Performance Comparison of Residual Related Algorithms for ToA Positioning in Wireless Terrestrial and Sensor Networks

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***Abstract*—Time of Arrival (ToA) is a popular technique for terrestrial positioning. This paper presents a comparison of ToA based residual related positioning algorithms in wireless terrestrial and sensor networks in both long range outdoor and short range indoor environments. Using ToA distance measurement error models in both environments, we compare the performance of the Least Square Estimation (LSE), Residual Weighting (Rwgh), Iterative Minimum Residual (IMR), Select Residual Weighting (SRwgh) and Lower-Computational-Cost Residual Weighting (LCC-Rwgh) algorithms. The latter three algorithms are inherited from the Rwgh algorithm for wireless sensor networks. Two aspects of the performance comparison are addressed: computational complexity and positioning accuracy. In performance comparison, the complexity comparison is done by analyzing the number of LSE iterations while the accuracy comparison is conducted through a set of simulations in both environments.**

***Index Terms*—performance comparison, positioning, ToA, residual.**

I. Introduction

In recent years, there has been a lot of interest in the positioning of mobile terminals. For instance, providing the accurate location information of the mobile terminal for Emergent 911 call has become mandatory in the USA and determining the physical positions of sensors is a fundamental and crucial requirement in many wireless sensor network applications.

Positioning techniques, such as Time of Arrival (ToA), Time Difference of Arrival (TDoA), and Received Signal Strength Indicator (RSSI) are often used to obtain distance information (range measurements) between transmitters and receivers [1]. The location estimation can be computed based on a set of range measurements. Of these, the ToA technique is the most widely used one. A fundamental assumption in applying this positioning technique is the receptions of signal propagation through Line of Sight (LOS) path. Violation of this assumption will introduce Non LOS (NLOS) errors in range measurements and will lead to erroneous location estimation [2].

In two dimensional cases, at least three reference nodes are needed for positioning in ToA method. If there are more than three reference nodes, redundancies of reference nodes can be adopted for NLOS error mitigation. There are several positioning algorithms which have the ability of NLOS errors mitigation by using redundancies of reference nodes without identifying NLOS errors, such as Rwgh [2], IMR [3], SRwgh [4], LCC-Rwgh [5] etc. With the redundancies of reference nodes, the algorithms can avoid reiterative transmissions to reference nodes to get enough information for NLOS error mitigation, which can reduce power consumption during the measurement stage and are suitable for power constrained networks. The traditional one of these algorithms is Rwgh which can mitigate NLOS errors efficiently. However, it is very computationally complex when the number of reference nodes is large and not suitable for wireless sensor networks where there may have more reference nodes available. Recently, there are some ramifications which need lower computational complexity based on conventional Rwgh algorithm, such as the IMR algorithm, SRwgh algorithm, LCC-Rwgh algorithm etc. Since the algorithms mentioned above are all based on residual and the principle of these algorithms is similar, the computational complexity and performance of these algorithms in different environments need to be investigated.

The comparison of the algorithms includes two aspects: computational complexity and positioning accuracy. In computational complexity comparison, the number of LSE is adopted because LSE is used iteratively in these algorithms and it is the major part in the positioning procedure. The positioning accuracy of these algorithms mentioned above is compared in three popular ToA-based NLOS error models, two long range outdoor environments and one short range indoor environment respectively. In long range outdoor environments, deterministic and random NLOS error models [2, 6] are adopted in our simulation. In short range indoor environment, the model [7, 8] is based on ToA-estimation techniques and characterizes the distance errors as a function of the bandwidth of the system in the presence of LOS, and Obstructed LOS (OLOS) propagation conditions respectively.

The rest of the paper is organized as follows. Section II undertakes a description of the algorithms compared. Section III is devoted to discussions of the computational complexity. Section IV presents the results of the performance evaluation and the paper is concluded in Section V.

II. Algorithms Description

In this section, we will give a brief description of all the above mentioned algorithms. Assume there are two kinds of nodes: reference nodes and target nodes. Reference nodes, equipped with Global Positioning System (GPS) or deployed in a known position in advance, know their positions accurately and can be used by the target nodes which do not know their positions during the positioning procedure. The position information received from the reference nodes, denoted as range measurements, are treated the same, which means that every reference node has the same priority. Assume that the number of range measurements is equal to that of the reference nodes, which means that the distances between target nodes and reference nodes are measured only once for each reference node.

A. LSE for Location Estimation

LSE is a basic technology in location estimation, which is based on the criterion of least square. Although this method does not use residual weighting and comparing, we still take it into account in order to compare with other algorithms as reference. The problem of distance-based location estimation can be defined as follows [9]: Assume the target node is located at some target location (x, y) and M reference nodes are deployed at known locations (x_i, y_i), $1 \leq i \leq M$, with range measurements R_i. For equation:

$$\mathbf{Y} = \mathbf{AZ} \tag{1}$$

where

$$\mathbf{Y} = \begin{pmatrix} R_1^2 - R_2^2 + x_2^2 + y_2^2 - x_1^2 - y_1^2 \\ R_1^2 - R_3^2 + x_3^2 + y_3^2 - x_1^2 - y_1^2 \\ \vdots \\ R_1^2 - R_M^2 + x_M^2 + y_M^2 - x_1^2 - y_1^2 \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \\ \vdots & \vdots \\ x_M - x_1 & y_M - y_1 \end{pmatrix}$$

$$\mathbf{Z} = \begin{pmatrix} x \\ y \end{pmatrix}.$$

The parameters in matrix $\mathbf{Y}$ and $\mathbf{A}$ could be found from reference node locations and the corresponding range measurements. $\mathbf{Z}$ is the value of location that we want to estimate. If $A^T A$ is nonsingular, the LSE is obtained as follows:

$$\hat{\mathbf{Z}} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{Y}. \tag{2}$$

LSE works well in LOS environments, but it can not mitigate the NLOS error.

B. Residual Weighting Algorithm

The Rwgh algorithm, the traditional algorithm, works as follows [2]:

1) Given M ($M > 3$) range measurements (from M different reference nodes), $N = \sum_{i=3}^{M} C_M^i$ range measurement sets are formed. Each set is represented by a reference node index set $\{S_i, i = 1, 2, ..N\}$.

2) For each set, compute the intermediate LSE $\hat{\mathbf{Z}}$ and normalized residual error, $\overline{Res}$:

$$\overline{Res}(\hat{\mathbf{Z}}, S_k) = Res(\hat{\mathbf{Z}}, S_k)/size\ of\ S_k, k \leq N, \tag{3}$$

where $Res(\hat{\mathbf{Z}}, S_k)$=$\sum_{i \in S_k}(R_i$-$\|\hat{\mathbf{Z}}$-$\mathbf{X}_i\|)^2$, $\mathbf{X}_i$ is the coordinate vector of the i-th reference node and R_i is the range measurement from the i-th reference node.

3) Find the final estimation, $\hat{\mathbf{X}}$, as the weighted linear combination of the intermediate estimations from step 2). The weight is inversely proportional to $\overline{Res}$ of the estimation. Mathematically,

$$\hat{\mathbf{X}} = \sum_{k=1}^{N} \hat{\mathbf{Z}}_k (\overline{Res}(\hat{\mathbf{Z}}_k, S_k))^{-1} / \sum_{k=1}^{N} (\overline{Res}(\hat{\mathbf{Z}}_k, S_k))^{-1}. \tag{4}$$

Because the set of range measurements with smaller $\overline{Res}$ has smaller chance contaminated by NLOS errors [2], the reciprocal of the $\overline{Res}$ is used as the weight.

C. Iterative Minimum Residual Algorithm

The IMR [3] which is developed for wireless sensor networks is an iterative algorithm aiming at finding the result from range measurements set with the minimum $\overline{Res}$ or when the difference between $\overline{Res}$es are small. Assume the parameters of target node, reference nodes and their corresponding range measurements are the same as in LSE. The steps of IMR are:

1) Initialization: Let $n = M$, $D = \{R_i, 1 \leq i \leq M\}$ and the tolerance δ to a small positive number. Define the number of iterations as N_i.

2) Conventional LSE: Find the LSE $\hat{\mathbf{Z}}$ using the observation data D and determine the corresponding normalized residual error of the estimator $\overline{Res}(\hat{\mathbf{Z}})$. Set $\hat{\mathbf{Z}}_i = \hat{\mathbf{Z}}$, $\overline{Res}_{min} = \overline{Res}(\hat{\mathbf{Z}})$.

3) Iteration: Find the LSE $\hat{\mathbf{Z}}^{(k)}$ and the normalized residual error $\overline{Res}(\hat{\mathbf{Z}}^{(k)})$, $1 \leq k \leq n$, for n sets of n range measurements in D taking n-1 at a time. Denote the estimator with the minimum normalized residual error in $\hat{\mathbf{Z}}^{(k)}$ with $\hat{\mathbf{Z}}_m$, the set of range measurements used in $\hat{\mathbf{Z}}_m$ with D_m, and $\overline{Res}_m = \overline{Res}(\hat{\mathbf{Z}}_m)$. If $\overline{Res}_{min} - \overline{Res}_m > \delta$, then $\hat{\mathbf{Z}}_i = \hat{\mathbf{Z}}_m$; else return $\hat{\mathbf{Z}}_i$. If $n > 4$ and M-$n + 1 < N_i$, then $n = n - 1$, $D = D_m$, $\overline{Res}_{min} = \overline{Res}_m$, repeat 3); else return $\hat{\mathbf{Z}}_i$.

In practice, tolerance δ can be determined based on location estimation accuracy requirements and the number of iterations N_i, can be determined based on the computational capacity of individual sensor node. In fact, the number of iterations specifies the maximum number of erroneous range measurements to be excluded by the IMR algorithm.

D. Select Residual Weighting Algorithm

The SRwgh [4] is based on Rwgh. It picks out the subsets of range measurements with minimum $\overline{Res}$ and then calculates the weighted mean, which can efficiently reduce the computational complexity. The steps are as follows:

1) Get all the range measurements sets which contain 3 range measurements from all the M range measurements. Let the number of the sets be K, obviously, $K = C_M^3$. Each set is represented by reference nodes index set $\{S_i, i = 1, 2, ..K\}$.

For example, if *M* = 4, we can get $K = C_4^3 = 4$ different sets, S_{1-4} are (1, 2, 3), (1, 2, 4), (1, 3, 4), (2, 3, 4). For each set, compute the LSE $\hat{\mathbf{Z}}$ and corresponding $\overline{Res}$. From the *K* sets, find the one that has the minimum $\overline{Res}$ and let the set *S* be S_{min}.

2) Let the number of elements in S_{min} be *P*, the complement of S_{min} be *Q*. Put every element in *Q* into S_{min} respectively to form *M-P* new range measurements sets, $\{S_i, i = 1, 2, ..M\text{-}P\}$. Now, the number of the elements in the new S_i becomes $P+1$. For the *M-P* new sets, compute the location estimations by LSE and the corresponding $\overline{Res}$, then find the one that has the minimum $\overline{Res}$ and let the *S* be S_{min}.

3) If $P < M$, go to 2); if $P = M$, go to 4).

4) For all the computed location estimations and the corresponding $\overline{Res}$ in steps 1) 2) 3), use the following equation to get the final estimation of location,

$$\hat{\mathbf{X}} = \sum_{k=1}^{V} \hat{\mathbf{Z}}_k (\overline{Res}(\hat{\mathbf{Z}}_k, S_k))^{-1} / \sum_{k=1}^{V} (\overline{Res}(\hat{\mathbf{Z}}_k, S_k))^{-1}, \quad (5)$$

where *V* is the number of the computed location estimations.

E. Lower-Computational-Cost Rwgh Algorithm

The LCC-Rwgh [5] is also based on Rwgh and has a different way in range measurement sets selection from that of SRwgh. The steps are as follows:

1) Get all the range measurements sets which contain *M*-1 range measurements from all the *M* range measurements. Let the number of the sets be *K*, obviously, $K = C_M^{M-1} = M$. For each set, computes the LSE $\hat{\mathbf{Z}}$ and the corresponding $\overline{Res}$. From the *K* sets, find the one that has the minimum $\overline{Res}$ and let the set *S* be S_{min}.

2) Let the number of elements in S_{min} be *P*. Pick out every element in *P* respectively to form *P* new range measurements sets, and the new index set is $\{S_i, i = 1, 2, ..P\}$. For the *P* new sets, compute the location estimations by LSE and the corresponding $\overline{Res}$, and then find the set that has the minimum $\overline{Res}$ and let the *S* be S_{min}.

3) If $P > 3$, go to 2); if $P = 3$, go to 4). Note that the minimum number of range measurements needed for localization is three in 2-dimensional cases.

4) Using LSE, compute the location estimation and its $\overline{Res}$ with all the range measurements.

5) For all the computed location estimations and the corresponding $\overline{Res}$ in steps 1) 2) 4), use the following equation to get the final estimation of location,

$$\hat{\mathbf{X}} = \sum_{k=1}^{V} \hat{\mathbf{Z}}_k (\overline{Res}(\hat{\mathbf{Z}}_k, S_k))^{-1} / \sum_{k=1}^{V} (\overline{Res}(\hat{\mathbf{Z}}_k, S_k))^{-1}, \quad (6)$$

where *V* is the number of the computed location estimations.

III. COMPUTATIONAL COMPLEXITY ANALYSIS

The position computation in LSE, based on all the range measurements available without any selection, has the lowest computational complexity because it only needs to use LSE once. Note that for other algorithms that are discussed above, LSE is always needed for each iteration. In the Rwgh algorithm, if three or more range measurements are available, i.e., $M \geq 3$, the number of all possible sets with three or more range measurements can be calculated as $C = \sum_{k=3}^{M} C_M^k$ [2]. We can see that the number of sets becomes very large as *M* increases. One LSE needs to be derived for each of the sets with three or more range measurements. Therefore, the computational complexity of this algorithm might be quite high when a large number of reference nodes are available. IMR [3] does not use residual weighting method. It is a suboptimal, iterative implementation of the minimum residual estimator (MRE) [2], which iteratively excludes erroneous range measurements one-by-one and searches for the MRE instead of conducting global search as in the MRE algorithm. The reason for low computational complexity of this algorithm is that it iteratively searches for the MRE among the LSEs derived from a subset of all possible sets instead of conducting global search. If more than three range measurements are available, i.e., $M \geq 4$, the possible maximum number of iteration is $M-3$ and the number of LSEs needed in the IMR algorithm can be derived as $C_i \leq 1 + (M - n + 1)(M + n)/2$ [3], where $n = max(M - n_i + 1, 4)$, and the variable n_i is the predefined number of iterations.

From the algorithms described above, we can observe that the computation of LCC-Rwgh algorithm is very similar to that of the IMR. The difference is that the LCC-Rwgh gets the residual weighted mean value of all the computed LSEs from the subsets of all possible sets while IMR aims to find the one with the minimum residual $\overline{Res}_{min}$. Therefore, the number of LSEs needed in the LCC-Rwgh algorithm can be expressed as follows: $C = (M^2 + M)/2 - 5$.

The method that the SRwgh algorithm uses to select subsets from range measurement sets is quite different from that of LCC-Rwgh. In the first step, the LCC-Rwgh derives *M* LSEs based on all possible sets of *M* measurements taking $M - 1$ at a time. Then, it determines the best estimator in terms of minimum normalized residual error $\overline{Res}$. The range measurement which is not employed in the derivation of the best estimator is eliminated from the observation data. Then it takes $M-2$ from $M-1$ measurements and so on. The SRwgh algorithm derives C_M^3 LSEs based on all possible sets of M range measurements taking 3 at a time. Then, determine the best estimator in terms of minimum normalized residual error $\overline{Res}$. By adding every rest measurement into the best estimator, it can form $M - 3$ new range measurements sets and derives the $M-3$ LSEs. By determining minimum normalized residual errors, we can get another best estimator and use the rest $M-4$ measurements to form $M - 4$ new sets and so on. Therefore, the number of LSEs needed in the SRwgh algorithm can be expressed as $C = C_M^3 + (M^2 - 5M + 6)/2$.

Table I shows the number of the iterations for range measurements in which the LSE needs to be derived in the Rwgh, SRwgh, LCC-Rwgh, IMR algorithms as a function of the total number of range measurements, i.e., reference nodes. Because IMR uses an iterative way to search the MRE, the number of LSEs of this algorithm is not an accurate

TABLE I
COMPARISONS OF NUMBER OF LSES BETWEEN DIFFERENT ALGIRITHMS.

Number of LSEs		Number of range measurements					
		4	5	6	7	8	9
Algorithms	Rwgh	5	16	42	99	219	466
	SRwgh	5	13	26	45	71	105
	LCC-Rwgh	5	10	16	23	31	40
	IMR	5	6-10	7-16	8-23	9-31	10-40

number but a range depended on δ and N_i. From Table I, we conclude that the number of LSE iterations of these algorithms ranking from largest to smallest with respect to the number of range measurements is: IMR, LCC-Rwgh algorithm, SRwgh algorithm, Rwgh algorithm. IMR and LCC-Rwgh algorithms may have the same number when IMR algorithm gets its largest number of LSEs.

IV. SIMULATIONS AND NUMERICAL RESULTS

The performance of the algorithms described in Section II is evaluated through simulations using Matlab 7. The simulation has been carried out for both long range outdoor and short range indoor environments. The performance criterion for the accuracy of these algorithms is chosen as the Root Mean Square Error: $RMSE = sqrt(E[(x-\hat{x})^2 + (y-\hat{y})^2])$, where x, y are the true value of position, and $\hat{x}, \hat{y}$ are the estimated value.

Considering that the IMR algorithm is a suboptimal implementation of MRE [3], which means that sometimes the performance is as good as that of MRE and sometimes it is not in sense of minimum residual, we use the result of MRE instead of IMR in the performance comparisons in order to achieve consistency.

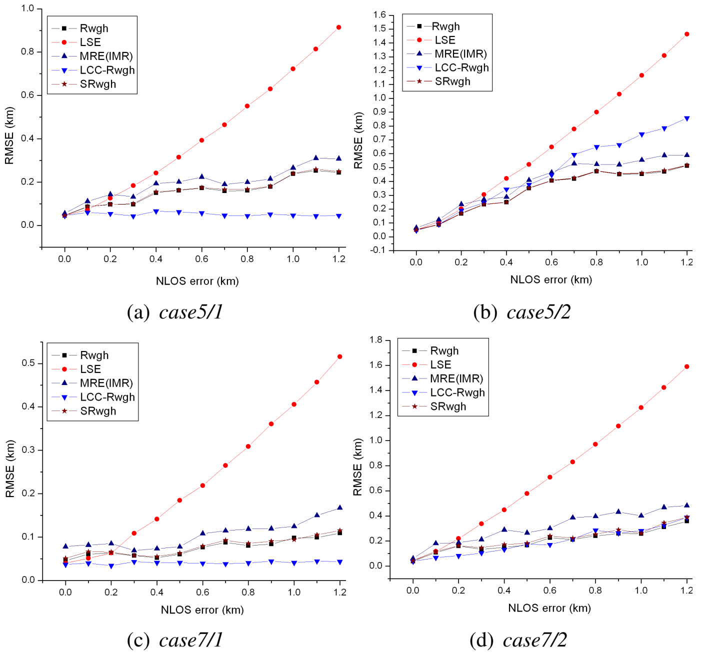
A. In Long Range Outdoor Environments

In long range environment, range measurement is contaminated by two types of errors [10]: measurement errors that can be modeled as a Gaussian random variable with zero mean and a small standard deviation and NLOS error that has a complicated distribution with positive mean and a relative large standard deviation which is verified in experiments [11]. Therefore, we model the measurement errors as a Gaussian distribution with zero mean and 30 m standard deviation. To remove the target nodes location dependence in the performance evaluation, all the results presented here are the average results of 100 target nodes that distribute uniformly in a 2500 m×2500 m square and the reference nodes are also distributed uniformly in the area. In the following discussion, the performance of different algorithms is given in both deterministic and random NLOS error models. We define "case m/n" to be that n out of m range measurements are contaminated by NLOS errors. For example, "case 5/1" means that one out of five range measurements is contaminated by NLOS error.

1) NLOS Error as a Deterministic Variable: In order to see the relationship between the RMSE of these algorithms and value of NLOS error(s), we give the RMSE comparisons of these algorithms with the NLOS error(s) as the same constant although in real environments the situation is quite different. We vary NLOS errors from 100 m to 1200 m and calculate the RMSE as a function of NLOS error(s).

Fig. 1 (a)-(d) shows the comparisons of RMSE as a function of NLOS error(s) among 5 different algorithms in four scenarios. From all figures, we can see that the LSE algorithm which is linearly proportional to the NLOS error(s) results in the poorest RMES performance because it uses all the range measurements available equally, without doing any NLOS error(s) mitigation procedure. The other algorithms have similar performance, but the MRE (IMR) is worse than Rwgh and SRwgh. That is because the results from the range measurements set with minimum $\overline{Res}$ are not always the best estimation. When the number of NLOS errors is small, it also misses the other sets of range measurements without NLOS error. The performance of SRwgh and Rwgh is almost the same and in some cases the performance of SRwgh is slightly worse than that of the Rwgh. From the description of SRwgh algorithm, we can see that it uses the range measurements set with minimum $\overline{Res}$ to generate the sets for next steps. Because the range measurements sets with smaller $\overline{Res}$ have larger weights in the process of weighted mean, SRwgh gets the main part in Rwgh. Even SRwgh needs only half of the LSE compared to Rwgh when there are 7 range measurements, the performance of SRwgh and Rwgh is close. The performance of LCC-Rwgh is much more complicated compared with other algorithms. In cases 5/1 and 7/1, the performance of LCC-Rwgh is much better than the others. In case 7/2, the performance of the LCC-Rwgh has a similar performance to that of the Rwgh, which is worse than that in cases 5/1 and 7/1. But in case 5/2, the performance of LCC-Rwgh descends dramatically. The smaller the numbers of the NLOS errors, the better performance of the LCC-Rwgh algorithm. When the percentage of NLOS errors becomes larger, the performance of LCC-Rwgh algorithm will not be stable, as shown in Fig. 1 (b). The reason is that the range measurements set choosing procedure of LCC-Rwgh is from large number of range measurements to small number of range measurements. If there is only one NLOS error, this method is quite efficient because the one it chooses in the first step is the one most likely not to be contaminated by NLOS error and the corresponding subsets of range measurements are also with LOS. So the sets of LOS range measures contain the major parts. While if there are more than one NLOS range measurements, this procedure is not that efficient.

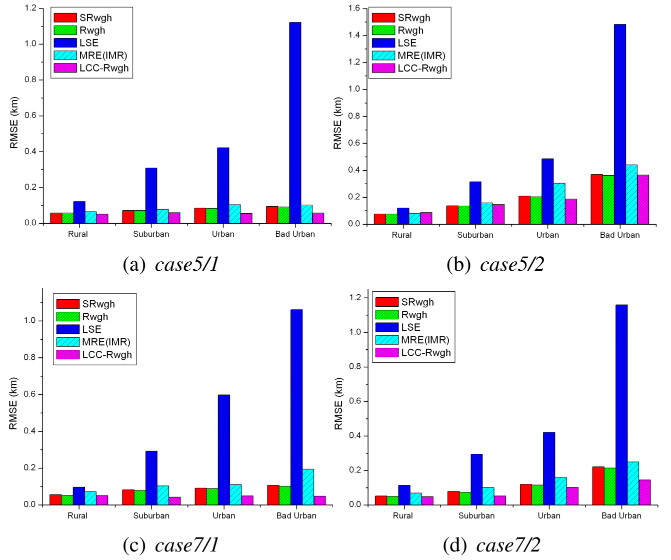
2) NLOS Error as a Random Variable: Random NLOS errors can be derived from the delay profiles described by a probability density function of excessive propagation delay with respect to a direct path [2]. By multiplying excessive delay τ and the speed of light, we can obtain the NLOS errors. Three frequently used delay profiles are an exponential, a uniform, and a delta random variable. In this study, the exponential model is adopted and the parameter of the exponential model is τ_{rms}, where τ_{rms} is the delay spread which depends on the propagation environments. τ_{rms} is log



(a) *case5/1* (b) *case5/2*

(c) *case7/1* (d) *case7/2*

Fig. 1. RMSE comparisons of different algorithms for deterministic NLOS error model in long range environment.



(a) *case5/1* (b) *case5/2*

(c) *case7/1* (d) *case7/2*

Fig. 2. RMSE with random delay model in long range environment.

normal distributed [6]:

$$\tau_{rms} = T_1 d^{\epsilon} \xi, \tag{7}$$

where T_1 is is the median value of τ_{rms} at $d = 1$ km, d is the distance between the reference node and target node in kilometers, ϵ is an exponent that lies between 0.5-1.0, ξ is a log normal variable and $10log\xi$ is a Gaussian random variable having zero mean and a standard deviation, σ_{ϵ}, that lies between 2-6 dB. For four typical environment types, Bad urban, Urban, Suburban and Rural, the value of T_1 are 1.0, 0.4, 0.3, and 0.1 [6] respectively. ϵ is 0.5 and σ_{ϵ} is 4dB in all above environments.

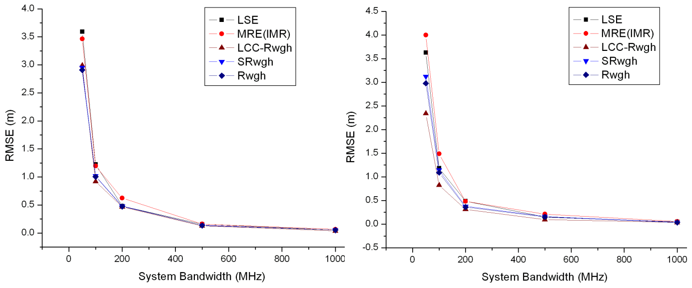
Fig. 2 (a)-(d) shows the simulation results in random delay model. From Fig. 2 (a)-(d), in different environments, the performance of the algorithms varies a lot with the number of NLOS errors. As demonstrated in Fig. 2, the more number of range measurements and less number of NLOS errors, the more accurate the estimated location is. In rural, the performance of algorithms is the best while in Bad urban where there are more severe NLOS propagations the performance is the worst compared to other cases. As for the algorithms compared and the NLOS error model used, the ranking order from the best to the worst is LCC-Rwgh, Rwgh, SRwgh, MRE(IMR), LSE in most cases. The performance of SRwgh and Rwgh is almost the same while in some cases the performance of SRwgh is slightly worse than that of the Rwgh. The performance of LCC-Rwgh algorithm is not stable in cases 5/2 and 7/2, namely, in some environments it is better than Rwgh but in some environments it is not, as shown in Fig. 2 (b) and (d). That is because in deterministic NLOS error environment, in cases 5/2 and 7/2, two NLOS errors are in the same value but in random cases which is from true propagation situations, the chance that 2 NLOS errors have the same value is small. So in random NLOS cases, LCC-Rwgh can also find out the range measurements set with smaller NLOS errors when the number of NLOS errors is larger than 1 thus the performance of it is better than the other method discussed above in some cases in 5/2 and 7/2.

B. *In Short Range Indoor Environment*

We adopt a model for the estimated distance from ToA of the first path in an indoor multipath environment typically used for WPAN applications [7,8]. It presents the effect of bandwidth on distance estimation error because increasing the bandwidth makes the channel impulse response closer to the ideal case and decreases the distance error [8]. The behavior of the channel under LOS and OLOS environment is very different. The range measurements can be modeled by using the equation: $\hat{d} = d(1 + r)$, where r is a random variable, whose distribution depends on the particular channel scenario. From [7] we can see that for the LOS case, r follows a Gaussian distribution with a zero mean, and a variance that depends on system bandwidth. For the OLOS case, it has been shown that r has a hybrid distribution, which is a linear combination of Gaussian and exponential distributions. The Gaussian parameters can be found in Table II. The exponential distributions parameter is 2.6 m^{-1} according to [7]. Since the ratio of Gaussian and exponential distribution is 504:137 when system bandwidth is 100 MHz in OLOS cases [8], we adopt 21% as the percent of exponential distribution in OLOS cases. The performance of the algorithms discussed is evaluated through the following simulations: All the 100 target nodes distribute randomly in a 15 m×15 m square, the reference nodes distribute uniformly in this area. System bandwidth varies from 50 MHz to 1000 MHz, as a parameter of distance error models.

TABLE II
TYPICAL ERROR PARAMETERS IN LOS AND OLOS CASES ACCORDING TO SYSTEM BANDWIDTH.

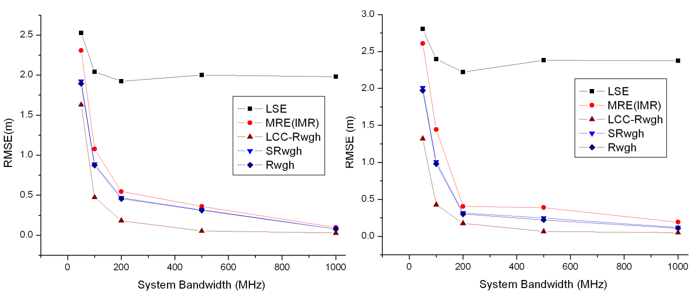
System Bandwidth (MHz)	50	100	200	500	1000
Standard deviations in LOS	19.06	6.48	2.6	0.83	0.27
Standard deviations in OLOS	9.27	2.67	0.78	0.29	0.15



(a) *5 reference nodes* (b) *7 reference nodes*

Fig. 3. Performance comparison in LOS channel.

Figs. 3 and 4 show the comparison of RMSE as a function of system bandwidth among 5 different algorithms for the 4 scenarios in short range indoor environment. Fig. 3 (a)-(b) shows the performance in LOS condition while Fig. 4 (a)-(b) shows the performance in OLOS condition with 5 reference nodes and 7 reference nodes respectively. From Fig. 3 (a) and (b), we can see that all the algorithms discussed have similar performance because the range measurements are all LOS cases. From Fig. 4 (a) and (b), the performance of LSE becomes the worst one while the others have similar performance. The performance of MRE (IMR) algorithm is worse than that of the Rwgh and SRwgh which have very similar performance. The LCC-Rwgh has the best performance in the OLOS cases. In OLOS cases, the larger OLOS measurement error is from exponential distribution. Since the ratio of exponential distribution in the OLOS cases is about 21%, which is close to the case 5/1 in deterministic NLOS error in long range environments, LCC-Rwgh works well in OLOS cases. From Figs.3 and 4, we notice that the wider the system bandwidth, the more coherent the trends of the performance of the algorithms except LSE. Therefore in system design, we can make a trade off between the system bandwidth and the algorithms. If the system bandwidth is wide enough, we can choose the algorithm with the smallest computational complexity except LSE, i.e. IMR.



(a) *5 reference nodes* (b) *7 reference nodes*

Fig. 4. Performance comparison in OLOS channel.

V. CONCLUSIONS

In this paper, we give a comprehensive performance comparison between different residual related positioning algorithms and get the following conclusions. The computational complexity ranges from high to low is Rwgh, SRwgh, LCC-Rwgh, IMR, LSE. Rwgh and SRwgh have close performance and the performance of SRwgh is slightly worse than that of the Rwgh. MRE (IMR) has the same trend as Rwgh and SRwgh, but it is not as good as the former two algorithms in most cases and they are all robust to both the number and the value of NLOS errors compared with LSE and LCC-Rwgh. LCC-Rwgh performs well in indoor environment and in outdoor environments when the number of NLOS errors is small. But its robustness to the number of NLOS errors is weak in deterministic model in outdoor environment. In indoor environment, since the OLOS error is a function of system bandwidth, we can make a trade-off between system bandwidth and positioning algorithms when designing a system.

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