The Facility Location Problem: Modeling and Solution Methods

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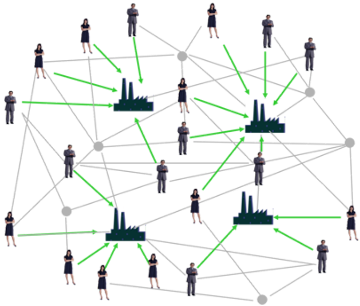
Outline

- Introduction
- Facility Location Models
  - The Set Covering Problem (SCP)
  - The Maximal Covering Location Problem (MCLP)
- Solution Methods
- Summary



- In a basic formulation, **the Facility Location problem** consists of a set of potential facility sites L where a facility can be opened, and a set of demand points D that must be serviced. The goal is to pick a subset F of facilities to open, to minimize the sum of distances from each demand point to its nearest facility, plus the sum of opening costs of the facilities. (*Wikipedia*)
- Facility location is a critical component of *strategic planning* for a broad spectrum of public and private firms (Owen and Daskin 1998).

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- Resources, e.g. Land, Energy, Materials, Labor.
- Infrastructure, e.g. Financial Institutions, Government (stability, tax, ...), Transportation.
- Proximity to market.

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Facility Location Models

- Many analytical techniques: Factor Rating; Cost-Profit-Volume analysis; etc.
- One of the most popular models among facility location models is covering problem (Farahani et al 2012).
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The Set Covering Problem (SCP) tries to minimize location costs satisfying a specified level of coverage.

$$\text{minimize} \sum_{j=1,\ldots,n} c_j x_j \tag{1}$$

$$s.t. \sum_{j=1,\ldots,n} a_{ij} x_j \geq 1, \quad for \quad \forall i (i = 1, \ldots, m) \tag{2}$$

$$x_j \in \{0, 1\}, j = 1, \ldots, n. \tag{3}$$

Location Set Covering Problem (LSCP) Implicit and Explicit

- *LSCP-Implicit* model assumes that each demand area can be covered not only by one facility but also by two or more so that *each facility covers a percentage of demand*.
- *LSCP-Explicit* model considers the coverage provided to a demand area by a *specific* set of facilities (facility combination).

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Capacitated SCP

- In the above models, no limitation for the capacity of a new located facility.
- Current and Storbeck (1988): capacitated version of SCP formulations.

Multiple Optimal SCP

- The optimal number of the new facilities needed for total coverage.
- A secondary criterion minimizes maximum time (or distance) for all the demand points to their nearest facility.

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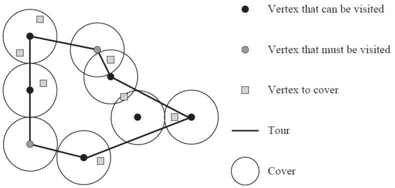
Covering Tour Problem



- The goal is determining a minimum length Hamiltonian cycle on a sub-set of V such that every vertex of W is within a prespecified distance from the cycle.
- Routing of a mobile medical facility.

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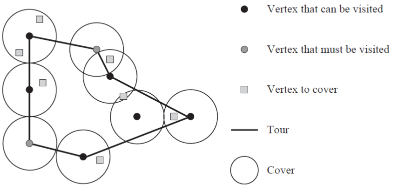

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Probabilistic SCP

- Consider dynamic aspect of location problems.
- In emergency facilities sometimes vehicles are not available when they are called.
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Stochastic SCP

- Stochasticity in demand, travel time, etc.
- Hwang (2002) applied Stochastic SCP in design a supply chain system with *random travel time* (Assume the located facilities are always available). The probability of each demand point being covered is not less than a critical level.

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Other Variants of SCP

- Multiple Coverage SCP: each demand must be covered by a number of facilities (Kolen and Tamir 1990).
- Backup Coverage SCP: air ambulance and ground ambulance combination (Erdemir et al 2010).
- Fuzzy SCP
- Multi-Criteria SCP
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The Maximal Covering Location Problem (MCLP) maximizes the amount of demand covered within the acceptable service distance by locating a given fixed number of new facilities.

$$\text{maximize} \sum_{i=1,\ldots,m} h_i z_i \tag{4}$$

$$s.t. \quad z_i \leq \sum_{j=1,\ldots,n} a_{ij} x_j, \quad for \quad \forall i (i = 1, \ldots, m) \tag{5}$$

$$\sum_{j=1,\ldots,n} x_j \leq P \tag{6}$$

$$z_i, x_j \in \{0, 1\}, i = 1, \ldots, m, \quad j = 1, \ldots, n. \tag{7}$$

MCLP Implicit and Explicit

- Each demand area can be covered not only by one facility but also by two or more so that each facility covers a percentage of demand (Implicit);
- Or have to be covered by a specific set of facilities (Explicit).

MCLP on the plane

- The potential sites for locating the new facilities are not on the network or discrete (and finite).

Capacitated MCLP

- Facility capacity restriction.

MCLP with a criticality index analysis metric

- Oztekin et al (2010): RFID network design methodology for asset tracking in healthcare.
- The objective function maximizes the total covered criticality indices of demand squares and also minimizes the reader collision.

MCLP with mandatory closeness constraints

- Multiple optimal solutions.
- The objective is to seek location for P facilities to maximize population covered within a desired time or distance.

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Probabilistic MCLP

- ReVelle and Hogan (1989): locate P facilities so that with the probability α it maximizes the covered population that can find an available server.

Other Variants of MCLP

- Maximal Covering Tour Problem.
- Partial Coverage Problem.
- Backup Coverage Location Problem.
- ...

- Both the SCP and the MCLP are hard to solve.
- NP-Complete.

Solution Methods for the SCP

- Exact Method.
- Approximate algorithm (Heuristics or meta-heuristics).

Exact Method

- Branch and bound algorithm
  - Linear programming (LP) relaxation: Church and ReVelle (1974) and Fisher and Kedia (1990).
  - Lagrangian relaxation: Etcheberry (1977) and Balas and Carrera (1996).

Approximate Method

- Heuristics
    - Lagrangian heuristics: Beasley (1990), Beasley and Jornsten (1992), Haddadi (1997), Caprara et al (1999), etc.
    - Greedy heuristics: Chavtal (1979) and Grossman and Wool (1997).
    - Local search heuristics: Yagiura, Kishida, and Ibaraki (2006).
    - Others.

Approximate Method

- Meta-heuristics
  - Simulated annealing, e.g. Jacobs and Brusco (1995)
  - Genetic algorithm, e.g. Solar, Parada, and Uttutia (2002)
  - Tabu search, e.g. Kinney, Barnes and Colleti (2004)[1]
  - Greedy Randomized Adaptive Search Procedure (GRASP), e.g. Bautista and Pereira (2007)
  - Others

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Summary

- Facility Location Problem
- Models (SCP and MCLP)
- Solution Methods

Major references

- Schilling D A, Jayaraman V, and Barkhi R (1993), A Review of Covering Problems in Facility Location. *Location Science*, Vol. 1, pp. 22–55.
- Farahani R Z, Asgari N, Heidari N, Hosseininia M, Goh M (2012), Covering Problems in Facility Location: A Review. *Computer & Industrial Engineering*, Vol. 62, pp. 368–407.

Thank you for your attention!