International Trade with Competitiveness Effects in R&D

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Abstract

In an oligopoly trade model where firms engage in R&D, international differences in market size allow for the emergence of endogenous asymmetries between firms. Concretely, firms located in countries with more demand become more competitive because they have strong incentives to perform R&D (“home market” and “competitiveness effects” in R&D). As a consequence, these firms have better access to export markets and the countries where they are hosted often also tend to run trade surplus in the oligopolist sector. This shows that cross-border differences at the level of R&D intensity can be a basis for international specialization.

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JEL Classification: F12, L13, O31.

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1 Introduction

The motivation for this paper comes from two inter-related questions that are at the basis of international trade analysis: i) why is international competition national in content? ii) why are some firms more competitive than others?

By tradition, the focus was mainly put on the first question. In fact, the answer for this can be found not only in classical perfect competition models (Ricardo and Heckscher-Ohlin), but also in modern imperfect competition models such as the “new” trade theory (Krugman, 1980 and Brander, 1981) or the “trade and growth” literature (Grossman and Helpman, 1995). Specifically, while the Ricardian model highlights international differences in technology as the basis for trade, the Heckscher-Ohlin model explains international specialization as a result of international differences in factor endowments. In turn, the “new” trade theory differentiates countries according to the local level of demand and trade costs. Conversely, in both Krugman’s (1980) monopolistic competition model and Brander’s (1981) oligopolist competition model, “home market” effects arise such that countries with more demand tend to have a proportionally larger share of industry. Finally, the trade-growth literature calls attention to the local level of technology. Specifically, Grossman and Helpman (1995) show that as long as R&D spillovers are local, countries with a higher stock of knowledge capital will tend to grow faster than rival laggard ones.

In what refers to the second question, due to the difficulty of dealing with asymmetry assumptions, this issue has only recently gained a central position in the international trade literature. This includes for example Neary (1994), Rosen (1991), Melitz (2003) and Bernard et al. (2003). Neary (1994) and Rosen (1991) model asymmetries between firms as an à priori difference in marginal costs (i.e.: it is assumed that some firms have lower marginal costs than others). Melitz (2003) and Bernard et al. (2003) generate firm heterogeneity by allocating productivity levels to firms randomly accordingly to some ex-ante statistical distribution.

In spite of the fact that the type of asymmetry assumed in all these papers is exogenous, it had allowed to approach questions not possible in symmetry set-ups. Neary (1994) shows that “winner” (low cost) firms should be preferred for government support relatively to “loser” (high cost) ones, given that the “profit-shifting” effect is larger in more competitive firms. Rosen (1991), in turn, concludes that “larger” (low cost) firms tend to invest
more than “smaller” (high cost) firms, but given that the former choose safer R&D projects they make fewer break-trough innovations than the latter. Finally, Bernard et al. (2003) and Melitz (2003) account for the empirical evidence that only the more productive firms tend to export (Roberts and Tybout, 1997).

This paper tries to answer the two above-mentioned questions together. Having this in mind, Brander’s (1981) model of two-way trade is extended to incorporate process R&D that reduces marginal costs but increases fixed costs as in Leahy and Neary (1997). The objective is to access in what ways R&D by individual firms affects international trade. As a consequence of the modeling strategy adopted, the answer to the first question is the same as in the “new” trade theory, i.e.: international competition is mainly “country against country” due to the role of local demand and trade costs. However, it is also possible to give an answer to the second question by saying that firms can become asymmetric as a result of R&D competition.

To be precise, it is shown that endogenous asymmetries between firms from different countries can arise as a result of interactions amongst innovative activities, demand and trade costs: i.e.: firms located in countries with more demand tend to invest more in R&D (“home market” effect in R&D) and are therefore more competitive than foreign rivals (“competitiveness” effect in R&D). This happens due to the fact that when firms choose how much to invest in R&D they face a trade-off between lower marginal costs and higher fixed costs that due to trade costs is more easily met in larger markets. Then, demand patterns can trigger strategic responses by firms on R&D investment, which in the end often affects trade patterns.

The remainder of the paper consists of seven sections. Section 2 introduces the base-line R&D model and Section 3 shows the production equilibrium. In Section 4, two central implications of the R&D model are derived, namely the “home market” and the “competitiveness” effects in R&D. Section 5 establishes conditions for trade to be profitable for firms, while Section 6 analyzes how R&D and demand affect firms’ access to international markets. In Section 7 the trade patterns of the oligopolist sector are studied and Section 8 concludes and discusses the results of our analysis.
2 The Model

This section introduces a simple oligopolist trade model in the line of Brander (1981) where firms perform process R&D as in Leahy and Neary (1997).

2.1 Basic assumptions

The economy is made up of two countries (home and foreign\(^1\)), two sectors (the oligopolist sector and the perfect competition sector) and one factor of production (labor).

Firms in the increasing returns oligopolist sector (IRS) compete on R&D and outputs to produce the IRS-good, which is subject to \textit{ad-valorem} trade costs when exchanged between countries. Quantity competition is Cournot, and R&D and output choices are made simultaneously. There are \(N\) firms in the oligopolist sector and \(s\) (with \(s \in (0, 1)\)) represents the share of firms at home, i.e.: home hosts \(sN = n\) oligopolist firms, while foreign \((1 - s)N = n^*\).

In turn the constant returns perfect competition sector (CRS) produces the CRS-good that can be freely traded between countries. This sector is kept in the background and its role is to represent the rest of the economy and to correct trade imbalances that can occur in the oligopolist sector.

Labor (\(M\)) is the only production factor and \(r\) denotes the share of workers located at home, i.e.: home hosts \(rM\) workers and foreign \((1 - r)M\). Since workers are simultaneously consumers, then \(r\) is also a country share of world demand.

Given that it is not desirable to have a country with no population, it is assumed that \(r \neq 0\) and \(r \neq 1\). For that reason, countries are modeled as having two population parts: a “core-mass” (\(A\)) and a “differential-mass” (\(L\)). The former guarantees that a country is a country and not a “desert”. Further, this “core-mass” has the same size at home and at foreign: both home and foreign have at least \(A/2\) units of population. On the contrary, the latter can make countries differ in size. Concretely, it is considered that home hosts \(uL\) units of this “differential-mass” of population (while foreign \((1 - u)L\), i.e.: \(u\) (with \(u \in (0, 1)\)) is the share of the “differential-mass” at home. The objective of this set-up is just to access the influence of different levels of domestic market size on trade and production patterns.

Then, \(M = A + L\), \(rM = A/2 + uL\) and \((1 - r)M = A/2 + (1 - u)L\).

\(^1\)An asterisk indicates foreign variables.
Also, $r$ is linear in $u$. Throughout the paper, results will therefore be mainly shown in terms of $r$ and $M$ (instead of $u$, $L$ and $A$).

2.2 Preferences and demand

Preferences are quasi-linear in the two goods, with a quadratic sub-utility in the good produced by the oligopolist sector:

$$U = aQ - \frac{b}{2}Q^2 + q_0$$  \hspace{1cm} (1)

Foreign has a similar expression, with $a = a^*$, $b = b^*$ and $q_0 = q_0^*$ (production and consumption of the CRS-good), i.e.: home and foreign are symmetric in terms of preferences and demand parameters. Also $Q = \sum_{i=1}^{n} q_i + \sum_{i=1}^{n^*} x_i^*$ is the total home consumption of the IRS-good, with: $q$ ($q^*$) sales of a representative home (foreign) firm to each consumer in the home (foreign) market; $x$ ($x^*$) exports of a representative home (foreign) firm to each consumer in the foreign (home) market.

Each individual is endowed with a unit of labor and $\bar{q}_0 > 0$ units of the CRS-good\(^2\). Consumers then have the following budget constraint:

$$PQ + q_0 = I + \bar{q}_0$$  \hspace{1cm} (2)

where $P$ and $I$ stand respectively for the price level and income at home.

From this maximization problem it is possible to derive the indirect demand:

$$P = a - bQ$$  \hspace{1cm} (3)

where $a$ is the intercept of the demand function and $b$ an inverse measure of market size.

2.3 Firms and technology

Turning now to firms, profits by a representative home firm (and by symmetry for a representative foreign firm) are defined as:

$$\Pi_i = (P - C_i) rMq_i + (P^* - C_i - t) (1 - r) Mx_i - \Gamma_i$$  \hspace{1cm} (4)

\(^2\)This is assumed so that the consumption of the CRS-good is always positive.
where \( t \) represents the specific per-unit trade costs (with \( t = t^* \), i.e.: home and foreign firms bear the same trade costs), \( C \) is the marginal costs of production and \( \Gamma \) is the fixed costs (with \( C^*_j \) and \( \Gamma^*_j \) for a foreign firm).

Technology is explained next. It is assumed that the perfect competitive sector uses one unit of labor per unit of output. This implies that as long as this sector produces positive output, the economy wide wages are fixed relative to the price of the \( CRS \)-output. As such, factor supplies are fixed to the economy as a whole, but not to the oligopolist sector.

Due to this, wages (\( w \) and \( w^* \)) are normalized to one in both countries: \( w = w^* = 1 \). This assumption is made for two reasons. First, for analytical purposes since the model becomes very cumbersome when wages are not fixed. Second, to abstract from income effects in order to consider only the impact of R&D in international trade: a country with higher income (i.e.: higher wages) has in principle more demand for imports, and this can counter-weight any export-promotion effect that can come through R&D\(^3\).

In turn, technology in the oligopolist sector is explained in terms of \( C \) and \( \Gamma \). Production costs are central because it is through them that R&D is introduced and that this model distinguishes itself from Brander (1981). More concretely, as in Leahy and Neary (1997) it is considered process R&D that reduces marginal costs but increases fixed costs:

\[
C_i = c - \theta k_i \tag{5}
\]
\[
\Gamma_i = \gamma \frac{k_i^2}{2} \tag{6}
\]

where \( k_i \) is R&D performed by a representative home firm (and \( k_j^* \) for a foreign firm); \( \theta \) is the cost-reducing effect of R&D; \( \gamma \) is the cost of R&D; and \( c \) is the initial marginal costs (i.e.: without R&D). Production costs for a representative foreign firm are symmetric, with \( \theta = \theta^* > 0 \), \( \gamma = \gamma^* > 0 \) and \( c = c^* > 0 \). This implies that home and foreign firms are symmetric in

\(^3\)Consequently, this model has a partial equilibrium nature. This is so for two different reasons. First the assumption of quasi-linear preferences (see equation 1) implies that income effects in demand apply only to the \( CRS \)-good, i.e.: the demand function for the \( IRS \)-good is unaffected by changes in real income. Second, factor markets are not explicitly modeled given that economy wide wages are fixed. Note however, that the economic literature has not yet totally solved the problem of the Cournot formalization in general equilibrium. In fact, like here, most Cournot models are in partial equilibrium. Only recently Neary (2002) has started to give a satisfactory answer to this problem by constructing full-fledged general equilibrium oligopoly models.
terms of technology parameters (θ, γ and c), i.e.: they have the same level of access to technology.

The type of R&D considered here has two main characteristics: first, it reduces marginal costs by θk; but increases fixed costs by γk²/2. The net effect depends on the relation between θ and γ, since the first increases competitiveness, while the second reduces profitability. Another way to interpret this is to say that when γ is high, R&D is costly since it greatly increases fixed costs (and the contrary for low γ); and when θ is high, R&D is very efficient given that it reduces greatly marginal costs (while the contrary for low θ). In other words, when a firm decides on how much to invest in R&D it faces a trade-off between lower marginal costs and higher fixed costs.

3 Production Equilibrium

In this section the model is solved for outputs and R&D levels.

3.1 Outputs

Outputs can be found by computing the first-order conditions (FOCs) in relation to q, x, q* and x*. The resulting expressions are:

\[
\begin{align*}
 bq_i &= \frac{D + \sum^n_{j=1} t_N \theta k_i - \theta \Sigma^n_{j=1} k_j - \theta \Sigma^n_{j=1} k^*_j}{N+1} \\
 bx_i &= \frac{D - N t + \sum^n_{j=1} t_N \theta k_i - \theta \Sigma^n_{j=1} k_j - \theta \Sigma^n_{j=1} k^*_j}{N+1} \\
 bq^*_i &= \frac{D + \sum^n_{j=1} t_N k^*_i - \theta \Sigma^n_{j=1} k^*_j - \theta \Sigma^n_{j=1} k^*_j}{N+1} \\
 bx^*_i &= \frac{D - N t + \sum^n_{j=1} t_N k^*_i - \theta \Sigma^n_{j=1} k^*_j - \theta \Sigma^n_{j=1} k^*_j}{N+1}
\end{align*}
\] (7)

Where D = (a - c) is a measure of firms’ initial cost competitiveness. The parameter space is restricted to 0 < t < D so that even without R&D investment all firms can face trade costs\(^4\). This is assumed in order not to restrict \(\text{à priori}\) who exports and who does not export. The objective is to make market access depend on endogenous forces in the model, namely: R&D and demand-competition issues.

\(^4\)Note however that this condition does not guarantee \textit{per se} that firms will be able to export. As shown in a subsequent section, the conditions for that to be the case are stricter than t < D.
Equation (7) shows that outputs from the home firm \( i \) \((q_i \text{ and } x_i)\) increase with own R&D efforts \((k_i)\) but decrease with rivals’ R&D, either domestic \((\Sigma_{j\neq i}k_j)\) or foreign \((\Sigma_j^*k_j^*)\). However, since firms are symmetric and markets are segmented, in equilibrium: \( q_i = q_j, x_i = x_j \) and \( k_i = k_j \) for \( \forall i \) and \( j \) home firms (and also \( q_i^* = q_j^*, x_i^* = x_j^* \) and \( k_i^* = k_j^* \) for \( \forall i \) and \( j \) foreign firms), but possibly \( k \neq k^* \). As a result, and as will be seen more clearly below, asymmetries can only arise between firms from different countries, and not amongst firms from the same country. This implies:

\[
\begin{align*}
 bq &= \frac{D+(1-s)Nt+(1-s)N+1)\theta k-(1-s)N\theta k^*}{N+1} \\
bx &= \frac{D-(1-s)Nt+(1-s)N+1)\theta k-(1-s)N\theta k^*}{N+1} \\
bq^* &= \frac{D+(sN+1)\theta k^*-sN\theta k}{N+1} \\
bx^* &= \frac{D-(sN+1)\theta k^*-sN\theta k}{N+1}
\end{align*}
\] (8)

Now, outputs for a representative home firm \((q \text{ and } x)\) increase with the domestic level of R&D \((k)\), but decrease with foreign R&D \((k^*)\). The contrary happens with \( q^* \) and \( x^* \). Conversely, a representative domestic firm benefits from positive R&D performance of other national firms and from weak R&D behavior by foreign rivals\(^5\). Then, as in Brander (1981), trade costs lead to “national market games”: a firm sees as its main rivals firms from the other country and not so much other local competitors.

### 3.2 R&D investment

R&D levels can be found by solving the FOCs in relation to \( k \) and \( k^* \):

\[
\begin{align*}
\gamma k &= \theta M (rq + (1-r)x) \\
\gamma k^* &= \theta M ((1-r)q^* + rx^*)
\end{align*}
\] (9)

Proof is in appendix. From equation (9) it can be seen that R&D increases with the number of workers in the world economy \((M)\), the size of the firm

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\(^5\)Gustavsson et al. (1999) present some empirical evidence that supports this result. They show that firm level competitiveness is not only determined by own R&D performance, but also by R&D of other local firms. This seems to indicate that there exists a very important domestic content in international R&D rivalry.
(measured by output levels \( q, \ x \) or \( q^*, \ x^* \)) and the cost-reducing effect of R&D (\( \theta \)), but decreases with the cost of R&D (\( \gamma \)). Furthermore, and most importantly, R&D levels depend on the relative size of the local and the foreign demand markets (\( r \)). Below, the exact relation between R&D and spatial-demand patterns will be analyzed.

The model can now be solved explicitly for \( q, \ q^*, \ x, \ x^*, \ k \) and \( k^* \):

\[
q = \frac{(N+1)((1-\eta)(D+1-s)N\tau-(1-r)\eta(2(1-s)N(N+2-\eta)+2sN(2sN+2-\eta))(1-\eta)(N+1))}{b(N+1)-\eta(1-\eta)(N+1)}
\]

\[
x = \frac{(N+1)((1-\eta)(D-t)-(1-s)N\tau)+\eta(2(1-s)N(N+2-\eta)+2sN(2sN+2-\eta))(1-\eta)(N+1))}{b(N+1)-\eta(1-\eta)(N+1)}
\]

\[
q^* = \frac{(N+1)((1-\eta)(D+sN\tau)-\eta(2sN(N+2-\eta)+2sN(2sN+2-\eta))(1-\eta)(N+1))}{b((N+1)-\eta)(1-\eta)(N+1)}
\]

\[
x^* = \frac{(N+1)((1-\eta)(D-t)-sN\tau)+(1-r)\eta(2sN(2sN+2-\eta)+(1-\eta))}{b((N+1)-\eta)(1-\eta)(N+1)}
\]

\[
k = \frac{\theta M((1-\eta)(D-t)-(1-r)+\eta(2sN(2sN+2-\eta))}{b((N+1)-\eta)(1-\eta)}
\]

\[
k^* = \frac{\theta M((1-\eta)(D-t)-sN\tau)}{b((N+1)-\eta)(1-\eta)}
\]

(10)

Like in Leahy and Neary (1997) \( \eta = \theta^2 M/b\gamma \) is defined as an indicator of the “relative return to R&D”. It will be shown throughout the paper that this relation is central to the analysis. Namely, a stability condition regarding the parameter \( \eta \) is required so that firms have no incentives to invest infinitely in R&D in order to attain negative marginal costs. Concretely, it is assumed:

\[
0 < \eta < 1
\]

(11)

Equation 11 says that the cost-reducing effect of R&D (\( \theta \)) weighted by market size (\( 1/b \)) and the number of consumers in the world economy (\( M \)) cannot be bigger than the cost of R&D (\( \gamma \)). Conversely, if \( \gamma \) is not sufficiently high relatively to \( \theta \) and \( M \) and \( 1/b \), the trade-off that a firm faces when investing in R&D (lower marginal costs \textit{versus} higher fixed costs) is not binding.\(^6\)

\(^6\)As a consequence, if equation 11 is not satisfied outputs and R&D levels may be negative. To see this make the following thought experiment: imagine that home hosts all world demand (i.e.: \( r = 1 \)), then if \( 0 < \eta < 1 \), \( q > 0 \); if \( \eta > (N + 1) \), \( q < 0 \); if \( 1 < \eta < (N + 1) \), \( q \) can be either positive or negative. Remember that it is assumed that \( r \neq 1 \) (and \( r \neq 0 \)), but the objective of this exercise is to show that even when a country hosts all demand, local sales might be negative in case \( \eta > 1 \). Also, as will be seen below, if equation 11 does not hold comparative static results and the model previsions do not make much economic sense.
4 Home Market and Competitiveness Effects in R&D

This section introduces the “home market” and “competitiveness” effects in R&D. These two effects are central in this paper, given that they explain how firms can become endogenously asymmetric. As will be seen, asymmetries between firms from different countries arise due to firms’ strategic responses in R&D to asymmetric spatial demand markets. Concretely, firms located in the country that hosts a higher share of demand perform more R&D. Then, there is a type of “home market” effect in R&D, since domestic demand conditions can affect the innovative behavior of local firms\(^7\). As a result, firms established in the larger country become more efficient, showing the presence of “competitiveness” effects resulting from R&D competition.

4.1 Home Market Effect in R&D

The existence of “home market” effects in R&D can be investigated by subtracting \( k \) to \( k^* \):

\[
k - k^* = 2 \left( r - \frac{1}{2} \right) t \frac{\theta M}{\gamma(1-\eta)}
\]

(12)

It can be seen from equation 12 that as long as equation 11 holds: if \( r = 1/2 \) (i.e.: demand is evenly distributed between home and foreign) \( k = k^* \); if \( r > 1/2 \) (i.e.: home hosts more demand) \( k > k^* \); and if \( r < 1/2 \) (i.e.: foreign hosts more demand) \( k < k^* \). In resume: firms located in the larger country invest more in R&D than firms from the smaller country.

This can be interpreted as the presence of “home market” effects in R&D, in the sense that firms established in the country with more demand conduct higher levels of innovative activities. The rational for this result comes from the fact that investing in R&D involves a trade-off between lower marginal costs against higher fixed costs. Conversely, this trade-off is more easily met (i.e.: R&D investment is more profitable) the larger the local market.

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\(^7\)This is labeled as “home market” effect in R&D to make an analogy with Krugman’s (1980) “home market” effect, since in both cases they are related with local demand. However, note that these two effects are different. In Krugman (1980), the “home market” effect states that countries with more demand have a proportionally larger share of industry. Here, the “home market” effect in R&D is not related with the size of the domestic industry, but with the level of R&D performed by local firms (in relation to foreign rivals).
Proposition 1 In an international oligopolist market, “home market” effects in R&D arise since firms located in countries with more demand invest more in R&D.

4.2 Competitiveness Effect in R&D

The “home market” effect in R&D implies that firms located in the larger country are more competitive than their foreign counterparts, since by investing more in R&D they attain lower marginal costs. In other words, the demand channel allows firms from different countries to become endogenously asymmetric, because it triggers strategic responses in R&D that can give rise to “competitiveness effects”.

To see this “competitiveness” effect in R&D at work, it can be helpful to look at the derivatives of local sales and exports per consumer (respectively \( q \) and \( x \)) in relation to the local share of demand \( r \):

\[
\frac{dq}{dr} = \frac{dx}{dr} = \eta l \frac{(1-\eta)(N+2) + (1-\eta)(N+1)}{b((N+1)(1-\eta)(1-\eta)(N+1))} > 0 \tag{13}
\]

It turns out that both derivatives are equal and unambiguously positive. As such, \( q \) and \( x \) increase with \( r \). This is never the case in either standard Cournot or monopolistic competition models (respectively Brander, 1981 and Krugman, 1980). There, demand patterns have no effects on output levels per consumer (i.e.: \( dq/dr = dx/dr = 0 \)). In fact, in Brander (1981) and Krugman (1980) demand only affects total output per firm, i.e.: \( qrM \) and \( x(1-r)M \) (see Head et al., 2002)\(^8\).

The most important insight of the R&D model, however, does not run from this difference relatively to standard imperfect competition models, but from explaining the reason for this to happen (especially in what concerns exports per consumer). In the case of the derivative \( dq/dr \) it is easy to see why this is positive. This follows, at least in part (and for the moment), from the traditional explanation. If local demand increases, domestic firms gain, since now they have a bigger local market that is protected from foreign competition due to trade costs.

\(^8\)Namely, as shown by Head et al. (2002) in these standard imperfect competition models only the marginal revenue of domestic sales increase with the local share of demand, but not the marginal revenue of exports. However, since the increase in local sales is bigger than the decrease in exports, the “demand” effect there is still positive.
In the case of the derivative $dx/dr$ the explanation needs further elaboration. The rational for $dx/dr > 0$ is that when $r$ increases, $k$ also increases (due to “home market” effects in R&D). Consequently, and as was mentioned above, home firms become more efficient and as a result out-compete foreign rivals. In fact, this “competitiveness” effect is so strong that it even allows the more efficient firms to surpass the trade cost disadvantage in the foreign market and increase exports per consumer.

It is possible now to return to complete the explanation for the derivative of $q$ in relation to $r$ to be positive. When $r$ increases it is not only domestic demand that increases, but also the cost competitiveness of local firms. Then, $q$ increases not only due to “demand” effects, but also because of “competitiveness” effects that help to keep less competitive foreign rivals away from the domestic market.

**Proposition 2** In an international oligopolist market, “competitiveness” effects in R&D allow firms located in the larger country to become endogenously more efficient than foreign rivals. As a result total sales per consumer of a representative local firm are positively related with the domestic share of demand.

Subsequent sections will analyze the consequences of this type of endogenous asymmetry on trade.

## 5 Overlapping Market Condition

The overlapping market condition ($OMC$) gives the threshold level of trade costs that makes trade profitable for firms (see Head et al., 2002). To be precise here the home $OMC$ and the foreign $OMC^*$ are defined in terms of an inverse measure of trade costs:

$$OMC = \frac{1}{\pi_{\text{OMC}}}$$

$$OMC^* = \frac{1}{\pi_{\text{OMC}}^*} \quad (14)$$

Note that the asterisk in $t$ for the foreign $OMC^*$ does not indicate that $t \neq t^*$, but simply that the autarchy threshold level of trade costs can be different for the home and the foreign firms, i.e.: symmetry at the level of trade costs continues to be assumed. Then if the $OMC$ ($OMC^*$) decreases
home (foreign) exports are promoted since home (foreign) firms can export for higher levels of trade costs. If instead the \( OMC \) (\( OMC^* \)) increases home (foreign) exports are discouraged since home (foreign) firms can only export for lower levels of trade costs.

The home \( OMC \) and the foreign \( OMC^* \) as a function of \( k \) and \( k^* \) can be obtained by respectively setting \( x = 0 \) at \( s = 0 \), and \( x^* = 0 \) at \( s = 1 \) in equation (8) and then solving for \( 1/t \):

\[
OMC > \frac{N+1}{D+θk(N+1)−Nθk^*} \tag{15}
\]

\[
OMC^* > \frac{N+1}{D+θk^*(N+1)−Nθk} \tag{16}
\]

First, for both the \( OMC \) and the \( OMC^* \), trade is promoted when \( D \) is high. Conversely the higher the initial cost competitiveness of firms, the more likely it is that they will be able to export. Second, the \( OMC \) and the \( OMC^* \) behave inversely in relation to \( k \) and \( k^* \): the home \( OMC \) decreases (i.e.: trade is promoted) with \( k \), and increases (i.e.: trade is discouraged) with \( k^* \). The reverse happens with the \( OMC^* \). The rational for this is straightforward: home firms have better market access the more they invest in R&D, and the less the foreign firms do so; and the contrary for foreign firms.

Then, in the R&D model it is possible that \( OMC \neq OMC^* \), i.e.: market access can be defined separately for home and foreign firms. However, this is never the case in conventional imperfect competition trade models (such as Brander, 1981 and Krugman, 1980) given that there, home and foreign firms always have the same \( OMC \) (i.e.: \( OMC = OMC^* \))\(^9\): home and foreign firms have the same level of access to export markets. This results from their assumption of symmetric firms. The opposite occurs here, because initially symmetric firms can endogenously differentiate themselves from foreign rivals as a result of innovation and spatial demand markets. That being so, home and foreign firms can also have different levels of international market access.

It is also possible to derive the explicit expressions for the home \( OMC \) and the foreign \( OMC^* \) by solving respectively for \( x = 0 \) at \( s = 0 \), and \( x^* = 0 \) at \( s = 1 \) in equation (10) and again substituting for \( 1/t \):

\(^9\)For example in Brander’s (1981) model: \( OMC = OMC^* > (N + 1)/D \).
\[
OMC > \frac{(N+1)^2 - \eta r(N+1)}{D(N+1)(1-\eta)} 
\]

\[
OMC^* > \frac{(N+1)^2 - \eta (1-r)(2N\xi - \eta (N+1))}{D(N+1)(1-\eta)} 
\]

(17)

(18)

Now the two OMCs besides \( D \) and \( N \) also depend on the R&D parameters \((\theta \text{ and } \gamma)\) and on spatial demand markets \((r)\). The next section studies in more detail the relation between market access (i.e., \( OMC \)), R&D and demand.

Before that, some closing statements for this section are in order. Following the tradition in the trade literature, the analysis carried out in subsequent sections focus only on parameter spaces that make trade possible. As such, cases where both OMCs are not satisfied are ruled-out, but cases where only one OMC (either the \( OMC \) or the \( OMC^* \)) is satisfied are accepted, given that in this last situation trade arises even if one-sided. Finally, note that to assume that at least one \( OMC \) holds, it is also sufficient to guarantee that \( t < D \), since this is a less stringent condition than either of the two OMCs.

### 6 R&D, Demand and Market Access

Given the results from the previous sections, a question arises: what is the relation between R&D, demand and market access? To investigate this, it is computed the difference between the home \( OMC \) and the foreign \( OMC^* \):

\[
OMC - OMC^* = -2t\eta \left( r - \frac{1}{2} \right) \frac{(2N\xi + 1) - \eta (N+1)}{N+1(1-\eta)} 
\]

(19)

This difference is negative for \( r > 1/2 \), positive for \( r < 1/2 \) and zero for \( r = 1/2 \). Then, when demand is evenly distributed \( (r = 1/2) \), the equality of the two OMCs observed in standard imperfect competition models is restored. The reason for this is that at \( r = 1/2 \), home and foreign firms invest the same in R&D and asymmetry therefore does not arise, i.e.: if home and foreign firms are symmetric, they also have the same level of market access. Instead, when \( r > 1/2 \) home firms penetrate the foreign market more easily, than the foreign firms penetrate the home market, since the \( OMC \) is smaller than the \( OMC^* \). The reverse happens for \( r < 1/2 \). This is so because firms located in the country that hosts a large share of demand invest more in R&D and are consequently more efficient than foreign rivals (“home market”
and “competitiveness” effects in R&D). Conversely, more competitive firms export more easily than less efficient firms.

**Proposition 3** In an international oligopolist market, firms located in the country with more demand have better access to international export markets due to “competitiveness” effects in R&D.

The former proposition states that R&D competition together with spatial demand markets can influence market access. Furthermore, R&D and demand do not limit their influence to the exporting performance of local firms but also of foreign firms, given that the competitive relation between firms from different countries is affected, i.e.: home and foreign firms’ competitiveness is inter-connected. In fact, market access works symmetrically in terms of demand for home and foreign firms. This can be seen by looking at the derivatives of the two OMCs in relation to \( r \):

\[
\frac{dOMC}{dr} = \eta \frac{(2N+1)-(2N+2)+1}{(1-\eta)(N+1)D} < 0
\]

\[
\frac{dOMC^*}{dr} = -\frac{dOMC}{dr} > 0
\]

Then, while the derivative of the home OMC in relation to \( r \) is negative, the contrary happens to \( dOMC^*/dr \). In other words, increasing the home share of demand (\( r \)) makes trade more difficult for foreign firms, but the reverse happens for home firms. This shows that domestic demand, by helping the exporting performance of local firms through R&D, deteriorates the exporting capacity of firms in the other country. As such, for very low \( r \) trade may not be possible for home firms, but is always possible for foreign firms (and the contrary for very high \( r \)). As corollary, firms located in the country with more demand are always more “protected” from foreign competition, and the opposite is true for firms from the country with less demand that are more “exposed”.

This is so due to the role of demand on R&D patterns. In other words: R&D can “promote” or “discourage” trade depending on a country’s share of demand: first, it “discourages” trade for firms located in the country with less demand (i.e.: less competitive firms); second, it “promotes” trade for firms located in the country with more demand (i.e.: more competitive firms). In the first case, the country disadvantage at the R&D level can act as a “barrier” to trade for local firms, since it has a similar role as trade costs in
making exports less competitive. In the second case, the country advantage at the R&D level can work as a “promoter” of trade for domestic firms, since it has a similar role as subsidies in making exports more competitive. In short, due to R&D the country with more demand sees exports “promoted” and imports “restricted”; while the contrary is true for the country with less demand.

**Proposition 4** *In an international oligopolist market, R&D can act as a “barrier” to or a “promoter” of trade depending on the country share of demand: it restricts trade for firms from the smaller country, given that they become less competitive; and promotes trade for firms from the larger country, once they become more competitive.*

## 7 Patterns of Trade

As in Head *et al.* (2002) the balance of trade in the oligopolist sector of the home country is defined as:

\[
B = MN \left( (1 - r) s x - r (1 - s) x^* \right)
\]  

(21)

Where \((1 - r) M s N x\) represents total home exports and \(r M (1 - s) N x^*\) total home imports of the IRS-good. Then, if \(B = 0\), trade is balanced; if \(B > 0\), home runs a trade surplus; and if \(B < 0\), home runs a trade deficit.

The balance of trade is first defined in terms of \(k\) and \(k^*\):

\[
B = MN \frac{s \theta k ((1-s)N+(1-r))-(1-s)\theta k^*(sN+r)+(D-t)(s-r)+2sNt(1-s)(r-1/2)}{(N+1)b}
\]  

(22)

It is easy to see that \(B\) increases with \(k\) and decreases with \(k^*\), i.e.: the more competitive the home firms are compared to foreign rivals, the more likely it is that the home country can run a trade surplus in the oligopolist sector.\(^{10}\)

\(^{10}\)Some empirical studies confirm the result that technologic competition has a significant impact on international competitiveness. For example, Fagerberg (1988) presents evidence that national differences on R&D activities influence export growth of countries more than traditional factors (as price differentials). In turn, both Lundberg (1988) and Magnier and Toujas-Bernatet (1994) find that high R&D expenditure relatively to foreign rivals increase exports shares. Also, Amable and Verspagen (1995) show that changes in bilateral market shares in OECD countries are positively related to relative bilateral R&D.
Proposition 5 In an international oligopolist market, the balance of trade of the oligopolist sector is positively related to the level of R\&D performed by local firms, and negatively related to R\&D performed by foreign competitors.

Equation 22 can also be solved for $k$ and $k^*$ to obtain:

$$B = MN\frac{(1-\eta)(N+1)(D-t)(s-r)q+\eta t(1-r)r(2s-1)+sNt(N+1)(1-s)(2r-1)}{k(N+1)-\eta(1-\eta)(N+1)} \tag{23}$$

The denominator of this expression is always positive as long as $0 < \eta < 1$. Then, to sign $B$ is just necessary to study the nominator: the first term in the square brackets is zero for $s = r$, positive for $s > r$, and negative for $s < r$; the second term is zero for $s = 1/2$, positive for $s > 1/2$ and negative for $s < 1/2$; the outside term is zero for $r = 1/2$, positive for $r > 1/2$, and negative for $r < 1/2$.

Conversely, the outside term is the effect of local demand on domestic firms’ competitiveness, the second term in the square brackets represents the effect of domestic industry size, and the first term in the square brackets captures the intercept effect between the domestic share of demand and the domestic share of industry. Relatively to the “new” trade theory, the “competitiveness” effect is novel, but the other two effects are already present there. As a result of these three effects, four cases can be identified.

Case 1: $s = r = 1/2 \Rightarrow B = 0$ All terms cancel out. As a result $B = 0$.

Case 2A: $s > 1/2$, $r > 1/2$ and $s \geq r \Rightarrow B > 0$ The “competitiveness”, the “size” and the “intercept” effects are positive. Then, $B > 0$.

Case 2B: $s < 1/2$, $r < 1/2$ and $s \leq r \Rightarrow B < 0$ Symmetric to Case 2A.

Case 3A: $s < 1/2$, $r > 1/2 \Rightarrow (i) B < 0$ or (ii) $B > 0$ The “competitiveness” effect is positive, but the “size” and the “intercept” effects are negative. Then, the sign of $B$ depends on the relation between what is inside and outside the square brackets. As shown in appendix, $B > 0$ (i.e.: the “competitiveness” effect dominates) if $s$ is not near to one or $r$ is not near to one-half (so that the outside term do not almost vanishes) and either: $\eta$ is close to one; or $N$, or $t$ are very large; or $D$ is very small. If the contrary holds, $B < 0$. 

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Case 3B: $s > 1/2$, $r < 1/2 \Rightarrow (i) B > 0$ or (ii) $B < 0$ Symmetric to Case 3A.

Case 4A $s > 1/2$, $r > 1/2$ and $s < r \Rightarrow (i) B > 0$ or (ii) $B < 0$ The “competitiveness” and the “size” effects are positive, but the “intercept” effect is negative. As such $B > 0$ or $B < 0$. Conversely, $B > 0$ (i.e.: the “intercept” effect is dominated) if $s$ is sufficiently near one; or $\eta$, or $N$, or $t$ are large; or $D$ is small. If the contrary holds $B < 0$. See proof in appendix.

Case 4B $s < 1/2$, $r < 1/2$ and $s > r \Rightarrow (i) B < 0$ or (ii) $B > 0$ Symmetric to Case 4A.

Proposition 6 In an international oligopolist market, the balance of trade of the oligopolist sector depends on the relation between competitiveness effects on R&D, domestic industry size effects and intercept effects between the dimension of the local industry and of the local demand. The first is positive when a country hosts more demand, the second when a country hosts more industry and the third when a country hosts a higher share of firms than that of demand. The reverse holds for the opposite situations.

The rational for these different cases are now considered. Case 1 is straightforward: if firms and demand are evenly distributed between countries, then the balance of trade of the oligopolist sector is also in equilibrium. Cases 2 (Case 2A and 2B) are also simple to follow: the country with more industry runs a trade surplus as long as the share of firms it hosts surpasses (or equals) the domestic share of demand. If the reverse happens, $B < 0$.

In turn, in Cases 3(i) (Cases 3A(i) and 3B(i)) the country that hosts a larger share of industry but a lower share of demand runs a trade surplus, i.e.: “size” and “intercept” effects are larger than the “competitiveness” effect. Instead, Cases 4 (Case 4A and 4B) tell that when a country hosts relatively more industry and demand but holds a small share of firms than demand, then, $B$ can either be negative (“size” and “competitiveness” effects are smaller than the “intercept” effect) or positive (if the reverse holds).

All these cases (1, 2, 3(i) and 4) replicate results already known from the literature. In fact, the models of Brander (1981) and Krugman (1980) can also predict similar trade patterns (see Head et al. 2002). This can be seen
in figures 1\textsuperscript{11} and 2 that depict the balance of trade in the \((s, u)\) space\textsuperscript{12}. Figure 1 (that includes Cases 1, 2, 3\((i)\) and 4) shows that the country that hosts more firms has a higher propensity to run trade surplus, but, and very important, “demand-for-imports” effects can counteract this tendency. Instead, in figure 2 (that covers Cases 1, 2, 3\((i)\) and 4\((i)\)) the country with more industry always has a positive trade balance.

![Figure 1: Balance of Trade (Cases 1, 2, 3\((i)\) and 4)](image1)

![Figure 2: Balance of Trade (Cases 1, 2, 3\((i)\) and 4\((i)\))](image2)

\textsuperscript{11}Given that \(r \neq 0\) and \(r \neq 1\), this figure and also the next ones are better defined in the \((s, u)\) space so that the two axes have the same origin.

\textsuperscript{12}The isoline gives values of \(s\) and \(u\) that make \(B = 0\).
The R&D model, however, differs from other standard imperfect competition trade models in Cases 3(ii) (Case 3A(ii) and 3B(ii)). Notably, the country with a larger share of demand can run trade surplus in the oligopolist sector even when it hosts less firms, if the “competitiveness” effect is larger than the “size” and the “intercept” effects. This is the case when either: the initial cost competitiveness of firms is very small; or the trade costs, or the number of oligopolist firms, or the return on R&D are very large\(^\text{13}\). These scenarios indicate strong competition: high \(t\) or low \(D\) make exports less profitable; high \(N\) makes both local and international competition fiercer; and large \(\eta\) creates strong incentives for firms to invest more in R&D in order to beat-up competition.

The balance of trade associated with these parameter configurations are shown in figure 3 that encompasses Cases 1, 2, 3(ii) and 4(i). As can be seen, \(B\) is positive to the left of the isoline and negative to the right. Then, for low values of \(u\), \(B < 0\) (except for \(s\) close to one where \(B > 0\)); while for higher values of \(u\), \(B > 0\) (except for \(s\) close to zero where \(B < 0\)), i.e.: the country that hosts more demand tends to run trade surplus in the oligopolist sector\(^\text{14}\).

Figure 3 might be think as an extreme case, but it can be representative of the effects at work in the R&D model. As shown above, when home hosts more demand, home firms invest more in R&D and are therefore more competitive than foreign rivals (“home market” and “competitiveness” effects in R&D). These two effects are amplified when competition is made fiercer by market and industry conditions, as is the case with the parameter values above. When that occurs, firms from the larger country gain a decisive competitive edge: this allows them to gain market shares on the rivals’ market and to deter exports from foreign competitors to their own domestic market\(^\text{15}\).

As a consequence, for high \(r\) home tends to run a trade surplus (and the reverse for foreign), because home firms have better market access than

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\(^{13}\)High return on R&D implies that market size is large, or R&D is not very costly or very efficient.

\(^{14}\)If \(\eta > 1\) results are the following. For \(\eta > (N + 1)\) the country with more industry tends to run trade deficits. For \(1 < \eta < (N + 1)\) results are analogous to Cases 1 and 2, except for \(\eta\) very close to one where the country with more demand runs trade deficits even when it hosts all the firms in the oligopolist sector. The strangeness of these results confirms the choice made for the parameter space where the game is valid.

\(^{15}\)Now it can also be understood the effects at work in Case 4(i). Under this case it is not only “size” effects that matter (as in the “new” trade theory) but also the “competitiveness” effects that gain more weight with the increase in competition.
foreign firms. The contrary happens for low $r$. This type of trade pattern cannot arise in standard imperfect competition models, because there, all firms (either from home or foreign) have the same level of market access. Accordingly, in those models only the share of demand and the share of firms matter for the balance of trade. Here however, also the competitiveness level of local firms can play an important role.

Finally, given that the R&D model predicts that the country that invest more in R&D in a given sector can be a net exporter on that sector, this shows that international differences at the level of R&D intensity can be a basis for international specialization. Blomstrom et al. (1990) present evidence that such is the case for US-Sweden bilateral trade: two of the most developed and technologically advanced countries of the world. This indicates that R&D is undoubtedly central for international competition dynamics.

8 Discussion

This paper has analyzed the influence of R&D on international trade in oligopolist sectors. It was showed that innovative activities affect international competitiveness dynamics. Concretely, firms from countries with more demand tend to be more competitive since they invest more in R&D ("home market" and "competitiveness" effects in R&D). In consequence, larger countries are also more likely to run trade surplus in R&D intensive sectors.

As such this, explanation complements the one given by the “new” trade
theory for larger countries to run trade surplus. In fact, while in the “new”
trade theory, countries with more demand can have a positive balance of trade
because they tend to host a larger share of industry (“home market” effect of
Krugman, 1980), in this paper that is so because firms from these countries
tend to be more competitive. In other words, what matters in terms of trade
patterns is not only the number of firms that a country hosts relatively to the
others, but also the competitiveness level of local firms vis-à-vis foreign
competitors.

In this sense the two main contributions of this paper are: i) it gives one
reason for firm heterogeneity: R&D investment; ii) it explains international
specialization based on asymmetric research intensity levels between firms
from different countries.

The first contribution is particularly important because the large bulk of
international trade literature deals only with symmetric firms, and when the
contrary is assumed the asymmetry is exogenous. Introducing endogenous
asymmetries between firms is relevant, not only due to the fact that firms are
in reality asymmetric, but also because if the R&D model in this paper does
not produce it, then, it replicates results already known from the literature.

Moreover, the “competitiveness” effect derived in this paper can be inter-
preted in a Ricardian “comparative advantage” way, since the country that
host firms that invest more in R&D has a “comparative-competitiveness ad-
antage” in the oligopolist sector. However, and contrary to what occurs in
the Ricardian model where it is not possible to know from where the “com-
parative advantage” comes, here this “comparative-competitiveness advan-
tage” is explained as the outcome of innovative activities and spatial demand
markets.

In what concerns the second contribution, note that this result is not
possible in standard Cournot or monopolistic competition models. Although,
a similar specialization effect may occur in the context of the Heckscher-Ohlin
model, even that there, through a different channel: a country tends to export
the commodity that uses intensively the factor with which it is relatively
well endowed. The R&D model, however, is built outside the Heckscher-
Ohlin framework (imperfect competition, factor prices are fixed and countries
have similar endowments), but it can still predict that in fierce competitive
environments international differences at the level of R&D intensity resulting
from innovative activities and spatial demand markets can conduce a country
to be a net exporter in that sector.

Results in this paper then carry an interesting policy implication: de-
mand as a catalyst for innovative activities by firms. For that reason, given that market access is central for firms’ international competitiveness, regional integration agreements may face an extra argument for support, especially in what concerns peripheral and small countries. In addition this poses an extra challenge for developing countries where an important part of the population is excluded from the market economy. The survival of firms in R&D intensive sectors in these countries may depend on the ability of local governments to bring back to the market more and more of its population.

Finally, the analysis carried out here also provides some suggestions for further research. First, the prevision that firms from countries with more demand have strategic incentives to invest more in R&D must be tested empirically. If that is confirmed, demand should not be treated solely as demand per se but also as something that can affect industry dynamics, namely the productive efficiency of firms. Second, the paper only identifies one factor that can promote some firms to invest more in R&D than others (demand) and one mechanism through which endogenous heterogeneity between firms can arise (R&D). However, there are certainly other channels through which this can happen. For that reason future work should focus on these issues, given the role of innovation in trade-production patterns and the central part that asymmetries between firms play in the competitive game.

9 Appendix

R&D First Order Condition The R&D maximization problem for a representative home firm is:

\[
\begin{align*}
Max_k \Pi &= (P - C) q r M + (P^* - C - t) x (1 - r) M - \Gamma \\
\text{s.r.} & \quad C = c - \theta k \geq 0 \quad \text{and} \quad k \geq 0
\end{align*}
\]  

(A1)

This can be solved using the Kuhn-Tucker method. First write the Lagrangian function (denoting the Lagrange multiplier by \(\lambda\)):

\[
L = \Pi + \lambda (c - \theta k)
\]  

(A2)

Since it is assumed that outputs and R&D levels are chosen simultaneously, the Kuhn-Tucker conditions are equal to:
\[
\frac{\partial L}{\partial k} = \theta M (rq + (1 - r)x) - \gamma k - \lambda \theta \leq 0, \quad k \geq 0, \quad \text{and} \quad k \frac{\partial L}{\partial k} = 0
\]
\[
\frac{\partial L}{\partial x} = \gamma - \theta k \geq 0, \quad \lambda \geq 0, \quad \text{and} \quad \lambda \frac{\partial L}{\partial x} = 0 \quad (A3)
\]

The non-negativity and the complementary-slackness conditions on \( \lambda \) (respectively \( \lambda \geq 0 \) and \( \lambda (\partial L/\partial \lambda) = 0 \)) imply that if \( \lambda = 0 \), \( k < c/\theta \); while for \( \lambda > 0 \), \( k = c/\theta \) (since \( \theta > 0 \)). Then, if \( \lambda = 0 \), \( k < c/\theta \) and \( k = \frac{\theta M}{\gamma} (rq + (1 - r)x) \), the complementary-slackness condition on \( k \) \((\partial L/\partial k) = 0 \) is satisfied and consequently the same happens for the remaining Kuhn-Tucker conditions. On the contrary, if \( \lambda > 0 \) and \( k = c/\theta \), the complementary-slackness condition on \( k \) is never satisfied, since \( k (\partial L/\partial k) \neq 0 \) (i.e.: there is no corner solution).

As a result, R&D for a representative home firm equals:

\[
k = \frac{\theta M}{\gamma} (rq + (1 - r)x) \quad (A4)
\]

R&D levels for a representative foreign firm are symmetric.

**Sign of \( B \) under Case 3A**

If \( \eta \) is close to one, the term inside the square brackets of equation 23 goes to zero and \( B \) then tends to be negative.

In what refers to \( N \), note that the derivative of the nominator of equation 23 in relation to \( N \) equals:

\[
(1 - \eta) (s - r) (D - t) + (1 - s) st (2r - 1) (2N + 1) \quad (A5)
\]

As long as \( s > 1/2 \) and \( r < 1/2 \) this derivative is negative. In other words, under Case 3A the larger the \( N \) the more likely \( B \) < 0.

For \( t \) this derivative is:

\[
(1 - \eta) (- (N + 1) (s - r) + \eta (1 - r) r (2s - 1)) + sN (N + 1) (1 - s) (2r - 1) \quad (A6)
\]

Under Case 3A this expression is negative. Then high \( t \) makes \( B < 0 \).

For \( D \) the correspondent derivative simplifies to:

\[
(1 - \eta) (N + 1) (s - r) \quad (A7)
\]

This expression is positive given that in Case 3A \( s > r \). Therefore, \( B < 0 \) only for \( D \) small.
Sign of $B$ under Case 4A  If $s$ is close to one the first term in the square brackets of equation 23 is small even when $r$ is at the maximum ($r = 1$), as such $B$ tends to be positive.

If $\eta$ is close to one, the term in the square brackets of equation 23 tends to vanish, i.e.: $B > 0$.

For $N$, see above the derivative of the nominator of equation 23 in relation to $N$ (equation A5). For $s > 1/2$, $r > 1/2$ and $s < r$ the first term of this derivative is negative while the second is positive. However, for $N$ sufficiently larger the second term surpasses the first one. As such $B > 0$ for high $N$.

In relation to $t$, see also the respective derivative above (equation A6). Note then that under Case 4A all terms are positive, i.e.: for high $t$, $B > 0$.

For $D$ the correspondent derivative (equation A7) is always negative given that in Case 4A $s < r$. As a result $B > 0$ only for low $D$.

Parameter Values Figures 1 to 3  Figures 1 to 3 can for instance be constructed with the following parameter values. Figure 1: $N = 100$, $t = 2$, $D = 5000$ and $\eta = 5/6$ (for example $M = 100$, $b = 1$, $\gamma = 3000$ and $\theta = 5$). Figure 2 by substituting figure 1 values to: $N = 800$, or $t = 17$, or $D = 600$ or $\eta = 50/51$ (for example $M = 100$, $b = 1$, $\gamma = 2550$ and $\theta = 5$). Figure 3 by replacing figure 1 values to: $N = 5000$, or $t = 50$, or $D = 20$ or $\eta = 500/501$ (for example $M = 100$, $b = 1$, $\gamma = 2505$ and $\theta = 5$). All these scenarios assure that trade is possible.

References


