Convex costs and the hedging paradox

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Abstract

Financial theory suggests that hedging can increase shareholder value in the presence of capital market imperfections, including direct and indirect costs of financial distress, costly external financing, and convex tax exposure. The influence of these costs, which are high when profits are low and low or negligible when profits are large, on the extent of firm hedging has not been consistently addressed in the finance literature. In Brown and Toft’s (2002) model, more convex costs imply that a firm will decrease the extent of hedging. At the same time, one version of Smith and Stulz’s (1985) tax hypothesis implies that a given firm is expected to increase the extent of hedging under a more convex tax exposure. I address this ambiguity in the literature by showing that, in incomplete markets, value-maximizing firms that stand to gain the most from hedging may in fact hedge less than otherwise identical firms with less to gain from hedging. This hedging paradox can partly account for the lack of conclusive evidence to suggest that convex costs can influence both a firm’s decision to hedge and the extent of the firm’s hedging. Finally, I introduce a new interpretation of empirical relations between potential hedging gains and the extent of hedging.

Keywords: Convex costs, deadweight costs, financial distress, costly external financing, tax convexity, nonfinancial firms, hedgeable risk, nonhedgeable risk, hedging.

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1. Introduction

Financial theory suggests that corporate hedging can increase shareholder value in the presence of capital market imperfections, including direct and indirect costs of financial distress, costly external financing, and taxes (Aretz & Bartram, 2009). The theoretical basis for this claim originates from an early attempt to resolve the apparent conflict between Miller and Modigliani's (1958) implicit hedging irrelevance proposition and the large number of firms that operate costly hedging programs. Smith and Stulz (1985) hypothesized that hedging can be a rational response to nonlinear risk exposure caused by taxes, financial distress, or investment distortions. Consequently, a firm facing a convex relation between tax liabilities and net profits might reduce the value of its expected taxes by hedging in order to reduce the volatility of its net profits. Brown and Toft’s (2002) model of firm hedging embodies this same idea in terms of exponential deadweight costs, which are "consistent with a firm that experiences high costs when profits are low and low costs when profits are large" (p. 1290). These authors also argue that "indirect bankruptcy costs at t = 1 (affecting revenues at time t > 1) could impact the hedging decision in a way that is well approximated by an exponentially declining cost function. Another example is a firm confronting costly access to external capital markets (Froot & Stein, 1993) where external financing costs increase exponentially in the amount of funds raised” (p. 1291).

More than two decades later, Mackay and Moeller (2007) contend that the empirical support for the various applications of Smith and Stulz's claim that nonlinearities justify hedging is "limited and mixed" (p.1379). As a result, the motives and value of corporate hedging are still in doubt, and positive and normative theory is underdeveloped (Mackay and Moeller (2007), pp. 1379-80). While these problems within the literature could be attributed to challenges and limitations associated with the empirical testing of these theories (Aretz &
flaws in existing theories on corporate hedging may at least in part account for the limited empirical consensus.

I find that convex costs influence firm hedging practices differently than what is currently believed by distinguishing two hypotheses often confused in the literature. First, convex costs increase the probability that a firm chooses to hedge. The more convex the costs are, the higher is this probability (H1). Second, firms with hedging programs will hedge more of their risk exposure as the degree of convexity increases (H2).\(^1\) I argue that while H1 is a viable hypothesis, this is not the case for H2. Contrary to the current consensus, in incomplete markets, value-maximizing firms that stand to gain the most from hedging may hedge less than otherwise identical firms with less to gain from hedging. This is the paradox of hedging. This finding applies to various deadweight costs that could motivate hedging by growing large in low-profit events. Hence, I refute the current consensus among financial scholars that, in theory, the extent of a nonfinancial firm’s hedging increases as its tax exposure becomes more convex (Aretz & Bartram, 2009; Graham & Rogers, 2002; Haushalter, 2000; Tufano, 1996).\(^2\)

The analysis is framed using Brown and Toft’s (2002) model, in which the extent of hedging may be defined as the number of linear hedging instruments or, equivalently, as the sensitivity of the perfect exotic hedge at the expected price under a risk-neutral measure. I first present new insights into the relation between the extent of hedging and cost function convexity using primarily well-known results. Next, I incorporate convex taxes seamlessly into Brown and Toft’s model to analyze how value-maximizing firms respond to small changes in a parameter that relates marginal tax rates to (pre-tax) net profits. The distributions

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\(^1\) Most early studies, including Mian (1996) and Nance et al. (1993), fall in the first category, while Graham and Rogers' (2002) study (except for the robustness procedures) and Tufano's (1996) research falls into the second category. However, this categorization should not hide the fact that financial scholars generally do not demonstrate any awareness that H1 and H2 are two distinct hypotheses. Thus, some fail to realize that the choice of a continuous measure for corporate hedging such as Tufano's "delta percentage," as opposed to a binary measure, has implications for the relation between hedging and cost function convexity.

\(^2\) The origin of this consensus could possibly be traced to the two possible interpretations of Smith and Stulz’s (1985) claim that “the tax reducing benefits of hedging increase if the function that yields after-tax income (tax liability) becomes more concave (convex). Thus, if...[there is an] increase [in] the convexity of the tax function, then such a tax will induce the firm to hedge more” (p. 395).
of net profits remain unaffected; changes in the tax parameter only affect after-tax profit distributions. While this superficially appears confined to tax function convexity, this analysis is relevant for any firm that experiences high costs when profits are low and negligible or zero costs when profits are large. The source of these costs could just as well be financial distress or costly external financing; thus, I essentially replace Brown and Toft’s exponential cost function with another convex cost function. Framed in an incomplete market setting, quantity risk implies that firms cannot hedge net profits directly but must condition hedge portfolio payoffs on the realized price. Under these conditions, value-maximizing firms facing costs that are more convex may hedge less than otherwise identical firms facing less convex costs.


One useful feature of Brown and Toft’s (2002) model that is unrecognized by its authors is that the optimal number of forward contracts converges to the minimum variance solution \( a^{MV} = -\mu Q - (\mu_p - s_i) \rho \sigma Q / \sigma_p \) as the curvature parameter \( c_2 \) approaches zero. For the exponential deadweight cost specification \( c_2 e^{-c_2 NP} \), the optimal number of contracts is \( a^* = a^{MV} + (1 - \rho^2) (\mu_p - s_i) \sigma Q^2 c_2 \). Thus, a higher \( c_2 \) means less hedging for a firm expecting a positive contribution margin, except for in some extreme cases characterized by strongly negative price-quantity correlation and predominantly unhedgeable risk. If a higher \( c_2 \) implies higher convexity, then higher convexity usually implies less hedging.

Assume that the distribution of net profits is exogenously given, and define a deadweight cost (DWC) function as some mapping from net profits \((NP)\) onto deadweight
costs. Now, there are at least two possible definitions of a "more convex function." First, DWC function A is more convex than DWC function B if and only if the expected deadweight costs given DWC function A are greater than or equal to the expected deadweight costs given DWC function B (definition 1). Second, DWC function A is more convex than DWC function B if and only if the expected hedging gain given DWC function A is greater than or equal to the expected hedging gain given DWC function B (definition 2).

Note that Brown and Toft’s (2002) exponential deadweight cost function does not satisfy definition 1. For a given $c_1$, a higher $c_2$ implies larger deadweight costs for negative profits and lower deadweight costs for positive profits. This tradeoff initially lowers expected deadweight costs, as $c_2$ increases for most parameterizations; see the upper right part of Figure 1. Thus, given two otherwise identical firms, the firm with the highest curvature parameter could be the firm with the highest pre-hedge market value. It may also face the highest potential hedging gain, as the exponential deadweight cost function seems to satisfy definition 2 for economically-interesting parameters. This is the case for the parameterizations $c_1 = 0.1, c_2 \in \{1, 2, \ldots, 10\}, \rho \in \{-0.5, 0, 0.5\}, \sigma_\rho = \sigma_\phi = 0.2, s_1 = 0.25, s_2 = 0.4$, which include Brown and Toft’s (2002) choice of parameters; this is also the case for the different parameter choices used in Section 3 with $c_1 = 0.1$.^4

Value-maximizing firms recognize that a variance-minimizing strategy does not minimize expected deadweight costs under a risk-neutral measure. Faced with exponential deadweight costs, firms usually decrease the extent of hedging linearly with the variance of the unhedgeable noise, $\sigma_\phi^2$, relative to the variance-minimizing strategy. The higher the contribution margin and the lower the absolute value of the price-quantity correlation (i.e., larger potential hedging noise), the higher is this downward hedge adjustment. In general,

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^4 In all the studied cases, expected deadweight cost savings increase monotonically with $c_2$. This is usually, but not always, the case for the relative increase in firm value; it is true for Brown and Toft’s parameterization.
firms respond to increased potential hedging gains resulting from a higher curvature parameter by reducing the extent of hedging.

Figure 1. The relation between the curvature parameter $c_2$ and the exposures faced by firms for Brown and Toft’s parameterization when $c_1 = 0.1$, $\sigma_P = \sigma_Q = 0.2$, $\mu_P = \mu_Q = 1$, $s_1 = 0.25$, and $s_2 = 0.4$. Upper left: The relation between economic profits and net profits for alternative parameter value $c_2$. Upper right: Expected deadweight costs assuming no hedging for various price-quantity assumptions. Lower left: The dollar reduction in expected deadweight costs relative to a no-hedge strategy resulting from forward sales in the amount defined by $a^*$. Lower right: The relative increase in firm value vis-a-vis a no-hedge strategy resulting from forward sales in the amount defined by $a^*$.

3. Convex tax exposure and the extent of hedging

A firm faces linear tax exposure when the value of the tax benefits arising from an arbitrary dollar loss equals the absolute value of the tax liability generated by a dollar income of the same magnitude. Even with unrestricted tax loss carryforwards and carrybacks, few firms are expected to face linear tax exposure, as interest is usually foregone due to the delay in future compensation. However, a convex tax exposure can be seen as the sum of a linear tax exposure and the value lost because the tax authorities do not fully accept shared responsibility should profits turn negative. These costs may be termed deadweight tax costs.
Using this two-step specification of marginal tax rates, I first incorporate linear tax functions into Brown and Toft's (2002) model. Note that all of their analytical solutions apply in this case. Next, I incorporate a deadweight tax cost function and set the parameters $c_1$ and $c_2$ of the deadweight cost function arbitrarily low. I then numerically calculate how the demand for hedging varies with deadweight tax costs using $M^{MV}$ as an initial value.5

3.1 A parsimonious representation of convex tax exposures

If the marginal tax rates can be specified to satisfy the following two requirements, then it will be possible to analyze how firms respond to tax function convexity. First, the cases of both linear tax exposure and no tax loss carryforward or tax loss carryback (i.e., zero marginal tax rates for negative profits) should be included as special cases. Second, a wide range of varying degrees of tax function convexity between these two extremes should be controlled by a single parameter. Both requirements are satisfied by the following marginal tax rate function given an appropriate choice of parameter $\theta$:

$$\frac{t_{CT}}{1 + \theta(e^{-\theta NP} - 1)}$$

(1)

For $\theta = 0.001$, this function approximates the statutory rate, $t_{CT}$, for positive net profits, $NP$, and attaches different values to the tax benefits associated with negative profits depending on the parameter $\beta$. Given an exogenous distribution of net profits and the function $CT$ that maps net profits onto taxes, higher $\beta$s are consistent with higher tax function convexity.6

**Proposition:** Assume that the statistical distribution of (pre-tax) net profits is exogenously given and that the marginal tax rates can be represented by (1). Then, $\beta_i$ greater than or equal

5 The assumption that the representative firm adheres to linear hedging instruments, which keeps the dimensionality of the optimization problem minimal for the numerical procedures, may be justified as follows. First, the dominance of linear hedging strategies is well documented (Gay et al. (2002), Huang et al. (2007), Benson and Oliver, (2004) and Bodnar and Gebhardt (1999)). Second, to the extent that this restriction introduces distortions, they are expected to apply only to firms facing a dominant nonhedgable source of risk, i.e., firms facing high-quantity risk relative to price risk. Third, Brown and Toft’s “exotic hedge” features the same sensitivity (i.e., slope) at $p = \mu_p$ as the optimal forward hedge (pp. 1294-1295). Although we cannot be certain this is the case for the perfect price hedge with the deadweight tax cost function used here, the significance of linear instruments is nevertheless indisputable.

6 A higher beta will also be associated with higher potential hedging gains; i.e., definition 2 is satisfied.
to $\beta_B$ implies that tax function $A$, $CT(NP;\beta_A) = \int_0^{NP} CT'(s,\beta_A)ds$, is more convex than tax function $B$, $CT(NP;\beta_B) = \int_0^{NP} CT'(s,\beta_B)ds$, in the sense that the expected tax liability given tax function $A$ is greater than or equal to the expected tax liability given tax function $B$.

**Proof.** Assume $NP \geq 0$. Then, $CT'(NP;\beta_A) \geq CT'(NP;\beta_B) \forall NP \Rightarrow E[CT(NP;\beta_A) | NP \geq 0] \geq E[CT(NP;\beta_B) | NP \geq 0]$. Now assume $NP < 0$. Then, $CT'(NP;\beta_A) \leq CT'(NP;\beta_B) \forall NP \Rightarrow E[CT(NP;\beta_A) | NP < 0] \geq E[CT(NP;\beta_B) | NP < 0]$. Since both expectations are negative with negative profits, the unconditional expectations must satisfy the inequality $E[CT(NP;\beta_A)] \geq E[CT(NP;\beta_B)]$.

Although $\beta$ cannot universally characterize tax function convexity except in the limiting cases $\beta \to 0$ (i.e., the linear tax function) and $\beta \to \infty$ (i.e., zero marginal tax rates for all negative profits), the parameter relates to tax function convexity in the context of a given firm's risk exposure. Thus, identifying how different firms respond to varying $\beta$ amounts to identifying how the extent of hedging varies with tax function convexity for the given firm.

Approximating $\lim_{\beta \to 0} CT(NP;\beta)$ by $CT(NP;0.00001)$ yields the deadweight tax cost function $DWTC(NP;\beta)$ implicit in Figure 2.

$$DWTC(NP;\beta) = \frac{\int_{CT}^{NP}}{\beta(1-\theta)} \ln[\theta - e^{\beta NP}(\theta - 1)] - \frac{\int_{CT}^{NP}}{0.00001(1-\theta)} \ln[\theta - e^{0.00001 NP}(\theta - 1)] (2)$$
Figure 2. The relation between pre- and after-tax profits for various tax function convexity assumptions $\beta$ for $t_{ct} = 0.28$ and $\theta = 0.001$. Deadweight tax cost is given by the vertical distance between the general function $CT(NP; \beta)$ and the limiting function $\lim_{\beta \to 0} CT(NP; \beta)$. These risk exposures resemble those faced by firms in the Brown and Toft (2002) model. In the former case, concavity is governed by $\beta$ in the sense of both definition 1 and definition 2, while in the latter case, it is governed by $c_2$ as in definition 2.

3.2 Convex costs and the extent of hedging

A value-maximizing firm facing negligible deadweight costs maximizes the risk-neutral expected value of after-tax profits conditional on linear tax exposure minus expected deadweight tax costs:

$$\max_{a \in S, c_1, c_2 \to 0} E^Q[\Pi(a)] \approx \max_{a \in S} \int_p \int_q \{NP(p, q; a)(1 - t_{ct}) - DWTC\{NP(p, q; a, \beta)\}\} f(p, q)dqdp$$

However, since no hedging strategy can change expected net profits under the no-arbitrage assumption, a value-maximizing firm effectively minimizes the expected value of deadweight tax costs.
Figure 3. The influence of tax function convexity, which is represented by the relative increase in firm value corresponding to $\beta = \{0.00001, 1, 5, 10, 25, 50, 75, 150, 250, 500\}$, on $a^*$. The markers furthest to the left approximate the limiting hedging strategy, $a^{MV}$, which minimizes the variance of taxable income. Every marker represents a numerical solution of equation (3) conditional on negligible deadweight costs ($c_1, c_2 \rightarrow 0$). Replacing the relative increase in firm value with the absolute or relative decrease in expected tax liabilities yields almost identical relations.

Figure 3 illustrates that value-maximizing firms initially respond to a gradual introduction of tax function convexity by downscaling hedging relative to $a^{MV}$, thereby incrementally increasing the volatility of taxable income. Since any response to more convex tax exposure must incrementally reduce expected deadweight tax costs, firms attempt to reduce the probability of lower-tail outcomes by reallocating negative taxable income facing some or full government tax relief to events in which less or no (i.e., zero marginal tax rate) tax relief is implicitly offered by the government. Firms will incrementally increase the volatility of taxable income to the point at which the incremental value of reduced deadweight tax cost in lower-tail outcomes is matched by the incremental value of increased deadweight tax cost for moderately negative profit realizations. Some firms facing high exposure to quantity risk (i.e., cases 1 and 2) eventually start trading off the probability of highly negative
outcomes for reductions in the probability of moderately negative taxable incomes by incrementally scaling up the size of their hedge portfolios. For these firms, the value of the incremental deadweight tax costs associated with moderately negative profit outcomes is more significant than the incremental value of lower-tail profit outcomes. However, firms with hedging programs usually respond to tax function convexity by scaling down their hedge portfolios, contrary to the standard assumption in the finance literature.

4. Empirical implications

My research shows that one version of the tax hypothesis is false (Aretz & Bartram, 2009; Smith & Stulz, 1985). While it may be true that more convex tax schedules can increase the probability that a firm chooses to hedge, it is not the case that, in theory, a firm will engage in higher levels of hedging under a more convex tax schedule. In contrast, firms with hedging programs that face unhedgeable risk typically respond to increased tax function convexity by reducing the extent of their hedging. More general lessons can be learned from this study, since costs that are high in low-profit events and low or negligible in high-profit events could arise for a variety of reasons. That is, convex tax schedules are just one example of convex costs. Thus, the key question is how such convex costs generally influence the extent to which firms hedge when the decision to operate a hedging program has already been made.

No universal relation between cost function convexity and the extent of hedging applies to firms that face convex costs. How a firm responds to higher cost function convexity depends on the underlying exposure faced by the firm, including the type of convex cost function. Therefore, unless firms with similar risk exposure can be identified, empirical tests

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7 Firms facing unhedgeable risk with small or moderate volatility relative to the hedgeable source of risk make relatively small adjustments to their hedges. In a complete market, i.e., when the volatility of the former source of risk tends to zero, there is no adjustment. However, firms facing high exposure to nonhedgeable risk respond to higher tax function convexity by making sizeable changes to their hedge ratios (i.e., cases 1 and 2). The adjustments also appear smaller for firms that expect low contribution margins. This is evident from cases 3 and 4.
cannot and should not be expected to provide crystal-clear evidence regarding hedging, even if such tests were to include high-quality data unavailable in most previous studies. A statistically insignificant influence of tax function convexity on the extent of hedging may result when firms that face high hedging gains, relatively low unhedgeable risk, low contribution margins, and a non-exponential type of deadweight cost function are considered. Such an empirical finding neither invalidates H1 nor implies that firms consider rebalancing their hedge portfolios in response to higher potential hedging gains unimportant. Rather, it indicates that such firms cannot reduce expected deadweight costs by significantly changing the extent of their hedging. Thus, a negligible or non-response might be consistent with value maximization in incomplete markets.

The finding that an identical increase in potential hedging gains could trigger divergent responses from two otherwise identical firms that face different patterns of deadweight costs is important. It means that empirical observations of the relation between the extent of hedging and some proxy for deadweight costs are more likely to uncover the true nature of deadweight costs than test for whether cost function convexity influences the extent of hedging. Given risk exposure characteristics and an exponential deadweight cost function, the extent of hedging is expected to decrease monotonically with dollar hedging gains for economically-interesting parameters. As per the alternative convex cost function used in Section 3, the relation could be negative, zero, or even positive in some intervals. Thus, the incremental hedging gain is simply not sufficient to pin down the expected change in the extent of hedging, even when controlling for other risk exposure characteristics.

The potential nonlinear influence of convex cost functions on the extent of hedging suggests that it may be difficult to specify testable hypotheses that apply to cross-sections of

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8 Although the relation between the extent of hedging and $c_2$ is linear, the relation between $c_2$ and potential hedging gains is not; see the lower-left section of Figure 1.

In another analysis, $s_1$ and $s_2$ were set to .75 and .1, respectively, for case 2. A qualitatively similar result was found; the responses were smaller than for case 1.
firms with hedging programs. The fact that most proxies for deadweight costs, e.g., credit risk spreads, cannot be observed on a pre-hedge basis for firms with hedging programs is another potentially insurmountable obstacle. Worse still, even if pre-hedge proxies can be parsed out from post-hedge observations using other data sources, defining the operational categories of deadweight cost patterns would probably require data that are unlikely to be available. However, some testable hypotheses may be derived from relations that seem to apply to the different patterns of deadweight costs.

First, there should be less hedge adjustment in response to more convex exposures when the volatility of the unhedgeable risk factor is low. One testable implication is that the average difference between actual hedging and the extent of hedging under the minimum-variance hedging strategy is larger for firms facing relatively high unhedgeable risk than for firms facing predominantly hedgeable risk. Second, it also seems that firms expecting low contribution margins will gravitate to the minimum-variance solution. In the same vein as above, the average deviation from $a^{MV}$ should be significantly smaller for these firms than for firms expecting high contribution margins. Finally, the closer that the correlation between hedgeable and unhedgeable risk is to zero, the more noisy the hedge payoffs will be. Consequently, actual hedge ratios are expected on average to deviate more from $a^{MV}$ as the absolute value of the correlation coefficient decreases.

Another possibility would be to identify two types of firms, with the first type subject to rapidly increasing costs in low-profit events and the other type facing convex costs increasing more slowly as profits turn increasingly negative, as was the case with the deadweight tax costs in Section 3. The extent of hedging is expected to be lowest in the first group of firms. Reversing the argument, empirical relations can in theory identify the type of convex costs faced by groups of similar firms. If the extent of hedging decreases monotonically with some proxy for potential hedging gains, this may indicate deadweight
costs growing at an increasing rate as profits turn increasingly negative. If this relation is less pronounced, convex costs may not grow so fast in low-profit events.

In conclusion, my research confirms Aretz and Bartram’s (2009) claim that the empirical testing of positive theories of corporate risk management is challenging. It thus is a reminder that researchers should be careful when interpreting empirical results regarding the relationship between the extent of hedging and cost function convexity. In particular, a finding that suggests a lack of relationship should not be allowed to feed back on H1. It may be true that the traditional theories of hedging are unable to explain the determinants of corporate derivatives usage (Bartram, Brown, & Fehle, 2009), but inconsistencies in existing theories and the way in which empirical findings have been interpreted could, at the very least, be partly to blame.

References


