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ORIGINAL ARTICLE

Mediated action in teachers' discussions about mathematics tasks

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Abstract This paper presents analyses of teachers' discussions within mathematics teaching developmental research projects, taking mediation as the central construct. The relations in the so-called 'didactic triangle' form the basic framework for the analysis of two episodes in which upper secondary school teachers discuss and prepare tasks for classroom use. The analysis leads to the suggestion that the focus on tasks places an emphasis on the task as object and its resolution as goal; mathematics has the role of a mediating artefact. Subject content in the didactic triangle is thus displaced by the task and learning mathematics may be relegated to a subordinate position.

Keywords Mediation · Mathematics teaching development · Didactic triangle · Tasks

1 Introduction

The design and use of tasks in classrooms and teacher education is receiving much attention in contemporary research and scholarship in mathematics education (e.g. Berg 2011; Boston and Smith 2009; Clarke et al. 2009; Shimizu et al. 2010; Zaslavsky and Sullivan 2011; Zaslavsky et al. 2007). The purpose of this paper is to contribute to this accumulating body of literature by drawing attention to the relationships between tasks, mathematics, learning and teaching that emerge as teachers consider tasks for use in their classrooms. Our aim is to demonstrate how an analysis of the relationships embodied in the so-called 'didactic triangle' exposes a transposition in teaching that subordinates learning mathematics to the engagement in, and resolution of, tasks. We report from our analysis of teachers' collaborative engagement in mathematical tasks within teaching development activity, focusing on how tasks and mathematics are mediated in teachers' discussions. The group discussions that we consider occurred within a series of related mathematics teaching developmental research projects over the period 2004–2010. Our focus on mediation is prompted by the socio-cultural framework within which the projects have been set.

We continue by examining how didactical relationships might be exposed through an exploration of the mediational means employed in teachers' discussions. A brief exposition of mediation is offered to elaborate the meaning of the term adopted within this paper. Cultural historical activity theory (CHAT) is the theoretical framework that forms the backdrop to this study; the development of this framework with respect to the projects is described elsewhere (e.g. Goodchild 2011; Goodchild and Jaworski 2005; Jaworski and Goodchild 2006).[1] Following this, the projects within which the teachers' discussions occurred are outlined and the analytic approach taken is explained. Two episodes of teachers discussing mathematics and mathematics-didactics tasks are then described. These episodes are interpreted with the aim of making inferences about what they reveal concerning the didactical relationships that teachers foster in their classrooms.

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[1] For a more general introduction to CHAT the reader is referred to Roth and Lee (2007), and in the context of teaching development see Engeström (1994).

2 Mediation and the didactic triangle

We consider teachers working together on given mathematical problem tasks, particularly their ongoing engagement with the tasks and their collaboration in the creation of a joint response to the task.

We focus on the question that can be directly addressed through the data collected:

- What are the characteristics of mediation evident in teachers' collaborative actions in mathematical tasks?

A second question that focuses on the interpretation of evidence, and is only indirectly addressed through the data, is also posed:

- What can be inferred about the didactical relationships teachers foster in their classrooms from the characteristics of mediation entailed in teachers' collaborative actions?

The word 'characteristics' used in these questions is rather vague and thus we operationalize it within the analytic framework adopted in our work. CHAT draws attention to two fundamental categories: agency and mediation. Didacticians[2] exercise their agency in the choice and design of tasks; albeit informed and constrained by their knowledge of project participants, teachers and teaching mathematics in general terms, mathematics and schools. Teachers also exercise their agency in the way that they choose to interpret the tasks, occasionally at odds with didacticians' intentions. It could be said that the teaching development projects described here were intended to empower teachers and increase their agency through the promotion of inquiry. Inquiry, 'as stance' (Cochran-Smith and Lytle 1999) or 'as a way of being' (Jaworski 2004), it is asserted, broadens the scope of action (that is, both teaching goals and mediational means). Inquiry sustains a disposition of awareness that supports transformation and improvement in both teaching and learning mathematics. In this way inquiry can be seen as a heuristic that can release the teacher from the constraints of routine practice and the pupil from a sense of helplessness when engaging with the problems of doing and learning mathematics. Making reference to Wenger's community of practice theory (Wenger 1998), Jaworski (2006) describes this disposition as being 'critically aligned' to the practice. Inquiry thus empowers and enables people in practice. Groups of teachers come together to work on an agreed task with an agreed object and shared outcome. In the analysis reported here, the 'agency' of participants is assumed; the teachers have considerable scope to choose their interpretation of the tasks devolved to them. Consequently, agency does not form part of this analysis. Rather, this paper focuses more sharply on mediation, and we thus explore this concept further.

[2] We refer to participants based at the university as didacticians. We avoid the more usual 'researchers' because in the projects we assert that both teachers and didacticians are researchers.

2.1 Mediation

By mediating (cultural) artefacts we refer to entities that have been taken, adapted or invented to enable human thought, action and communication. In this sense, 'artefacts' include language, physical tools, and intellectual and institutional products (such as mathematics and the curriculum). In school, pupils do not engage with mathematics and mathematical ideas directly but through means of (that is, mediated by) the tasks, activities and explanations they encounter—the artefacts of teaching. Pupils will also experience mathematical ideas being used to mediate other knowledge, such as the interpretation of a straight line graph when learning about elasticity in science lessons. The significance of mediation and the role of mediating tools or cultural artefacts in human activity lies at the heart of socio-cultural theory and Vygotsky's writing (Wertsch 2007). Artefacts take on functions and meanings as they are deployed in cultural settings, hence the reference to 'cultural artefacts'; a bow and arrow, for example, have quite different meanings for a primitive hunter and a modern Olympic sportsperson. Wertsch notes the absence of a 'single unified definition' of mediation in Vygotsky's work (2007, p. 179). The interpretation taken in this paper is consistent with Wertsch's account of 'explicit' mediation, that is when mediating artefacts are explicitly included in a discourse, and the artefacts tend to have a materiality that is obvious and a form of permanence (as opposed to social and inner speech which Wertsch refers to as 'implicit mediation'). This interpretation, although limited in scope, avoids the need to impute unwarranted or vaguely supported assertions about the meanings intended, or embedded within, various utterances. We seek evidence of the cultural artefacts that explicitly mediate action.

In our analysis we adopt the notion of tool–sign functions of mediating artefacts explained by Vygotsky as a "complex mediated act" (1978, p. 40). In this, the direct link relating subject and object (or, stimulus and response) of action is redirected through two links connected by mediating (cultural) artefacts, which have tool function (acting on the object) and sign function (psychological, acting on the person). For example, the school mathematics curriculum is a cultural artefact: the 'tool' function of the curriculum is explicit, the curriculum resolves the teachers' 'problem' about what should be taught and when. The curriculum also takes on a 'sign' function because it informs explicitly about what knowledge is important for pupils to learn. The sign function of the curriculum can also be implicit when, for example, teachers view pupils within a curriculum model—as 'normal', as high or low attainers, as deficient of fundamental knowledge or understanding, and so on. Thus the curriculum takes on an intrapsychological, or cognitive, and normative function as teachers think about the subject content and pupils for whom they have responsibility. Such is described by Wertsch (1991, pp. 36–37) when he explains how the introduction of legislation in the USA regarding the categorization of children with special learning needs, and associated political-economic constraints, served to define children and their needs.

As a framework for analysing teaching and teaching development, the usefulness of Engeström's (1994) extension of Vygotsky's complex mediated act, which includes socio-cultural mediators of 'community', 'rules', and 'division of labour', is explained elsewhere (e.g. Engeström 1994; Jaworski and Goodchild 2006). Teaching may be mediated by constitutional rules (the curriculum, school plan, and policies embracing legal frameworks of language policy, inclusion, etc.) and local rules, such as an explicit regulation to set homework tasks or an implicit requirement such as 'ask questions rather than provide explanations' (Fuglestad and Goodchild 2010). The 'community' mediates in terms of making judgments about what will 'work' in this class (the pupil community), or through a conception of a 'generalised other' (Mead 1934) that reacts to a belief about how a teacher should behave in a given situation (professional practitioner community). The division of labour has, possibly, a determining role in the formation of didactical relationships as it concerns, within the classroom, who does the mathematics, and expectations about behaviour and actions of pupils, teachers and didacticians in different settings. Each of these can serve (as a tool) in the preparation and implementation of lessons; they also have a psychological function as they inform teachers' reflection on teaching and lessons—what is 'possible', 'appropriate' or 'acceptable', etc.

2.2 The didactic triangle

In the usual representations of the didactic triangle the vertices represent mathematics, pupil and teacher. In other papers in this special issue authors have extended the concept to a didactic tetrahedron to provide a fourth vertex to represent didactical technologies (Ruthven) or didactician (Jaworski). We prefer to hold to the simple triangular model in which attention is on the mediation between the mathematics and those engaged in mathematical activity (pupils or teachers). The third vertex we take to represent a complex of didactic, technological and social mediators (including the mediators of Engeström's model, and teacher, task and resources). We will therefore refer to this third vertex as a *mediating complex*; it includes, rather than replaces, the teacher who remains the principal component. The focus on mediation draws attention to complexities and ambiguities embedded in the relationships represented in the didactic triangle. Tasks are designed and chosen to support pupils' learning of mathematics, in which sense the task is intended as a cultural artefact that mediates mathematical knowledge, but to engage in the task pupils use mathematics, and mathematics is a cultural artefact that mediates the tasks.

Our analysis, within a CHAT framework, leads us to focus on the object and goals of the activity in which people engage. The didactic triangle is usually taken to model learners engaging with mathematics and learning mathematics through the mediation of teaching and all the resources associated with teaching. However, it is possible that the object and goal of actions can be the resolution of classroom tasks, in which mathematics is just one of several cultural artefacts that comprise the mediating complex used to understand (sign function) and work on (tool function) the task. The characteristics of engagement in mathematical tasks that we wish to expose are evident in the cultural artefacts that are used within the group discussions to work on the task/object. In the discussions, participants externalize or bring into view the cultural artefacts in their communication as the 'tool' is applied in action—which provides the evidence to address the first research question stated above. We infer (speculatively) the internal sign (or psychological) function of the mediator to address the second research question. Our assumption is that these signs are coincident with the meanings the cultural artefacts have in the didactical relationships espoused by the teachers.

3 Projects' description

The episodes reported here arose in projects that were founded on principles of community and inquiry. Inquiry has the potential to make a crucial impact in learning mathematics, teaching mathematics, and the development of teaching mathematics (Jaworski 2004). Two projects, learning communities in mathematics (LCM) and ICT and Mathematics Learning (ICTML), ran from 2004 to 2007. A follow-on binary project,[3] Teaching Better Mathematics/

[3] It was a binary project in the sense that although conceived and led as a unity, the funding for the two elements, research and development, came from different sources. TBM was funded as a research project partly by the Research Council of Norway and partly by the Competence Development Fund of Southern Norway (SKF). LBM was funded as a development project, steered by school authorities and teachers, but operationalized by didacticians. LBM was funded by SKF.

Learning Better Mathematics ran from 2007 to 2010. These were developmental research projects in which developmental activity with teachers was both informed by, and used to inform, research (Goodchild 2008; Gravemeijer 1994). Development was based upon the principle of inquiry with roots in action research, Japanese lesson study, learning study and design research. Research was framed within socio-cultural theory; initially the projects were conceptualized within communities of practice theory (Wenger 1998) and later CHAT was found to be of value when operationalizing the notions of critical alignment and development. The projects set out to support the development of communities of inquiry comprising school teachers and university didacticians. These inquiry communities are asserted to be the basis for sharing, support, innovation, risk-taking, challenge and mutual critical engagement—actions that underpin the developmental goals concerning the improvement of teaching and learning mathematics.

The projects included a variety of developmental activities such as workshops, lesson observation by didacticians and teachers, school-based team meetings and reflective discussions between teacher(s) and didactician(s) in the context of lessons observed. As far as possible all events within the projects were recorded, either audio or video, and related textual material was collected and stored in digital format. There were usually six workshops each year and other joint activity, such as preparing for project conferences and publication (e.g. Jaworski et al. 2007; *Tangenten*[4] 1/2007, 4/2008, 4/2010); thus the teachers were well acquainted with each other. Most events in which teachers were included were planned for development rather than research purposes. The recordings from these events form the corpus of data analysed in the research. We consider the data to be 'naturally occurring' or 'naturalistic' because it records events that were arranged for other than research purposes.

Workshops within the projects included plenary presentations by didacticians, whose expositions focused on both mathematics and didactical issues, and teachers who reported experiences from their own classrooms. Another important element of the workshops was group discussions in which teachers of pupils at similar grade levels worked together on tasks. Tasks were chosen or designed by didacticians to engage participants in mathematical activity and planning for classes. The word 'task' is used at a variety of levels and includes several possible actions. Thus workshops are concerned with the *classroom-tasks* that are given to pupils in school, and with the *didactical-tasks* that are proposed to stimulate teachers' discussion on didactical issues that arise from the use of the classroom-tasks. In the following we take a broad interpretation of a task as some form of devolved intentional stimulus for intelligent action. The task is 'intentional' because it has been chosen or designed with some rationale or purpose; the task is intended (by the person or group that devolves the task) to stimulate action, in which it is assumed stimulation motivates action to achieve an outcome embedded in the task. The word 'intelligent' is inserted to distinguish tasks from stimuli that result in a form of spontaneous reflex. It is perhaps necessary also to add that the stimulus is intended to challenge, to provoke wondering, curiosity, questioning and reflection, and provide a context for collaborative effort. Thus we refer to *inquiry-tasks* (inclusive of both classroom- and didactical-tasks) which are intended to elicit these actions.

This report is a product of our ongoing review of data that has been collected through the 6 years of the projects. These projects have been extensively reported elsewhere.[5] The data corpus now facilitates longitudinal inquiry into the projects' impact, and this present paper reports from an early phase in this longitudinal analysis. Here, we report from the analyses of teachers working on mathematical tasks in small groups that took place within project workshops. The analysis seeks to expose characteristics of the teachers' engagement in mathematics and teaching mathematics.

4 Methodology relating to the empirical basis of this paper: analytic framework and rationale for the choice of episodes

Our purpose in the study reported in this paper is to expose categories of engagement. We have made a purposeful selection from the data available and we do not make any claims about representativeness, generality or development. The episodes have been chosen to include: adaptations of regular tasks and non-routine tasks; engagement with mathematics and didactical issues; workshop group activity that combines teachers from more than one school; and different domains within mathematics. We explore teachers' engagement, to develop analytic categories that can be used to enable an analysis that systematically explores the extensive corpus of data collected over 6 years. In this paper we have chosen to limit the focus to discussions among upper secondary teachers (of pupils grades 11–13). The episodes in which mathematical tasks (classroom- and didactical-tasks) have been at the centre of the discussion have been selected on the basis that they were marked as being especially rich in terms of teacher engagement, response or inquiry.

4 A Norwegian professional journal for mathematics teachers.

5 An extensive list of publications from the projects can be found at http://prosjekt.hia.no/tbm/.

Limiting our analysis to upper secondary teachers will, we believe, reduce the amount of 'noise' in the data. However, we admit that it might also prevent us from seeing all possibilities, and thus a similar study of teachers working at other levels is required. For example, teachers in upper secondary school are expected to be able to teach in two subjects, they have strong subject knowledge, and are generally qualified to at least master's level in one of the subjects they teach. However, usually their professional education is not as extensive as that of compulsory school teachers. We conjecture that upper secondary and compulsory school teachers would expose the same categories of engagement but possibly differ in respect of the properties and dimensions of those categories. Mathematics as a cultural artefact is likely to be a significant mediator in teachers' actions at all levels—given that the tasks are mathematical. However, the significance, or strength, of mathematics as a mediator might differ between groups of teachers that represent different educational and experiential backgrounds.

The tasks were proposed by didacticians in order to stimulate groups comprising teachers and didacticians to engage in inquiry into mathematics, teaching mathematics and, crucially, pupils' learning. In the analysis we especially consider how mathematical tasks are mediated in teachers' discussions. In particular we look at the choices teachers make, and the arguments articulated for those choices. Further, we look for teachers' expressions of their didactical priorities, goals and conditions, because these reveal the juxtaposition of pupil, mathematics and didactical mediators in their practice. We make conjectures about what the characteristics of mediation might imply for the didactical relationships that teachers develop with their classes. We believe that focusing on teachers' discussions about mathematics, and within their own mathematical activity, may offer an insight into the way they relate to the mathematics, and how they wish to set up the relationship between mathematics and their pupils. We assert that observation in classrooms and teachers' workshop activities provide significant complementary perspectives of the didactical relationships, and here we focus on workshop groups and teachers' planning for classes.

The process of analysis includes data reduction (a basic summary and indexing of each data item) and transcription of complete recordings or key episodes that are identified as being informative through the data reduction process. The unit of analysis used to address the research questions of this paper is defined by the entities and mediating relationships represented by the augmented didactic triangle, outlined in Sect. 2, exposed in teachers' dialogues. We focus, especially, on the mediational links between the components (task, teacher, didactical technologies, rules, community and division of labour) and the subject–object of the actions that are chosen by teachers and made explicit in the small-group dialogues. An open coding approach (Strauss and Corbin 1998) to data analysis is adopted in this exploratory stage as we try to keep our minds open to teachers' introduction of mediational means. We report from an early stage in the process of data analysis in which we explore the data to expose mediation; we have not yet been concerned to operationalize different forms of mediation, and thus we do not describe a priori coding schemes or consider the reliability of coding decisions.

5 Example 1: angles and parallels—the 'M' task

The first episode that we consider took place in a workshop about 15 months into the life of the first (LCM) project. Here the focus is on the discussions that took place among three upper secondary teachers (Olav and Stefan from Kongens Upper Secondary School and Kristin from Dronningens Upper Secondary School)[6] and two didacticians (Dag and Roy) as they worked on a geometry task (Fig. 1). About 1 week in advance of the workshop a set of three tasks was distributed with the request that participants do some work on the tasks in preparation for the group activity in the workshop. On this occasion one task was inspired by an article in a mathematics teachers' journal (Hancock 2005). The group activity took place in two sessions separated by a short break for refreshments. In the first session the group was required to choose and work on at least one of the tasks that had been sent in advance as a *mathematical activity*. In the second session the group was required to work on the same task taking a didactical perspective and discuss how they would present and manage the task within their own classes. The group agreed that the task inspired by Hancock's article (ibid.) was the most appropriate for their pupils and chose to work on this in both sessions.

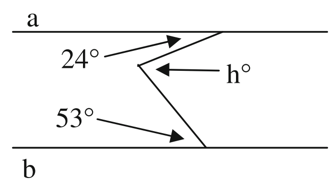
There were several goals for taking a mathematical focus on the task. Teachers and didacticians collaborating on mathematics tasks together was presumed to support community building (Eriksen 2007). Also, the ultimate goal is to stimulate teachers into developing mathematically rich learning experiences for their pupils. The underlying assumption is that if the teachers themselves can experience the rich possibilities of tasks they will be better prepared to share them with their classes.

Our analysis of the group's engagement with the task focuses especially on the participants' comments about the task—its difficulty, value, usefulness, etc.; and on the mathematical activity—the content knowledge used, and the challenge created and assumed in the resolution of the

[6] All names (teachers, didacticians and schools) have been changed.

Task 2

Consider pupils working on the following task (a and b are parallel).



What is the value of h?

The pupils are stuck. To help them you give them a hint: "Try drawing a help line." Which line would you have them draw? What other lines can the pupils draw? Of all the possible lines, which can be used to answer the task? What type of geometry must they know?

How can this task (or similar tasks) be adjusted for use with your class?

Fig. 1 Task used in group discussions: the 'M' task

task. As explained in Sect. 2.1 this focuses on 'explicit mediation and the way cultural artefacts are used to communicate meaning and solve the tasks'. In Sect. 2.2 we indicated that from this explicit evidence we make inferences about the sign (psychological) function of the artefact. In this first example we include detail from the conversation to illustrate how we use participants' conversation to make inferences about the mediation of the tasks.

5.1 Session 1: engaging in mathematics

A didactician (Dag) started the first session by signalling that the focus was to be on doing some mathematics through engaging with the tasks. Dag suggested: "*Let's throw school overboard and enjoy the problems!*"

The group chooses to work on Task 2 (Fig. 1) because Task 3[7] would take more time, Stefan remarks: "*possibly it [Task 3] could be used as a short project.*" The group recognizes that their school context is focusing their thinking: Olav remarks to Stefan: "*now you are thinking school again.*" Stefan agrees but observes that it is difficult to "*let go of school here.*"

Stefan looks for tasks with "*transfer value*", "*if 'we' will use it*", and believes that possibly Task 2 has better transfer value. Stefan claims that he "*could make use of [task 2] in class, thinking practically.*" Olav contributes by naming areas of geometry: "*parallel lines, corresponding angles*", and Stefan adds: "*In that way it can be applied more directly in the curriculum.*"

The conversation then turns to consider what their pupils will be able to do. Stefan: "*So then our question was: how many [pupils] will manage without us giving any hints?*" And he expresses his opinion that "*possibly the more clever pupils will manage*" and then the consideration is about which line the pupils "*possibly will naturally draw.*" Olav admits: "*we relate the whole time to the pupils; we did that when we worked with the tasks.*" Stefan adds: "*We thought of our class situation.*" Olav then adds: "*This with right angles, angle sum of triangles.*" Stefan then offers a solution using these properties by drawing a transversal at right angles to the parallel lines through the vertex of the angle h (Fig. 2i). Stefan observes: "*I could think that it is possible that the clever pupil could have managed.*"

Dag suggests extending one of the lines subtending the angle h° (Fig. 2ii), and calculates h as the exterior angle of the triangle formed, and alternate angles between parallel lines. He then asks each member in the group to suggest a different line to draw and the group to suggest how this line would support a solution. Kristin at first suggests drawing the parallel lines as in Fig. 2iii, and decides this will not be productive. She then makes another suggestion, a line parallel to the two given lines (Fig. 2iv), and calculates the value of h by adding 24 and 53, the sum of the alternate interior angles produced by the construction. Stefan asks: "*Do you think they [pupils] would have managed that? It was elegant.*" Olav replies by saying he will try it with his class the next day, but Kristin reflects that her class would not manage it; possibly only one pupil in the class would. And Stefan remarks: "*I have seen their [pupils'] thinking is very often hooked onto triangles.*" Stefan moves on to

[7] Three tasks were proposed: Task 1 was about folding an 'A4' rectangle to produce regular plane figures (square, equilateral triangle, etc.) and then proving the validity of the construction. Task 3 was based on a similar diagram as Task 2 (illustrated) but required the replacement of the 'dog-leg' transversal with a straight line so that the areas (considered bounded) to the left and right of the transversal remain unchanged.

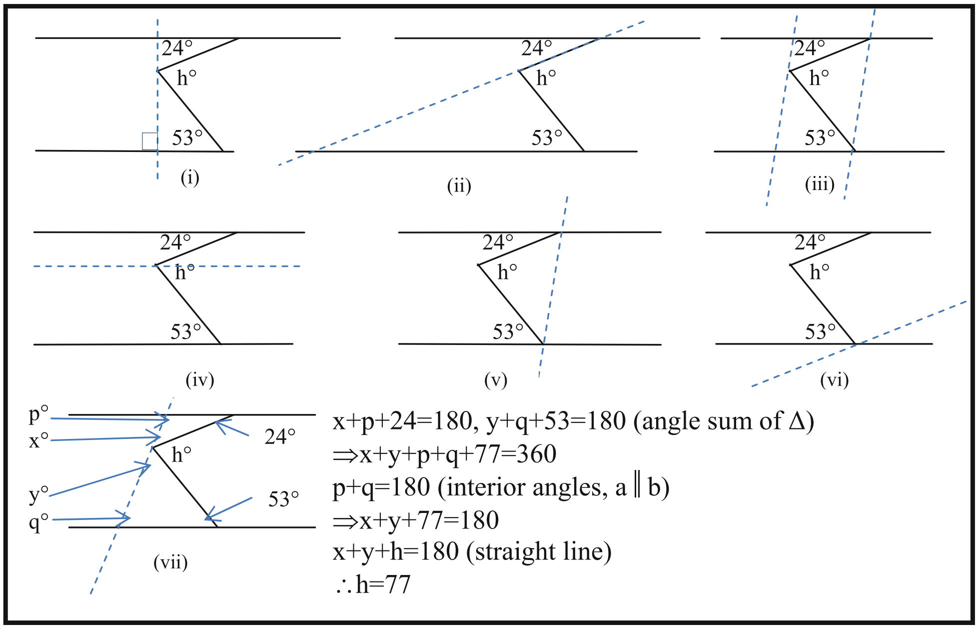


Fig. 2 Suggestions for 'help lines'. (*iii*) was not, at first, productive; (*vii*) constructs any line through the vertex of the angle h°

consider the types of geometry tasks that are presented in the textbook, and the conversation moves on to consider which of the elementary angle properties pupils know, and can possibly recall from their time at lower secondary school.

Up to this point the engagement with the task appears to be constrained by the teachers' experience of the curriculum content, textbook presentation of tasks, pupils' prior knowledge and competencies and the regular school conditions of time available. The solutions produced are consistent with the knowledge that participants presume pupils will apply: angles of right-angled triangles and angle properties related to parallel lines and a transversal. (Note that the solution using the exterior angle of a triangle and the sum of the opposite interior angles was offered by a didactician.) The teachers volunteer that it is difficult for them to engage with the task free from the influence of their knowledge and experience. The inquiry stimulated by the task appears to be limited to the application of familiar knowledge, rather than pushing the boundaries and asking, 'What if a line were drawn which does not immediately connect with experience?' In the instance when Kristin proposed a construction that looked superficially related to routine with two pairs of parallel lines (Fig. 2iii) it was fairly quickly dismissed, *perhaps* because it did not 'look like' figures that had been encountered in the textbook used.

An interpretation of the foregoing in terms of didactical relationships and mediation suggests that the goal is to 'find' resolutions of the task that lie within the range of pupils' knowledge, experience and competencies. Mediation then, as work *on* the task, is through the mathematical facts and skills, curriculum rules, resources and knowledge of pupils as a 'mediating complex'. We then infer from this interpretation the sign function of the mediating complex. It *appears that* the task is meaningful as an opportunity to apply already known mathematical ideas and mathematics is used as a tool to solve the task—some confirmation for this emerges in the didactical discussion in the second group session reported below. An alternative could be that the task is taken as an opportunity to develop problem-solving skills and strategies, and it takes on this character as Roy, the second didactician, is invited to contribute.

Roy poses a question with an admission that he does not know the answer. It appears that his question is accepted as an authentic inquiry; that is, an inquiry in which the inquirer is genuinely interested and personally committed to pursuing, rather than seeking to fit in with another person's agenda:

Roy: Well, I am going to take time here, because I have been intrigued by this one, is it possible to use those two parallel lines? [Roy refers to the construction that had been proposed by Kristin, Fig. 2iii.] I can't do it, so therefore I want to work on it. …Or, if it is not possible, should it be obvious looking at the diagram that it is not possible?

From this point, in terms of our analysis, the 'mediating complex' becomes dominated by the participants' mathematical knowledge; the goal of their discussion becomes to produce an answer to the question posed by Roy: "*is it possible to use those two parallel lines*?" At this stage the nature of the object about which they discuss *appears* to be transformed; it *appears* to be more than just a task that might be found in a school text, rather, it is a source of mathematical exploration. In the discussion it is not merely the task that is being mediated by mathematics and other artefacts but the resolution of the task, as re-articulated by Roy's question, that engages the group in mathematical activity.

All the participants in the group engage in the discussion. It is possible that Roy's admission that he couldn't do it makes it safe for all to engage without necessarily exposing their own mathematical knowledge to too much scrutiny. The shared object of the discussion is to explore the situation to address Roy's question, which demands conscious attention to both task and the application of appropriate mathematical knowledge. Stefan suggests drawing an additional line (Fig. 2vi), and Roy responds by observing that the other two would not be necessary. For nearly 3 min the group works in silence, until Roy offers an explanation for why he believes it is *not* possible to solve the problem with the pair of parallel lines shown in Fig. 2iii. However, Dag contradicts Roy's assertion and brings the inquiry to an end when he declares that he has a solution: "*On the other hand it might be that there is this parallelism* [referring to the parallel lines in the original statement of the task] *that is the important thing, and I think I have a proof for it.*" Dag then proceeds to share a proof of his resolution of the task (Fig. 2v) that uses only one of the lines Kristin constructed originally. For Dag, at least, the resolution is not just an answer to a task but an opportunity to develop a proof. The discussion then proceeds to explore whether the second line of Kristin's suggestion could be used, now, equipped with the argument Dag shared in his proof that it is established that *any* transversal through the vertex of the angle h will facilitate a solution (Fig. 2vii).

5.2 Session 2: preparing for the classroom

After a short break the group reconvened to consider the tasks from a didactical perspective: how they might be presented and used in class. At this point the object of the discussion becomes rather complex as it focuses on an imagined action, and thus concerns the interacting entities and relationships of the didactic triangle: pupils, mathematics and the mediating complex. These are the same things that appeared to constrain the first part of the discussion in the first group session. The organization of the sequence of group sessions was intended to provoke the participants to reflect on how their experience of the mathematics in the tasks might be shared with their pupils. The group begins by discussing how they will facilitate their pupils' entry into the problem. In particular they are concerned with how specific they will need to be in suggesting that the pupils draw a help line in the diagram:

Olav: I mean, it depends a little on what kind of class and what level you are on. I think that in my first class, but they are very clever then, because it is that sort of class. Actually I would just say, try to draw a help line, so I would not indicate any help line for a while.

…

Olav: and then, it might be that you, after some time, must step in and help them with the actual assisting lines.

Stefan: and it is not necessarily certain that we shall introduce that line, we must be a little flexible and, can you say, they have drawn a line and then we can build on what they have drawn.

Olav: The point is how much we are going to tell the pupils in advance. Are we going to give them tools, just lines for them to draw?

As we review the teachers' discussion we look for evidence of the outcome they intend for their pupils, the object of pupils' actions and the components of the mediating complex.

The teachers' discussion in the workshop is informed by their knowledge of the curriculum, the normal desirable state of pupil activity (Brown and McIntyre 1993) which they seek to achieve with their classes, and their knowledge of their pupils. Although the teachers work in classrooms independently of each other the discussion is pursued with a strong sense of 'taken as shared' understanding of classrooms and pupils; that is, there appears to be little need for them to explain these elements. Didacticians are largely excluded from this conversation as it relates to the teachers' domain. The discussion is principally concerned with the support that the teacher can offer pupils to ease their way into and through the task. The support is seen as comprising a sequence of hints that are framed to gradually close the gap between the pupil and the task, each hint reducing the challenge to a degree. There is no suggestion about how pupils might be led to a point of authentic inquiry, such as with an open challenge: 'How many different help lines can you find?'

The process suggested by the teachers resonates with the didactical 'effects' described by Brousseau (1997, pp. 25–26). Further, it has long been observed that teachers can reduce the cognitive challenge of tasks as they are implemented in classrooms (Henningsen and Stein 1997). We do not dispute that it is a teacher's role to facilitate pupils' engagement with challenging tasks and we thus reflect on what alternatives the teacher might have. The types of hints described in the dialogue above reduce the challenge in order to facilitate pupils' engagement. Alternatively a teacher might support by leading the pupil to access his or her own cognitive resources and knowledge of problem-solving strategies. However, the observation that teachers can reduce the cognitive demand of tasks, intentionally or otherwise, is not the main point emerging from our analysis. Within the developmental research activity, we have focused on the tasks that are used in mathematics classrooms, because tasks are one of the core technologies that teachers use to mediate the mathematics. However, we observe in the foregoing how the didactical relationship is transposed and mathematics, as a cultural artefact, becomes the mediator between the pupil and the task, in other words a tool to work on the task rather than the object of action. Such transposition is unintended and unwanted.

One of the didacticians tried to elicit from the teachers the purpose they would have in using this task with their classes, and prompted the teachers to consider a learning goal:

Roy: Could I ask a question? …Does this task fit in with your curriculum, and what would be your purpose in using the tasks in your classes, with the relation to the curriculum?

Stefan: I think all these exercises, none of them are exactly into the syllabus [*mm*], but we thought that maybe exercise two [the 'M' task] is what,

Olav: We use…corresponding angles…and parallel lines and use…with similar triangles.

This prompt did not generate a deeper discussion of the educational purpose for using the task. There was no reflection by the teachers on their experience in the first phase when they had engaged in authentic mathematical inquiry and what that had meant for them.

6 Example 2: inquiry tasks—a non-routine probability task

The second example is a group discussion in a workshop nearly 5 years into the projects. The group in the episode we consider here comprises six teachers, two from upper secondary schools and four from lower secondary schools; a didactician is also present but makes no contribution to the discussion. The group was required to evaluate three probability tasks for their potential to stimulate inquiry activity in their classroom and support pupils' understanding of mathematics. Following this, the group was asked to adapt the tasks so that they would be more effective in stimulating inquiry and developing pupils' understanding, and more suited to the teachers' classes. The first two tasks were taken from school texts; the third, a non-routine task, was created for the workshop (see Fig. 3). In our analysis we focus on the teachers' discussion about the third task. It is mainly the two upper secondary teachers who engage in the discussion, because the task is quickly judged to be too difficult for lower secondary pupils. It needs to be emphasized that the participants within the group were not required to work on the tasks mathematically, nor was it intended that the tasks be prepared for classroom use. The task devolved to the groups was intended to stimulate a meta-discussion about the nature and value of tasks; it was the didacticians' intention that the group would focus attention on the task as something to mediate mathematics, rather than on doing mathematics or teaching mathematics. Nevertheless, it was expected that the teachers' knowledge of mathematics, of the school curriculum and of their pupils and classes would mediate their discussion and evaluation of the tasks.

The task created for the workshop was not intended as an exemplar of a 'perfect inquiry task', if such a thing could ever exist. It was created as a non-routine problem that combined several domains within mathematics (probability, function graphs, decimals and percentages), within a realistic context.

The analysis of the discussion, as in the previous example, is based on the teachers' reflections (we omit much of the detail here to restrict the overall length of the paper). The first reaction of the teachers is that the task appears 'exciting' and open. Despite the suggestion that the group did not attempt to work out the answers, the two upper secondary teachers worked their way through the task, engaging critically with the ambiguities, and exposing the mathematical content and solution to the other teachers present. It appears that the task became a context for authentic mathematical inquiry. However, it became apparent to the group that the challenge in the task lies in making sense of all the text and its complexity, rather than the mathematical demand. The task was mediated by their knowledge of mathematics and linguistic and textual comprehension competencies that enabled them to make sense of the task requirements. Given that the task is non-routine it is not surprising that the teachers wanted to work through the task before making a judgment about its value or usefulness for their pupils.

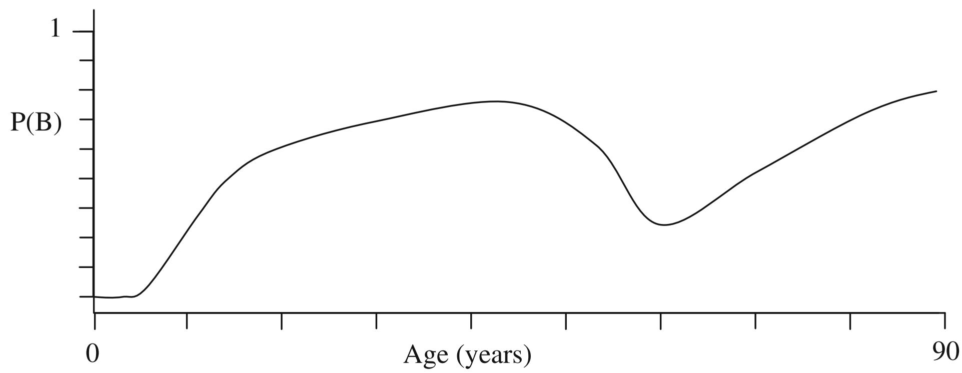
The teachers appear to value the task because of its realistic context, although, they argue, the context may not be so motivating for 16- and 17-year-old pupils who rarely experience serious illness. They perceive the task as being rich because of the range of mathematical concepts called upon in its resolution, such as the interpretation of graphical representations and reading two graphs in relation to each other, decimal and percentage representations, and probability. However, the opinion expressed by one and supported by others was that the task had little to do with probability, and that the major challenges came from ambiguity within the task (the relationship of P(A) and

Task 3

A potentially fatal disease can occur at any time in a person's life. The illness can be treated with a major operation. Whether the treatment is successful or not varies with the person's age when the illness first occurs.

Let P(A) be the probability for a successful result of the treatment. The graph below shows how P(A) varies with age.

Some patients get well without treatment. Let P(B) be the probability that the patient recovers (or dies of other causes) without treatment. This also varies with age of the patient when the illness occurs, as the graph below shows.



P(A) and P(B) are expected to be independent.

There is also a risk with major operations, also independent of A and B described above. Let P(C) be the probability to recover after a major operation, P(C) = 0.95.

Suppose that the patient is 40 years of age and is diagnosed with this illness. What action should the patient's doctor recommend? What is the reason for your suggested recommendation?

The hospital administration is concerned to use resources efficiently and decides to restrict the treatment of this illness to those with the following criteria:

$P(A) > 40\%$ and $P(A) > P(B)$

For what age groups (when the disease occurs) will the treatment be recommended? What is the probability of recovery?

Fig. 3 Probability task: evaluate for classroom use

P(C) and interpretation of the textual presentation) rather than the mathematics.

Criticisms of Task 3 were articulated and suggestions were made to adapt it or about how it might be presented to a class. For example it was suggested to delete the element referring to P(C) (i.e. removing the ambiguity), and that the two graphs might be prepared on acetate sheets and laid over each other. When the tasks were considered with regard to which the group thought would be most useful in developing pupils' understanding, Task 3 was the clear leader, but unlikely to be used! One of the upper secondary teachers, Frank, explained:

> If one comes to get a grip on task three, it is the most useful in the sense that it is used as a group task and use some time on it. But I feel it is too extensive and too many unknowns. …On the other hand it is perhaps the best task in relation to understanding the mathematics, even if there is a lot on graphs and functions and less probability in the task.

Earlier in the discussion Frank had reflected aloud on how his pupils might react to the task:

> I did use a relatively long time to understand that we ought to look at the relation between the two graphs together, and I at least think…if it is a task I use a long time on, my pupils would use even more time. If we were to give this task, firstly I think there would be new questions, and secondly a lot of frustrations and such, because they are not used to these kind of tasks. And I see the problem that we are used to tasks that they can find straightforward answers to, but it is, it is very much inquiry in this task here and a lot of interesting thinking really.

Frank's reflection on the task, which is mediated by an evaluation of his own engagement, leads him to a critical appraisal of the regular diet that pupils encounter in their mathematics lessons—short, straightforward tasks, to which answers can found relatively quickly. Frank appears to accept that more demanding tasks might better support pupils' inquiry and the development of pupils' understanding, but that the conditions of the classroom (time, pupil expectations, custom) preclude the use of such tasks.

7 Discussion: synthesis, what can be learned from this analysis

The group discussions presented above have been stimulated by a focus on tasks introduced by didacticians. Groups of teachers have been encouraged to work on tasks using their own mathematical knowledge, prepare tasks for teaching and evaluate tasks for class use. It should not be surprising therefore that the tasks have been central in the object of groups' actions and discussions. The resolution of the tasks has been mediated by mathematical knowledge, but also the resolution of the tasks has entailed, at times, making the mathematics the object of action, and mathematics then becomes both mediating artefact and object. The stimulus for mathematics to become the object, it appears, has been when the task becomes an object of authentic inquiry. This was most noticeable in the first episode above when the nature of the discussion changed as one participant shared his sense of uncertainty, and what had, up to that point, been considered to be a routine question became a focus of authentic inquiry.

In both episodes the teachers' perception of their own role in mediating tasks for pupils in their classes is evident. Teachers seek to facilitate entry into tasks, give hints and answer questions. It appears that teachers perceive their role to be in closing the gap between pupils and the challenge of tasks—rather than to support pupils in engaging with the challenge. Thus it is possible that the teachers' experience of authentic mathematical inquiry in the workshops is not passed on to their pupils. However, it becomes evident from the discussion in the second episode that teachers are aware of the unwanted implications of their 'gap closing role' but are also conscious of time constraints, the curriculum and pupils' expectations. The transformation of teaching entails more than just a change of behaviour on the part of teachers or the tasks that are used in the classroom.

As long as the focus is on the task, the object of discussion will be the task and the goal will be the resolution of the task. Given that most tasks in mathematics classrooms have no value beyond the classroom, the outcome of successful resolution is merely progression to the next task. Mathematics is one item in the toolbox of cultural artefacts that are used to mediate the given tasks. The focus on tasks does not appear to invite the exchange of mathematics and task as mediator and object in actions. This is not to say that a focus on tasks does not lead to purposeful mathematical activity—the tasks demand mathematical thinking and the application of mathematical concepts, skills, competencies, etc. The tasks provide important opportunities to practise routine skills and procedures. The tasks can make challenging demands on mathematical understanding. Nevertheless, given that the purpose of mathematics teaching is that pupils will encounter and learn mathematics, one is challenged to envisage situations in which learning mathematics becomes the central object of action and the task an item in the toolbox of cultural artefacts that mediate the mathematics.

The analysis we report here set out to expose characteristics of mediation in teachers' discussion. We first note the range of mediators introduced. The curriculum is, unsurprisingly, a major tool in teachers' actions as they work on tasks, it constrains their actions on the task, as in the first episode discussed where the teachers' engagement with the task was, at first, limited to the curriculum knowledge of their pupils. The curriculum was also used to evaluate whether tasks were appropriate for their use. However, as highlighted in the second episode, it is the syllabus of content knowledge that pupils are expected to acquire that appears to be uppermost in teachers' consideration even though they recognize the value of tasks in developing pupils' understanding. The curriculum is also implicated in the rules of action, such as the amount of time that can be spent on any task. When planning for class use, the complexity of teaching and learning is clearly evident. Teachers consider their pupils' competencies, pupils' expectations and the normal desirable states of pupil activity in their classrooms. The analysis also draws attention to the fact that in the tasks considered there appears to be a switch in roles between mathematics and task. It appears that mathematics is the mediating artefact in working on the task, and the goal is to resolve the task. This is contrary to the assumed position in teaching mathematics that the task will mediate mathematics and the goal is to learn mathematics.

8 Conclusion

It was suggested at the outset that this analysis would identify and characterize mediational means used by teachers. The analysis leads us to the conclusion that the focus on tasks appears to emphasize the resolution of tasks as the goal of actions in classrooms rather than the mathematics to be experienced and learned. Authentic inquiry within tasks can focus attention on mathematics; nevertheless, it appears that the goal continues to be task resolution rather than learning mathematics. Further consideration of the didactic triangle leads to the perception that with teacher (and other components of the mediating complex) and pupils at two of the vertices, mathematics subject content is replaced by the task at the third vertex.

An intended outcome of teaching development activity is that didactical relationships in teaching will change (evolve, expand or develop) over time. We assume that change will be related to sustained developmental activity. Hence, an important question in terms of the impact of developmental activity is: What *changes* in the didactical relationships can be observed over the course of the developmental activity? This present paper, which focuses on the operationalization of analytic categories, will contribute to the tracking of such changes. Additionally, the focus in this paper on the characteristics of teachers' engagement in mathematical tasks reveals affordances and constraints of tasks and is thus useful in the design and use of tasks in future mathematics teaching development.

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