commercials (ad-lovers). Car ads in automobile magazines or perfume ads in beauty magazines may well be appreciated by readers (Depken II and Wilson, 2004). In what follows, we continue to relate the model to the media market.

Equations (9) and (10) still hold when consumers are ad lovers, but with the important difference that the last terms in both equations turn from positive to negative, that is,

\[
\frac{dx}{dt} \bigg|_{q_x > 0} = \frac{dx}{dt} \bigg|_{q_x = 0} + \left( \frac{1}{1+t} \right)^2 \frac{\pi cc q_x}{H} \tag{11}
\]

\(^{15}\)With \( q_c < 0 \) and \( q_x < 0 \) it follows immediately from equation (4) that \( dq/dt < 0 \) if \( dx/dt > 0 \) and \( dc/dt > 0 \), and that the price reduction is larger the more consumers dislike ads.
If $q_x > 0$ is small, the last term is insignificant relative to the first one, so that $\text{sign}(dx/dt) = \text{sign}(dx/dt|_{q_x=0})$ and $\text{sign}(dc/dt) = \text{sign}(dc/dt|_{q_x=0})$. As shown above, we then have that quantities of both goods increase $(dc/dt > 0$ and $dx/dt > 0)$ if $xp_c > k_c$. However, if $q_x$ is sufficiently high, it follows from equations (11) and (12) that the sales of newspapers and advertising are decreasing in taxes. To see why, notice that the newspaper has more commercials than the quantity which maximizes profit on the advertising side when consumers love ads (c.f. equation (2)). An increase in VAT, though, implies that it becomes less profitable for the media firm to attract readers by having a large advertising volume. Instead, the media firm will have incentives to reduce the level of advertising, and approach the volume that maximizes profit on the advertising side. If $q_x$ is sufficiently high, both the level of advertising and the demand for the media product will therefore fall.

Finally, note from equation (4) that the signs of both $dp/dt$ and $dq/dt$ are ambiguous when output on both sides of the market is decreasing in $t$.

We can now state:

**Proposition 2:** If $q_x > 0$, but is relatively small, a higher value-added tax on good $C$ may increase output on both sides of the market. If $q_x > 0$ is sufficiently high, a higher VAT reduces output on both sides of the market. The effects on prices of higher taxes are ambiguous.

## 3 Policy Implications

In most countries newspapers are subject to a reduced value-added tax rate. In Germany they are subject to a rate of 7% (16% is the regular rate) while in e.g. the UK and Denmark they are exempted from value-added taxation all together (European Commission, 2004). Newspapers are also either fully or partially exempted from
sales taxes in a number of U.S. states. The reason for the preferential tax treatment is that governments consider newspapers to be essential for the dissemination of vital information regarding for instance culture, politics and international affairs. A lower tax rate is thought to reduce the newspaper price and, more importantly, to increase the circulation of the media product. The reasoning is in line with tax incidence analysis in a one-sided market, but the analysis above shows that this need not hold for the newspaper industry, which typically operates in a two-sided market. On the contrary, a lower VAT may reduce the sales of newspapers.

The analysis provides a framework for thinking about taxation also in other industries. Credit card services, which are VAT-exempted in the European Union, is one example. As the value of holding a credit card is increasing in the number of merchants accepting it, and vice versa, we have \( p_c > 0 \) and \( q_x > 0 \). If, for historical reasons, governments have wanted to increase the usage of credit cards, the policy of exempting the use from VAT may have been effective. Presently, though, the use of credit cards has become so widespread that the network effects presumably are more or less exhausted (which in particular means that \( q_x \) is small) in most European countries. Since the marginal costs of a transaction for the platform (the credit card company) are close to zero \( (k_c \approx 0) \) the analysis suggests that abolition of preferred VAT treatment of credit card services need not have a large negative impact on its use. In fact, we cannot disregard the possibility that the opposite may happen; imposing a VAT may further expand the network size on both the credit card holder and the merchant side of the market. This illustrates that the effectiveness of reducing the VAT-rate may depend crucially on whether or not we

\[ 16 \text{ Auerbach and Gordon (2002) discuss the desirability of taxation of financial services. Challenging the current European practice, they recommend taxation of financial transactions. Their analysis resort to the standard one-sided market view.} \]

\[ 17 \text{ This certainly does not mean that credit card companies would welcome a VAT. On the contrary, differentiating the equilibrium value of equation (1) with respect to } t, \text{ and using the envelope theorem, we find } d\pi/dt = -q(x, c)(1 + t)^{-2} < 0. \text{ The profit level is thus strictly decreasing in the tax rate.} \]
consider a mature industry.

The discussion above makes it clear that it is difficult to ascertain the effects of VAT changes in two-sided markets without detailed knowledge about marginal costs and externalities. The difficulties are particularly large in the media industry. Depken II and Wilson (2004), for instance, find that advertising is considered to be a good in some paper magazines and a bad in others. Presumably, we find a similar variety in the public’s attitude to advertising in the newspaper industry. So what can the government do if it wants to spur output of newspapers? A more accurate policy than changing the VAT rate, would be to subsidize newspapers. To see this, let the profit level of the media firm be given by

$$
\pi = \max_{c,x} \left[ xp(c, x) + \left( \frac{q(c, x)}{1+t} - \tau \right) c - k(c, x) \right],
$$

where $\tau$ is a specific tax on newspapers. The first-order conditions for the platform are the same as before, except that the specific tax now appears as an additional cost term in selling newspapers (c.f. equations (2) and (3)):

$$
\pi_x = 0 \implies \left[ p + xp_x \right] + \left[ q_x c (1 + t)^{-1} \right] = k_x \tag{13}
$$

and

$$
\pi_c = 0 \implies \left[ (1 + t)^{-1} (q_c + q) \right] + xp_c = k_c + \tau. \tag{14}
$$

Totally differentiating (13) and (14), holding $t$ fixed, we find

$$
\frac{dc}{d\tau} = \frac{\pi_{xx}}{H} < 0 \text{ and } \frac{dx}{d\tau} = -\frac{\pi_{cx}}{H} < 0. \tag{15}
$$

Equation (15) makes it clear that specific taxes unambiguously have a negative impact on output in both markets, independently of consumer preferences for ads. The reason is that higher specific taxes are equivalent to increased unit costs, as shown by equation (14). Since higher unit costs lower the marginal proﬁtability for any given output, it is optimal to reduce sales of newspapers ($dc/d\tau < 0$). As a result, the advertising level falls ($dx/d\tau < 0$).
As discussed in the introduction, Edgeworth’s tax paradox asserts that a higher specific tax on one of two goods under certain conditions may reduce the price of both.\(^{18}\) To see that this holds in our context of a two-sided market, note first that for the consumer price we have

\[
\frac{dq}{d\tau} = q_c \frac{dc}{d\tau} + q_x \frac{dx}{d\tau}. \tag{16}
\]

Equation (16) is unambiguously positive if consumers dislike ads \((q_x < 0)\). However, with ad-lovers \((q_x > 0)\) the second term is negative, reflecting that the consumers’ willingness to pay for the newspaper falls when the level of advertising decreases. If this effect is sufficiently strong, we obtain \(dq/d\tau < 0\).

We likewise find that

\[
\frac{dp}{d\tau} = p_x \frac{dx}{d\tau} + p_c \frac{dc}{d\tau} \tag{17}
\]

is negative if the fall in readership, \(p_c (dc/d\tau)\), dominates the increase in ads, that is \(p_x (dx/d\tau)\). Equations (15) - (17) thus show that an increase in \(\tau\) may reduce output and prices of both goods.\(^{19}\)

To summarize:

**Proposition 3:** A higher specific tax on good \(C\) reduces output of both goods. If \(p_c\) and \(q_x\) are positive and sufficiently large, prices fall (Edgeworth’s tax paradox).

The analysis in Sections 2 and 3 makes it clear that raising ad valorem taxes and specific taxes may have opposite quantity effects. The reason for this is that with \(^{18}\)See also Bailey (1954) for a discussion.

\(^{19}\)An example that yields this result is the following. Let \(p = -x/10 + c\), \(q = z - c/10 + x\) and \(\pi = xp + (q - \tau) c - x^2 - c^2\). Then we have that \(\partial^2 \pi / \partial c \partial x = 2 > 0\). It is easily verified that all second-order conditions are satisfied. Solving \(\partial \pi / \partial c = \partial \pi / \partial x = 0\) we find \(p = x = 50 (z - \tau)/21\), \(q = 131z/42 - 89\tau/42\) and \(c = 55 (z - \tau)/21\), from which it is immediately clear that a higher tax rate reduces all prices and quantities. Related to the media market, we may intuitively regard the reduction in readership (resp. advertising) as a quality reduction of the newspaper from the advertisers’ (resp. readers) point of view. Other things equal, this leads to a lower willingness to pay for the newspaper on both sides of the market.
specific taxes, there is a one-to-one relationship between tax payments and quantity, while there is no direct link between output and the burden of taxation under ad valorem taxation. Indeed, subsequent to a higher ad valorem tax the firm can in principle both reduce tax payments and increase the quantity by lowering the price. Put differently, in this case the firm has two instruments at its disposal, compared to only one under specific taxes. In a multi-sided market this difference has profound implications for firm behavior, and thus for public policy in a context where output per se is considered to be important.

4 Conclusion

Traditional analysis of tax incidence has focused on conventional (one-sided) markets, where the sale of one good does not directly affect the sale of other goods. In such markets a general insight is that indirect taxes are partly shifted (or even overshifted) onto consumers, resulting in lower sales of the taxed good. Our analysis has shown that this result is challenged in a two-sided market. If demand for the taxed good matters for sales of a product in another market, the incidence of taxation changes. In a two-sided market an increase in an ad valorem tax may, under certain conditions, lead to lower prices for both goods as well as to higher sales. The results obtained under ad valorem taxation are in sharp contrast to our findings under specific taxation, where a higher tax unambiguously has a negative effect on output.

Our study has been carried out in a monopoly setting. An interesting path for future research would be to check the robustness of our results under different market structures. However, we believe that the main results in this paper would survive under oligopoly as well. As long as firms have some market power, a tax increase on one side of the market implies that the firms will have incentives to shift profit to the other side of the market. In an appendix, available from the authors upon request, we show that this conjecture holds in a simple duopoly model with
linear demand functions.

Even though our discussion is related to the media market, we have not incorporated any of the particularities of the media market or the advertising market into the model. The reason is that we have used a model sufficiently general in structure to highlight the most common mechanisms in two-sided markets. We have also abstained from welfare analysis. Such analysis would hinge on specific characteristics of the industry in question. In the media market, this would for instance require that we make assumptions about whether advertising is persuasive or informative. This said, we believe that there is also a need for industry-specific analysis, both theoretically and empirically.

5 Appendix

Derivation of the relationship between quantities and ad valorem taxes

We assume that the second order conditions hold with non-negative prices and quantities, so that the equilibrium is characterized by first order conditions (2) and (3). To find how a higher value-added tax affects prices on the two sides of the market, we totally differentiate (2) and (3). We then find

\[
\pi_{xx} \frac{dx}{dt} + \pi_{xc} \frac{dc}{dt} = \left(\frac{1}{1+t}\right)^2 cq_x
\]

\[
\pi_{xc} \frac{dx}{dt} + \pi_{cc} \frac{dc}{dt} = \left(\frac{1}{1+t}\right)^2 (q + cq_c).
\]

Making use of the first-order condition (3), the effect of the tax on quantities is now given by

\[
\frac{dx}{dt} = \left(\frac{1}{1+t}\right)^2 \frac{\pi_{xc} (1 + t)(xp_c - k_c) + \pi_{cc} cq_x}{H}
\]

and

(18)
\[
\frac{dc}{dt} = -\left(\frac{1}{1+t}\right)^2 \pi_{xx} \frac{(1 + t) (xpc - kc) + \pi_{xc} q_x}{H}.
\] (19)

Consequences of relaxing the assumption that \(\pi_{xc} > 0\)

Suppose that \(\pi_{xc} < 0\) and \(q_x = 0\). From equations (6) and (7) we see that a higher ad valorem tax still increases sales of the newspaper and reduces its price if \(xpc - kc > 0\): thus the media firm’s incentive to sell a larger number of newspapers in order to shift revenue to the advertising side is unaltered. However, from equation (8) we find that \(dx/dt < 0\) if \(\pi_{xc} < 0\).

If \(q_x < 0\), we know that there will be less advertising than the volume which maximizes profit on the advertising side of the market. If the ad valorem tax rate on sales of newspapers increases, the media firm will care less about the revenue it captures directly from the readers. This is true independent of whether \(\pi_{xc} > 0\) or \(\pi_{xc} < 0\). The second term in equation (9) shows that this effect makes the media firm sell more advertising space if \(t\) increases. However, the second term in equation (10) makes it clear that this tends to reduce the sales of newspapers.

To grasp the intuition for this result, assume that \(\pi_{xc} < 0\) because \(k_{xc}\) is large. In order to save costs, the media firm will then have incentives to reduce the circulation of the newspaper when the advertising volume increases.\(^{20}\)

The case where \(q_x > 0\) has a similar interpretation. If the consumers are ad lovers, the newspaper has more ads than the level that maximizes profit on the advertising side of the market. Independent of the sign on \(\pi_{xc}\), the newspaper will therefore reduce the advertising level if \(t\) increases \((dx/dt < 0\)). However, a lower advertising level means that the marginal profit of selling newspapers increases if \(\pi_{xc} < 0\), which induces the newspaper to sell more newspapers \((dc/dt > 0\)).

The effects of assuming \(\pi_{xc} < 0\) when we consider specific taxes are analogous,\(^{20}\)

\(^{20}\)For the same reason, we see from equation (15) that a higher specific tax on newspapers - which always reduces sales of newspapers - increases the advertising volume if \(\pi_{xc} < 0\).
and seen from equations (15) - (17).

References


