Incentives for Debt and the Impact on Economic Stability

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Abstract

The global financial crisis came as a surprise for many economists, governments and populations around the world. Most economists relied on models of how the economy functions which did not have the capability to produce or predict breakdowns and crises, because long term stability was taken for granted and short term deviations were expected to largely resolve themselves by the normal functioning of free markets. However, other economists have developed theories and models which predict that excessive economic fluctuations with the potential for breakdowns can occur. This thesis describes and contrasts two of these theories and their associated models; the Financial Instability Hypothesis developed by Hyman Minsky, and later modeled by Steve Keen, and the Leverage Cycle developed by John Geanakoplos. What becomes evident from their analyses is that conditions in financial markets can have a significant impact on the real economy. On this background it is argued that individuals, firms and nations under certain conditions are intrinsically motivated to incur a level of debt, which is sustainable under the tranquil economic conditions that prevailed when it was undertaken, but leaves them vulnerable and sensitive to unexpected negative shocks.
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1 Introduction

When the financial crisis hit the global economy in 2007, it was to the great surprise of the general public and the vast majority of economists. Most media outlets portrayed it as though there were no theories which could describe or predict the calamity which unfolded. There are indeed such theories and models, however they have been given little attention beyond narrow academic circles until now. This thesis explores two of these theories and their differences to the mainstream models then compares their policy recommendations with the actual policy responses to the financial crisis in some selected countries.

It has been understood since Sismondi (1829) that the economy moves between periods of increasing and decreasing growth and it has been the central theme of economic debates throughout the centuries why this occurs and how to best deal with it. One of the main purposes of macroeconomics is to understand the forces and connections which give rise to economic fluctuations and propose policies which reduce them and the problems which arise as a consequence. The existence and costs of fluctuations in economic activity are well known and uncontroversial. A boom (extraordinary growth) period being problematic might seem paradoxical, as we generally think that growth is good. Even so, if the growth rate is unsustainable, there is likely to be a costly recession later. In booms there will be investment in production facilities and training of personnel for which there is insufficient demand in later periods. The dismantling and retraining which follows involves losses for the individuals and the society as a whole. Booms also involve price inflation as both inputs and outputs for goods and services are in high demand, pushing their prices up. Recessions, on the other hand, are characterized by low investment and high unemployment which suppresses demand and lowers activity further.

Although the existence of economic fluctuations and/or cycles are uncontroversial, there is a vivid debate about why they occur and in turn how to deal with them. The mainstream explanation for fluctuations comes from Keynesian economics, where the short run discrepancies between supply and demand leads to periods of insufficient supply (incentivizing expansion) and excess supply (motivating contraction of production). The problem reinforces itself particularly in a contraction because when firms lay off workers to reduce production, the unemployed workers can no longer purchase as many goods, causing a further fall in demand. The standard policy response to this problem is to curtail investments in expansionary periods by having the central bank increase interest rates and suppress demand by running government fiscal surpluses. Conversely, in a depression the central bank will encourage investment by lowering the interest rate and the government will run deficits to increase demand.
The reason why economists were surprised was that they believed that the modern tools of counter-cyclical policies for managing fluctuations were so effective that the possibility of bubbles and crashes negligible. In the first years of the new millennium, several leading economists argued that the preceding twenty years of unusually low macroeconomic volatility was evidence that the prevailing system of fiscal and monetary intervention used to stabilise economy had successfully brought fluctuations under control (Stock & Watson, 2002; Lucas, 2003; Bernanke, 2004).

1.1 The role of credit

The role of credit as a contributor to fluctuations in the economy has been largely discounted as unimportant because it was viewed that debt only redistributes wealth from one agent to another without having significant macroeconomic effects (Bernanke 1995). The current crises in the U.S. and Europe has forced economists to reconsider the role of credit because their roots are in precisely inflated debt levels; in the U.S. through an incredible inflation in mortgage loans and derivative trading in them - in Europe there was government debt with no collateral. In light of the recent crises, a better understanding of the role of debt in the evolution of the economy is needed.

In classic macroeconomic theory money is neutral. Agents' priorities do not change based on the number of coins they have, only how many goods they can buy with those coins. If there is twice the number of coins, but the same amount of goods and services which can be bought in the economy, we should expect a doubling of prices. Most economists agree that this must be the case in the long term, but there are different schools of thought on whether money affects the economy in the short run, which are discussed in detail in the literature review section.

In modern economic systems, there are two channels through which the supply of money (M2) increases: The first is when it is simply manufactured by the central bank and the other is through lending in the fractional reserve banking system. There are differences in the two conceptually, but for the end user of funds, they are indistinguishable. The most important difference is that the money created by the central bank is fixed by a single authority, while the money generated through the banking system varies with market conditions. When banks lend out money, they are not actually creating additional money, but are in fact lending out someone else's savings. The creation of money by banks is a temporary change to the money supply. Lending increases the money supply at the time of issue and repayment reduces it at a future time. After the loan is fully repaid, the money supply is back where it started, et ceteri paribus. This is rare the case, however, as new loans are continuously issued and old ones repaid. The net change to the money supply from the banking sector is thus the aggregated difference between new loans and repayment of old ones.
Along with moving money through time, the borrower is also moving their consumption or investment through time. An increase at the time the funds is borrowed entails a reduction in a future as some income must be set aside for interest and principal. As mentioned, when credit expands from loans, this is not new money, but making use of other agents' savings. Thus, there is not more money chasing goods. What really occurs is that the borrower increases the demand for some goods, because he is no longer bound by the credit constraint which prevented him from spending the way he desired. There is not an equivalent reduction in demand from the lender. Hence, the equilibrium price should increase, all else being equal.

The correlation between credit expansion/contraction and the business cycle is argued and empirically tested by Borio and Löwe (2004). They found that an excessive expansion in credit which is accompanied by an asset price bubble is a good indicator leading fluctuations in output. Further, they found that traditional monetary policy which targets inflation with a short time horizon will not optimally counteract the resulting economic cycles.

1.2 The financial instability hypothesis and the leverage cycle

This thesis explores two of the theories which are starting to gain momentum in academic circles to capture the possibility of economic cycles endogenously caused or amplified, in part, by interaction between the real economy and credit markets. In particular it will focus on a model by Steve Keen (1995), based on Minsky's financial instability hypothesis (1977) and on the leverage cycle developed by John Geanakoplos (2003; 2010).

A striking aspect of these two models is that they explain the phenomenon of economic cycles, with an astounding number of similarities in argumentation and dynamics. Both argue for heterogeneity among investors, causing asset prices to be bid upward by optimistic investors, at high leverage. It is argued that credit conditions are loosened over time when markets exhibit low volatility or trending good returns, allowing optimistic investors to take on additional leverage. High leverage eventually makes positions so volatile to unexpected shocks that when a negative turn comes, the results are catastrophic for some agents leading to fire sales which drives prices further down and creates a landslide of sales and prices.

That the two models have a number of similarities would not be particularly surprising if it was not for the fact that they are developed independently, from completely different theoretical backgrounds and employing very dissimilar sets of modeling tools. On the one hand, the financial instability hypothesis is argued as a heterodox interpretation of Keynes works (1936; 1937)
focusing on elements which the neo-classical synthesis did not. Keen takes his cue from Minsky, who argued that it was primarily the unwillingness or inability of investors to consider profitability over longer time periods which leads to "euphoric" periods because agents extrapolate current rising trends in asset prices, but also the irreducible uncertainty of future asset prices which investors face and an endogenous nature of money with real impacts on economic activity. The model traces its roots to Goodwin's growth cycle (1956), using the dynamic modeling employed by Lotka (1925) and Volterra (1926) in order to describe predator and prey systems. On the other hand, the leverage cycle argues a very similar sequence of developments through the introduction of collateral as a key factor in the determination of equilibrium in credit contracts. When markets are less volatile, lenders will allow more leverage, as the probability of default is lower. With higher leverage, the most optimistic buyers can hold a larger proportion of the assets and will bid up the prices of assets. The tools used to model the leverage cycle are taken from game theory and finance with simultaneous clearing of all markets.

1.3 Positive feedback

A main point where the financial instability hypothesis differs from traditional economics is in that it allows at least parts of the economy to be governed by positive feedback as opposed to the diminishing or dampening dynamics of equilibrium based models. Positive feedback means that a system will serve to reinforce the effect of an initial shock. For example, we may argue that investment leads to high employment, higher incomes and higher demand, which in turn motivates further investment to meet the increased demand.

In finance, the prevailing explanation of prices for the last six decades argues that markets are efficient in determining prices. Excessively optimistic or pessimistic investors will be punished by paying too much or selling too cheap, respectively. Markets will reward the rational investors eventually when assets deliver their actual fundamental value. If prices diverge from fundamental value, then there are arbitrage opportunities which rational investors will eliminate by trading in the asset -- buying underpriced assets and selling overpriced ones thus returning prices to their fundamental value through the forces of supply and demand (Fama, 1970). On the other hand, if the economy is subject to positive feedback, the equilibrium is not stable; if certain variables are displaced sufficiently in the short run, the economy will not necessarily return to equilibrium, but might well propel itself further away. Delong et. al. (1990) pointed out, that speculators who chase a trend for short-term gains, buying when markets are rising and selling when they are falling, will reinforce the initial trend. This is in reality a pyramid or Ponzi type logic which cannot persist.
indefinitely. Still, in the short run it is rational for an investor to "jump on the bandwagon" while the trend persists.

For example, when housing prices are rising, some investors (and lenders!) might extrapolate the trend and include further increases in their estimated ability to repay the loan. If they do, they will conclude that there is a near zero probability of incurring losses even in the case of a default on interest and principle payments as the value of the house will be sufficient to settle the debt and consequently lenders will allow borrowing with a very small proportion of the borrowers own capital as collateral (the margin). Neither borrower nor lender have a problem with taking/extendng a loan at say 5% interest, if the price of the house is expected to increase by 10% per year, because even if the borrower cannot repay the loan in cash he can sell the house which is expected to be worth sufficient to settle the debt, including interest. The result of the rising housing prices is thus a further increase in demand and prices because more loans can be issued against the collateral, reinforcing the initial movement. This type of investment strategy is rational in the short-run (buying and selling on a upward trend), but obviously prices cannot keep rising at astronomical rates forever, so eventually someone will have to take the losses when prices eventually flatten out or start falling.

Other examples of positive feedback can be found in microeconomics. Brian Arthur (1989) explained that especially high-tech industries which require heavy investment in R&D and service industries which rely on reputation and experience are characterized by increasing returns which causes increased market share and vice versa. E.g. once the Windows operative systems attained a certain amount of popularity, it became the standard for subsequent products gained further market share and feeding back into more people using their system. Once the company departs from the equilibrium market share, rather than return to it, the advantage they gain reinforces their chances of conquering additional share.

In the setting of the financial instability hypothesis, a period of prudent investment strategies causes most investments to be successful and the leveraged ones to be rewarded the most. This causes some overconfidence which leads to increasingly leveraged investment levels which in turn inflates asset prices, validating such leveraged investments and encouraging more of it. The positive feedback between increasing leverage and prices produces a boom phase in the economic cycle and also amplifies the speed and severity of the subsequent collapse. When prices are falling fewer agents are willing to invest, reducing demand and prices even further.

The leverage cycle does not describe positive feedback explicitly, because it is focused on the price
determination in equilibrium rather than describing the economy in a continuous dynamic model. According to the theory, the financial dimension causes the economy to endogenously produce excessively leveraged positions, which will default in the case of a negative shock. Thus a shock will be amplified by the amount of leverage incurred by the agents, but it will not feed back into the economy repeatedly in this model. However, positive feedback is an immediate consequence of the pricing mechanism it describes, because when wealth is redistributed toward the agents who correctly predict future price movements, it will increase their purchasing power and cause their valuation to weigh heavier in subsequent periods. E.g. when prices move upward (from unexpected good news), the optimistic investors are rewarded and allow them to purchase more assets, bidding up prices further, validating optimistic expectations.

The next section outlines the literature relevant to the thesis, traces the evolution of the central ideas toward their current expressions and places the topic within a bigger picture of economic debates. Section three and four outlines the theory and modeling of the financial instability hypothesis (FIH) and the leverage cycle (LC), respectively. Section five compares and contrasts the two theories. Section six provides some evidence for why these models are now relevant and discusses whether actual policy is consistent with the policy recommendations provided by the FIH and the LC.

2 Literature review

2.1 Business cycles and schools of thought in macroeconomics

"We find [that the state of trade is] subject to various conditions which are periodically returning: it revolves apparently in an established cycle. First we find it in a state of quiescence, — next improvement, — growing confidence, — prosperity, — excitement, — overtrading, — convulsion, — pressure, — stagnation, — distress, — ending again in quiescence." (Lord Overstone 1857, p. 44)

The phenomenon of economic fluctuations and cycles has been observed for presumably as long as there has been trade, but the concept has only attracted scientific scrutiny in the last two centuries. Jean Sismondi (1819) is credited with the first theories explaining economic crises which appeared periodically. He focused on the disconnection between production and demand as the main forces causing periods of excess supply (crises) and others of excess demand (booms). Clement Juglar (1862) showed empirically that production and prices rose during expansion phases and fell in
recessions. Schumpeter (1929) identified the four stages of a cycle as expansion, crisis, recession and recovery. He then imagined that the economy were subject to several overlapping cycles of different lengths, named after their discoverers; Kitchin, Juglar and Kontratieff (Zarnowitz, 1991).

While few contest the historically observed variations in economic output, which the National Bureau of Economic Research has identified as 33 business cycles (NBER 2012), there is a vivid debate about why they occur, their characteristics and what policies the society can undertake to minimize them and their associated costs. Because the problem of business cycles goes to the heart of our very understanding of how the economy operates on the macroeconomic level, there are as many theories of the business cycle as there are schools of economic thought and each prescribes a different set of policies to remedy them.

### 2.1.1 Classical

In the first half of the 19th century, mainstream economists subscribed to the classical view which draws on the works of Adam Smith (1776), Jean-Baptiste Say (1803), David Ricardo (1817) and Thomas Malthus (1798). This view explains economics through the decision making of the individual agent or firm as optimizers according to the conditions they find themselves in. Investors will allocate capital to where they believe they will receive the greatest return. This will cause assets to be priced fairly by the competitive market as mispriced assets will quickly be bought or sold by opportunistic investors. Fluctuations in the economy are thus not predictable cycles, but simply the adjustment agents make to capture better opportunities as they are discovered. There is then no need for the government to intervene, as competitive markets are already allocating resources efficiently.

In the second half of the 19th century, the classical view was modified to incorporate the insights of William Jevons, Carl Menger, Leon Walras and Alfred Marshall who distinguished between the cost of production and utility in consumption when determining the price and value of a product. This view entails that producers cannot always accurately predict what demand will be in the future, but will still adjust over time to capture changing demand.

In the early 20th century, the classical view lost popularity with the advent of the Keynesian revolution in economics. Nevertheless, following the stagflation of the 1970s, when Keynesian ideas were discarded by many economists, classical ideas were brought back and reworked by, amongst others, John Muth (1961) and Robert Lucas (1976) into what is now known as new classical economics. This school of thought emphasizes the rational expectations of individual
agents and flexible prices to ensure that all markets will clear (Barro, 1989). All agents act to maximize utility given the information available to them given. If at any time, the supply and the demand of goods are not aligned, wages and prices must adjust to restore equilibrium. A popular explanation of economic fluctuations presented in the new classical framework is the "real business cycle" (Kydland & Prescott 1982), which states that the fluctuations we see are in fact the rational responses of individuals and firms to continuous shocks in market conditions, technology and information.

In classical economics the disequilibrium caused by shocks are quickly absorbed. This is also the case for shifts in the supply of money. It is argued that prices will adjust swiftly to restore the real value of money (money supply divided by price level) to previous levels. The result is that money is neutral in this theory: unable to affect the output of the real economy.

2.1.2 Keynesian

Keynesian economics describe a different scenario underlying business cycles. Keynes emphasized that under certain conditions the market would not automatically restore equilibrium. Particularly, if the demand for money is in the low end of the scale where interest rates are not responsive to changes in output and further spending is lower than required for full employment of resources, then price and wage reductions will not restore supply-demand equilibrium (Heijdra & Van der Ploeg 2002, pp. 19-21). Following this line of reasoning, when markets operate with less than full employment, the government can stimulate the economy by demanding goods and services. Samuelson (1939) applied the Keynesian framework to produce oscillations as a result of an initial shock to output, and Goodwin (1967) showed how the interplay between wages and employment can produce repeated economic cycles without dampening.

In Keynesian economics, prices adjust slowly as opposed to classical economics where they adjust quickly. The consequence is that supply and demand shocks are not absorbed immediately, but have real effects on the economy in the short run. This is an important distinction because it means that it is possible for governments to enact policies which will have a real effect on the economy in the short run. For example, in a recession the government can increase spending to raise demand to normal levels or the central bank might increase the money supply to lower interest rates and encourage spending/investments.

2.1.3 Monetarism

Monetarism as developed, amongst others, by Milton Friedman (1963; 1983) argues that inflation is
solely a monetary phenomenon and fluctuations in the economy result when the change in output differs from the change in money in circulation. The main distinction is made between the nominal amount of money in circulation and the amount of goods said nominal amount can purchase. Intuitively, if we double the supply of money and not the supply of goods, then all the money can still buy the same amount of goods, so prices will have to double. However, the individual does not know at any given time what the exact amount of goods and money are, so price changes do not happen instantly. This entails that markets are not always efficient if there are shocks to either goods or the money supply. The goal of monetary authorities should then be to maintain stability and predictability in the purchasing power of money, primarily by keeping the supply of money relatively stable. He further asserted that fiscal policies are ineffective tools to deal with supply-demand disparities, as increasing government spending will simply "crowd-out" private spending and not have any net impact.

2.2 Positive feedback and financial acceleration

2.2.1 Debt deflation

A recurring theme in economic theory is the idea that shocks are absorbed and their effects dampened over time as the economy adjusts in a rational manner. Certainly this is true for goods, as no producer can survive for long if there is no demand for his goods. Even so, the great depression in the 1930s prompted economists to explain how the economy could crash to such an extent. During this period Irving Fisher had lost a great deal of his personal wealth and professional reputation by believing and publicly stating in the early stages of the depression that the market would soon shrug off the slump and recover to produce new heights in asset prices (Teach, 2007). As a response to the experience from the great depression, Fisher (1933) produced a different theory, explaining the interaction between debt and economic activity in an original way. In his debt-deflation theory he likens the majority of products, prices or sectors in an economy to a ship, which under normal circumstances will tend to stabilize itself after a lesser shock, but if it receives too great a blow to the side, will capsize and not return to "equilibrium". Further he explained that there are so many and continuous shocks of various sizes that we can never really say that any significant portion of the economy is ever in equilibrium: "It is as absurd to assume that, for any long period of time, the variables in economic organization ... will 'stay put' in economic equilibrium, as to assume that the Atlantic Ocean can ever be without a wave" (Fisher 1933, p. 339). Fisher then considered which of the mechanism of the economy which had in the past or at least the potential to capsize the economy. In his view the traditional explanation of over-production or under-consumption could cause minor disturbances, but alone they were not strong enough to
cause significant problems. On the other hand, the positive feedback between debt and price-levels (the purchasing power of the monetary unit) could significantly disturb the economy. He proposed the following sequence of events:

1. Suppose there is a state of the world in which many agents have taken on excessive debt to finance their investments. Suppose further that in this state some bad news which increases the volatility of future cash flows occur. As a response, creditors become nervous about the ability to repay the debts and will seek to reduce the leverage of investments in order to make their loans safer and will also increase the interest rates on new loans. In addition, some borrowers will sell to close the speculative positions.

2. Fewer new loans being issued than old ones repaid causes a contraction of the money supply in a fractional banking system and a fall in the velocity of circulation of money.

3. Nervousness to buy and less money chasing the same amount of goods, causes a deflation in prices. Falling prices, means a devaluation of firms' assets and they will record losses. It also increases the burden of the remaining debt (as money is more valuable when it is in shorter supply), making other agents relatively more indebted and prompting or forcing more liquidations.

4. Lower profitability, causes a reduction in investment, employment and output. It also causes more pessimistic outlooks and hoarding.

5. Careful agents further slow the velocity of money reinforcing the initial downward trend.

Once the spiral is started, the effects are difficult to trace in sequence, but the main point is that the effect is reinforcing itself. In modern language, there is positive feedback. Fisher's debt-deflation theory fell in the shadow of John Maynard Keynes ideas which the mainstream economists embraced at the time. The debt-deflation theory was predominantly ignored by the mainstream, but the work of Hyman Minsky, which is explored in some detail later in this thesis, builds crucially on it.

2.2.2 Financial acceleration

The understanding that economic downturns are amplified by a worsening of credit markets have received little attention by the mainstream economic literature, however there are some exceptions beyond Fisher (1933) and Minsky (1977; 1990): In Eckstein & Sinai (1986), the authors analyze the
eight cycles observed between the end of the second world war and 1986. They find that although the duration, amplitude and causes of fluctuations vary, there are recognizable stages, which have been observed by previous authors. Further they observe that along with fluctuations in the real economy there were equivalent movements in financial markets with every one of the eight cycles in the post-war period. The stages thus identified have a real part and a financial one which are intimately connected and inseparable: (1) recovery/credit expansion, (2) boom, (3) peak/tighter credit, (4), recession/contraction of credit. The effect is less obvious in the expansionary phases, but the authors observe that every crisis entails a "credit crunch" (Eckstein & Sinai, 1986). The authors point out that the financial aspect accelerates the trend, particularly in the credit crunch stage:

At the crunch stage of the business cycle, the degree of financial risk that is present constrains sectoral spending, limits the availability of credit, can result in bankruptcies, default, and failures, and intensifies the downturn in the economy.

(p.59)

Bernanke, Gertler & Gilchrist (1996) also pointed out that economic downturns can be amplified by worsening credit conditions. However, it is argued in a different manner: The authors point out that there is a cost to obtaining external finance, attributable to information asymmetry and agency problems. This cost is inversely proportional to the agents' net worth. A shock, either positive or negative, affects the smaller units to a larger extent, but nevertheless amplifies a macroeconomic trend.

Kiyotaki & Moore (1997) modeled the positive feedback mechanism between the real and financial markets. A temporary negative shock to output to credit constrained firms will cause them to cut back future investments. The lower revenue in each successive period means investment must also be reduced in all subsequent periods. For markets to clear prices must fall, causing a further reduction in firms net worth. In particular for the highly leveraged firms, the effect is dramatic. The initial shock is thus subject to an "intertemporal multiplier process" (Kiyotaki & Moore 1997, p.213).

The central point of difference between positive feedback and financial acceleration is that the former creates endogenous economic cycles, whereas the latter only amplifies an existing shock and does not affect the probability of a subsequent movement in the same direction.
3 Financial instability hypothesis

3.1 The theory

The financial instability hypothesis (FIH) was developed by Hyman Minsky (1977) as a response to the failure of the standard neo-classical synthesis to explain how and why financial crises and large economic fluctuations occurred. In the neo-classical models the economy constantly gravitates towards equilibrium, although it is understood that it will never actually rest in equilibrium because of frequent shocks and the inability of firms to adjust completely in the short run. It is understood that these dynamics will give rise to fluctuations over time, but there are no dynamics which indicate that the economy can in fact be inherently unstable and tend toward breakdown rather than equilibrium in the long run.

Minsky's financial instability hypothesis takes its cue from the debt-deflation theory developed by Irving Fisher (1933). Fisher found that in the central characteristic of the great depression was that the economy had come from a period of debt fueled asset price inflation. Subsequently, an initially small number of nervous borrowers or lenders decided to liquidate their assets to lower their leverage. More sellers led to falling asset price and meant more investors were forced to sell to meet margin calls. This initiated a spiral of debt deflation and falling asset prices. Fisher emphasized that this spiral was caused by endogenous forces, contrary to the more popular view that the economy is inherently stable.

Whereas Fisher focused on describing the anatomy of debt-deflation that takes place after the upper turning point is reached, the financial instability hypothesis extends the theory with a corresponding explanation for the way in which the economy will produce a state of high indebtedness, completing the circle. The theory explains that when investors are optimistic, increasing leverage amplifies returns. Such optimism combined with a liberal access to credit from lenders who are likewise optimistic fuels a debt financed asset price bubble.

3.1.1 The economics

For the main proposition of his hypothesis, Minsky (1977) points to Keynes response (1937) to professor Jacob Viner (1936) at the university of Chicago who posed that "[Keynes] General theory really did not make a sharp break with traditional economics and that Keynes achieved novel results because velocity was allowed to vary and prices and wages were assumed to be rigid." Minsky argues that Keynes response is the key to understand the break between the old and the new and that
it indicates a theory which gives explanations of the anomalies of crisis and large fluctuations we observe which standard theory cannot provide.

The new theory attempts to explain the determination of two sets of prices. Capital and financial assets is one set and output and wages is the other. In order to understand how these prices emerge, Minsky focuses on the decisions to be made by investors or investment banks. These agents must make decisions about purchases and sales of assets whose future cash flows are uncertain. The price is also affected by the opportunity cost of holding it. Hence the price of an asset will also change when alternative investments become more or less profitable. For example, a bond whose cash flow is fixed in the contract terms will still fluctuate in price depending on the interest rate on new bonds and the aggregate demand for holding bonds instead of other assets, like stocks.

In Minsky's view, agents face irreducible uncertainty when making investment decisions, meaning the probability of future events cannot be accurately estimated such that risks can be hedged. The consequence is that the price of assets whose majority of cash flows are some time ahead, reflect the fleeting opinions of buyers and sellers about future profitability and opportunities. The opinions of individuals and firms are subject to sudden, significant changes when unexpected news arrive. They might also be influenced by widespread overconfidence in boom periods or fear and panic in crashes. Investors also face uncertainty with respect to the cost of raising money by borrowing. The market conditions will determine the debt structures that lenders and investors will agree upon, in terms of interest rates and collateral. Thus, both investment levels and lending practices depend crucially on the agents' views which, as argued above, can change rapidly.

A central argument to the theory is that the opinions of agents are affected by their experiences in the near past. For example, when there has been an economic crisis in the near past, investors and lenders are cautious to act and more prudent in their estimations of future earnings. When a long time passes without any crisis occurring, in which there is steady growth, those same agents will be more optimistic. Because agents' estimations in turn determine price, this dynamic gives rise to cycles.

3.1.2 An illustration of the cycle

1. To illustrate the FIH, Minsky starts with a cyclical economy which has in the near, but not immediate, past been in a slump or recession and is now recovering. In this economy the agents will be cautious with their money and only invest in the most promising ventures. This is reinforced by banks which are also restrictive in their lending, so that debt financing
is limited to only a few ventures with virtual guarantee of repayment. In modern terms we would describe this by a strong risk aversion and agents requiring a high premium to take on risk.

2. As time progresses in this atmosphere, the economy will grow slowly, but almost all of the projects undertaken will be successful, because the riskier ones were simply not financed into existence. The successful projects which were leveraged, returned amplified profits to its investors (Fama, 1970).

3. The reaction to this is that investors and bankers reconsider their risk aversion. There is no landslide change, but as time goes by with a great number of successes in the market, the agents start to consider less prudent strategies. Investors look for new projects to invest in or revalue existing ones, finding that with less discounting from risk, valuations are now higher. They then turn to bankers for financing, who have similarly determined that because they had a low default rate, they could have increased their income by lending out more funds. As a result, lenders become more willing to allow for debt structures which involve higher leverage. The last decade would certainly be a good example of investors eager to purchase assets and banks lowering their lending standards.

4. The increasing will and ability to invest translates into rising asset prices, which again serves to validate the inflated valuations and for some to argue for even higher leverage. This self-reinforcing cycle of increasing leverage and rising asset prices pushes leverage and prices ever upwards. On top of this, it is a characteristic of growth or boom periods that there is financial innovation which serves to increase further the amount of available financing (Minsky, 1977). Over time, the steadily growing economy characterized by prudent agents turns into a booming economy where agents borrow funds to finance investments at inflated prices.

5. High leverage leaves agents with low liquidity and high sensitivity to interest rate changes. As the market soars towards its peak, eventually some investments will not live up to expectations and/or fail completely. The most optimistic investors will be hit first because they calculated further price increases into their valuations. Without asset price appreciation, they are unable to meet their obligations and the banks which financed their investments will reevaluate the risk premium on all their loans and increase interest rates to cover their losses.

6. Once the rising trend is broken, the increasing number of sellers in the market causes the price to drop. At a lower price, more leveraged agents are forced to sell to meet margin calls
on their loans or do so in fear of further price crash. Repayment of loans also means a contraction of the money supply, which suppresses prices and makes the debt burden heavier for the remaining borrowers, motivating them to reduce their leverage. The result is a downward spiral of leverage and prices, which is exactly the reverse of the previous rising price and increasing leverage.

Minsky argues that the crisis can be aborted by either the central bank flooding the market with funds and/or the government running large deficits to maintain income and profits. Even so, with the central bank solution there are known dangers of stagflation and with the public deficit solution there is a danger of preserving the asset price bubble or creating a new one.

### 3.2 The model

Australian economist Steve Keen has produced a model of Minsky's financial instability hypothesis. The model, first published in 1995, builds upon the framework produced by Goodwin (1967). The sections below outline the history and a brief discussion on the methodology.

#### 3.2.1 Dynamic modeling

Key features in dynamic modeling which were later used by Goodwin (1967) were originally introduced by biologists Alfred Lotka (1925) and Vito Volterra (1926) who simultaneously and independently produced what is today known as the predator-prey system. The model describes the populations of two species in an ecological system where, as the name suggests, one species are predators which prey on the other. If there are plenty of prey, the predator population will grow, but if the predator population is too great compared to the population of prey, there is not enough food for them to thrive and the predator population will shrink. Conversely if there are few predators, the prey population will be able to grow and if there are many predators, the prey population will shrink. More formally, let the populations of prey and predator be represented by $x$ and $y$, respectively. Further $a$ is the growth rate of prey, $b$ is the rate at which prey is hunted, $c$ is the mortality rate of predators without prey to eat and $d$ is the growth rate of predators per prey eaten. Then we have the following model for the development of the two populations over time:

\[
\frac{dx}{dt} = ax - bxy \tag{1}
\]

\[
\frac{dy}{dt} = -cy + dxy \tag{2}
\]

\[a, b, c, d, x, y \geq 0 \tag{3}\]
Equation (1) describes the rate of change for the prey population. We see that in the absence of any predators \((y=0)\), the population grows exponentially at rate \(a\), but when predators are present, the growth rate is reduced proportional to the size of the predator and prey population. Equation (2) describes the rate of change for the predator population. If there are no prey \((x=0)\) then the predator population will diminish off at rate \(c\), but if there are prey, then the population growth is greater than \(-c\).

An important finding of the Lotka-Volterra type system is that the evolution over time of the two populations will for certain values of the parameters \(a, b, c\) and \(d\), as well as initial conditions \(x(0)\), \(y(0)\) give either a stable population for both species, known as a fixed point or equilibrium or a forever repeating cycle of growth and decline in both populations, known as a limit cycle. In particular, if we choose initial conditions such that:

\[
\begin{align*}
x(0) &= \frac{c}{d} = x^*, \quad y(0) = \frac{a}{b} = y^* \\
\frac{dx}{dt} &= \frac{a}{d} - \frac{ac}{bd} = 0 \\
\frac{dy}{dt} &= -\frac{a}{b} - \frac{ac}{bd} = 0
\end{align*}
\]

Illustration 3.1: Goodwin growth cycle dynamics

The figure illustrates that to the left of the vertical dashed line \((x < x^* = c/d)\), \(y\) is increasing, as
there is enough prey for predators to eat to increase their population. To the right of it ($x > x^*$), $y$ is decreasing, because there is insufficient prey for them to eat. Similarly, below the horizontal dashed line ($y < y^* = a/b$), $x$ is increasing, as the prey population is reproducing quicker than the predators slay/eat them. Above the horizontal line ($y > y^*$), $x$ is decreasing because high predator population eats prey quicker than they reproduce.

The point $(x^*, y^*)$ is the fixed point of the system, where no change in populations occur over time. If we start observing the system when it is sufficiently close to this point, the system will repeatedly trace the same closed orbit around this point. If a one-time disturbance throws the balance off the original orbit, the system will move to another orbit and stay upon it. Every point that is sufficiently close to the fixed point is a part of another orbit which the system can trace. A system which has this characteristic is called conservative.

### 3.2.2 Goodwin's growth cycle

In 1967 Richard M. Goodwin applied the Lotka-Volterra's system to explain the cyclical nature of the economy by modeling wages and employment as taking the roles of predator and prey, respectively. In this model, there are two agents, capitalists and laborers who will share the output of production according to the relative strength of their bargaining position. Specifically, laborers will demand a wage increase when employment is very high and conversely accept a wage reduction when employment is low. When wages are low, the profitability for capitalists is high and so they will hire more labor, increasing the employment rate. With increased employment, wages again increase. Once wages reach the point where it is no longer profitable to increase investment, the trend reverses. Because wages are sluggish to respond to increased and decreased employment, the model does not rest in equilibrium, but rather traces a limit circle as portrayed in the previous section.

#### 3.2.2.1 Model assumptions

A1. There is a single sector in the economy which produces a single good. The good is used for paying labor and the residual is left with the capitalist to invest in the subsequent period.

A2. After paying labor, all profits are invested.

A3. The ratio of output to capital invested is constant, but the required labor to achieve a given production level (and equivalently the labor to capital ratio) falls over time.

A4. Regardless of what labor is paid, all wages are consumed and do not affect incentives for
investment through secondary channels such as increased demand for consumer goods.

A5. There is a constant rate of growth in the population and the productivity of labor.

A6. All capital is of the type which cannot be disinvested when profitability is low. In order to reduce capital, they have to "wait" for depreciation to lower it.

### 3.2.2.2 Model definitions

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>K(t)</td>
<td>Capital</td>
<td>Stock</td>
</tr>
<tr>
<td>σ</td>
<td>Capital per unit of output</td>
<td>Parameter</td>
</tr>
<tr>
<td>q(t) = K(t)/σ</td>
<td>Output</td>
<td>Flow</td>
</tr>
<tr>
<td>L(t)</td>
<td>Employed labor</td>
<td>Stock</td>
</tr>
<tr>
<td>a(t) = q(t)/L(t)</td>
<td>Labor productivity</td>
<td>Variable, endogenous</td>
</tr>
<tr>
<td>N(t)</td>
<td>Population</td>
<td>Stock</td>
</tr>
<tr>
<td>v(t) = L(t)/N(t)</td>
<td>Employment rate</td>
<td>Ratio</td>
</tr>
<tr>
<td>w(t)</td>
<td>wages</td>
<td>Variable, endogenous</td>
</tr>
<tr>
<td>u(t) = w(t)*L(t)/q(t)</td>
<td>Labor's share of output</td>
<td>Ratio</td>
</tr>
<tr>
<td>Π(t) = q(t) - w(t)*L(t)</td>
<td>Profit</td>
<td>Flow</td>
</tr>
<tr>
<td>α</td>
<td>Labor productivity growth rate</td>
<td>Parameter</td>
</tr>
<tr>
<td>β</td>
<td>Population growth rate</td>
<td>Parameter</td>
</tr>
<tr>
<td>γ</td>
<td>Autonomous wage change</td>
<td>Parameter</td>
</tr>
</tbody>
</table>

Note: Time subscripts will be suppressed for simplicity in the following formulae and the following shorthand notation for rate of change over time will be employed:
\[ K = \frac{dK}{dt}, \quad L = \frac{dL}{dt}, \text{ and so on} \]

### 3.2.2.3 Dynamics

\[ I = \dot{K} = \Pi = q - wL \quad (4) \]
\[ \dot{q} = \frac{1}{\sigma} \dot{K} = \frac{1}{\sigma} (q - wL) \quad (5) \]
\[ \frac{\dot{q}}{q} = \frac{1 - u}{\sigma} \quad (6) \]

\[ L = \frac{d}{dt} \left( \frac{q}{a} \right) = \frac{1}{a^2} (q \dot{a} - a \dot{q}) = \left(\frac{1 - u}{\sigma}\right) L \quad (7) \]
\[ \frac{\dot{L}}{L} = \left(\frac{1 - u}{\sigma}\right) - \alpha \]

\[ \dot{w} = (\gamma + \rho \lambda)w \quad (8) \]
\[ \dot{a} = \alpha e^{\alpha t} \quad (9) \]
\[ \dot{N} = \beta e^{\beta t} \quad (10) \]

Equation (4) incorporates assumption 2: The net profit, after labor is paid, is reinvested. Equation (5) states that the change in output scales directly with the change in capital. Equation (6) highlights the percentage change in output in terms of the labor share of output \( u \). Equation (7) expresses the change in employed labor. Equation (8) says that the rate of change for wages are a linear function of the employment rate \( \lambda \). Equation (9) describes the growth of labor productivity. Equation (10) the growth rate of the population.

From these dynamics, we can derive the change in wages share of output and employment:

\[ \dot{u} = \frac{d}{dt} \left( \frac{w}{a} \right) = \frac{1}{a^2} (\dot{w} a - a \dot{w}) = \frac{w}{a} (\rho \lambda - \gamma - \alpha) = -(\alpha + \gamma) u + \rho u \lambda \]

\[ \dot{v} = \frac{d}{dt} \left( \frac{L}{N} \right) = \frac{1}{N^2} (L \dot{N} - \dot{L} N) = \frac{L}{N} \left(\frac{1 - u}{\sigma} - \alpha - \beta\right) = \left(\frac{1}{\sigma} - \alpha - \beta\right) v - \frac{1}{\sigma} u \lambda \]

Equations (11) and (12) are recognizable as the standard Lotka-Volterra system with workers share of output \( u \) and the employment rate \( v \) representing predator and prey, respectively. When the employment rate (prey) is high, the workers share (predator) is increasing. When the workers share low, employment is rising. The fixed point can be found at:

\[ \frac{\dot{v}}{v} = \left(\frac{1}{\sigma} - \alpha - \beta\right) - \frac{1}{\sigma} u = 0 \]

\[ u^* = \frac{\left(\frac{1}{\sigma} - \alpha - \beta\right)}{\frac{1}{\sigma}} = 1 - (\alpha + \beta) \sigma \]

\[ \]
\[
\begin{align*}
\frac{\dot{u}}{u} &= -\left(\alpha + y\right) + \rho v = 0 \\
v^* &= \frac{\alpha + y}{\rho}
\end{align*}
\]  

(14)

As with the predator-prey system, for reasonable parameter values and initial conditions sufficiently close to the fixed point, this system is conservative and the trajectory [u(t),v(t)] constitutes a limit cycle around the fixed point (u*,v*).

### 3.2.2.4 Cycles

Given the population size (N) and the productivity (a) at a point in time, there exists a unique level of capital stock which will employ the equilibrium level of the population:

\[
v^* = \frac{1}{N} L^* = \frac{1}{Na} q^* = \frac{1}{Nav} K^*
\]  

(15)

It is important to note that the cycles occurs (unless the economy finds itself in the fixed point), because the capital stock surpasses the "sustainable" amount K* through the investments of capitalists in the time it takes for wages to travel from its lower turning point to the equilibrium level. If capitalists had stopped investing at that point and waited for wages to adjust, eventually wages would reach the point where the incentive to invest further had disappeared (a small investment would be required every period, as K* is unique for every point in time). Because investors overshoot the sustainable level, there are cycles in the model. The amount of time this overshooting occurs, and consequently the amplitude of the cycles, depend on the speed of adjustment of wages. If the speed of adjustment is very quick, then the cycles are equivalently short and there is less over- and undershooting of investment. In the extreme, where wages adjust immediately, there are no cycles. This is because if the economy was out of equilibrium, the incentive to invest would be exactly proportional to the distance between prevailing wages and the equilibrium wage. On the other hand, if the speed of adjustment is slow, capitalists will keep investing beyond the equilibrium level, because wages are still low enough that profitability is high, at that point. The longer time it takes wages to reach the equilibrium level, the longer and more severe the cycle becomes.

A word of warning is warranted at this point, concerning the model described above and the one below. Depending on the parameters chosen, it may be the case that the capitalists will want to hire an amount of labor which is greater than the population, causing the employment rate to exceed one. There is not built in a constraint which prevents this from happening in the model, although it is possible to specify the wage adjustment and/or investment function such that it will not happen.
3.2.3 Modeling Minsky

Steve Keen (1995) extended the Goodwin model in order to model the financial instability hypothesis. Keen modified the investment function to increase exponentially with profitability and added a banking sector which will allow investors to finance the excess of desired investment over previous profits by incurring debt. Lastly a government sector is added to intervene and attempt to dampen the cycles. In a forthcoming paper, he takes the model one step further and incorporates endogenous money in the same model. While the two-dimensional Goodwin model with only wages and employment exhibits stable cyclical dynamics, introduction of a third (banking sector), fourth (government) and fifth dimension (money) opens for the possibility of instability and economic breakdown.

There is a slight change of notation and some new definitions:

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>Capital</td>
<td>Stock</td>
</tr>
<tr>
<td>I = dK/dt</td>
<td>Net investment, including</td>
<td>Flow</td>
</tr>
<tr>
<td>v</td>
<td>Capital required per unit of output</td>
<td>Constant</td>
</tr>
<tr>
<td>Y = K/v</td>
<td>Output, determined by capital and the accelerator</td>
<td>Flow</td>
</tr>
<tr>
<td>a(t) = a(0) * exp(αt)</td>
<td>Labor productivity</td>
<td>Variable, endogenous</td>
</tr>
<tr>
<td>L = Y/a</td>
<td>Labor required determined by productivity</td>
<td>Stock</td>
</tr>
<tr>
<td>N(t) = N(0) * exp(βt)</td>
<td>Population</td>
<td>Stock</td>
</tr>
<tr>
<td>λ = L / N</td>
<td>The employment rate</td>
<td>Ratio</td>
</tr>
<tr>
<td>w</td>
<td>The wage rate</td>
<td>Variable, endogenous</td>
</tr>
<tr>
<td>r</td>
<td>The interest rate on debt</td>
<td>Variable, endogenous</td>
</tr>
<tr>
<td>D</td>
<td>The stock of debt held by capitalists</td>
<td>Stock</td>
</tr>
</tbody>
</table>
3.2.3.1 Assumptions

A1. There is a single sector in the economy which produces a single good. The good is used for paying labor and interest on debt. The residual is left with the capitalist to invest in the subsequent period.

A2. Investment is based on the rate of profitability per unit of capital.

A3. If there are insufficient profits to finance desired investment, the capitalist can borrow the balance from the banking sector.

A4. The flow of output to laborers is consumed, regardless of the wage rate and the flow to bankers leave the system as well.

A5. Bankers can supply any amount of funds the capitalist requires.

A6. The ratio of output to capital invested is constant, but the required labor to achieve a given production level (and equivalently the labor to capital ratio) falls over time.
A7. Regardless of what labor is paid, all wages are consumed and does not affect incentives for investment (there is no demand curve or prices).

A8. There is a constant rate of growth in the population and the productivity of labor.

A9. All capital is of the type which cannot be disinvested when profitability is low. In order to reduce capital, they have to "wait" for depreciation to lower it.

### 3.2.3.2 Dynamics

We begin with the main driver of the system which is the investment decision made by capitalists. The main difference between Keen's model and the Goodwin model is that Keen allowed capitalists to invest any amount they see fit and assumed that they will invest more when the rate of profitability is higher. If the desired investment exceeds profits from the previous period, the firm will borrow the necessary funds from the banking sector and pay the prevailing interest rate in any given period. Net investment is defined as an exponential function $k[.]$ of the rate of profitability less depreciation of capital:

$$I = \dot{K} = \frac{dK}{dt} = k\left[\frac{\Pi}{K}\right]Y - \gamma K = \left(\frac{1}{v} k\left[\frac{\pi}{v}\right] - \gamma\right)K$$

(16)

The function $k[.]$ will be left in its general form. The important characteristic is that it is increasing with the rate of profitability of capital.

Further, the change in output, labor and employment follows directly from the change in capital and their definitions:

$$\dot{Y} = \frac{d}{dt}\left(\frac{K}{v}\right) = \frac{1}{v} \dot{K} = \left(\frac{\dot{K}}{K}\right)Y$$

(17)

$$\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K}$$

(18)

Because output is always proportional to capital, the percentage change in output matches exactly the percentage change in capital, as we can see from equation (17) and (18). In other words the elasticity is one.

$$\dot{a} = \alpha a$$

(19)

$$\dot{L} = \frac{d}{dt}\left(\frac{Y}{a}\right) = \frac{1}{v} \left(\frac{d}{dt} \frac{K}{a}\right) = \frac{1}{v a^2} \left(\dot{K} a - \dot{a} K\right) = \left(\frac{\dot{K}}{K} - \alpha\right) L$$

(20)

$$\frac{\dot{L}}{L} = \frac{\dot{K}}{K} - \alpha$$

(21)
Labor is also determined immediately from capital, however as the productivity of workers increase with time at rate $\alpha$, a lower labor to capital ratio is demanded by capitalists. Equations (20) and (21) state that if there is no change in capital, labor employed will decline at a rate of $\alpha$. On the other hand, if we image an instantaneous change in capital (no time passes), labor would match it exactly (in percent).

\[ \dot{\lambda} = \frac{d}{dt} \left( \frac{L}{N} \right) = \frac{1}{N} (\dot{L} N - \dot{N} L) = \frac{1}{N} (\dot{L} - \beta L) = \left( \frac{\dot{K}}{K} - \alpha - \beta \right) \lambda. \]  

\[ \frac{\dot{\lambda}}{\lambda} = \frac{\dot{K}}{K} - \alpha - \beta. \]  

The employment rate $\lambda$ displays a similar dynamic to labor. It replicates the percentage change in capital and is reduced by the increased productivity. The difference is that the rate of employment will also decline in time because the population grows at rate $\beta$. Equation (24) makes it clear that if there is no change in capital (and thus labor employed), the employment rate declines at a constant rate as a result of increased productivity and a population growth.

K, Y, L and $\lambda$ are all linked together explicitly, which means that if we know one of them at a given time, we can determine the others. It is in this sense that all four of these variables only account for one dimension of the system and we only need one to represent this dimension. Goodwin (1956) chose to use the employment rate $\lambda$ to represent the variables K, Y and L, because it is $\lambda$ which determines the change in the wage rate, but any other will do along with the appropriate transformation.

Wages are a function of the employment rate $\lambda$:

\[ \frac{dw}{dt} = w[\lambda] * w. \]  

As with the capital function, the wage rate function $w[\lambda]$ will be left in this general form. The important aspect is that, as opposed to the previously discussed variables (K, Y, L and $\lambda$), wages do not scale directly with other variables in a given period. In order to determine wages, we must know the entire history of another variable: the employment rate. The important characteristic is that there will be some level of employment $\lambda = \lambda'$ where wages are in equilibrium, when $\lambda > \lambda'$ wages will increase at some rate proportional to the distance $\lambda - \lambda' = \Delta$ and conversely decline if employment is below the equilibrium level.
Because the employment rate falls naturally, et ceteri paribus, as the population and productivity increases over time, profitability of capital increases. There is equilibrium in workers' and capitalists' share of income if workers demand a wage increases exactly proportional to their increased productivity.

### 3.2.3.3 Adding the banking sector

Up until this point, the model matches the model outlined by Goodwin very closely. The difference from his model occurs because where Goodwin chose arbitrarily to model the scenario where capitalists will invest all their residual profit after paying labor, Keen allows the investor to choose the desired investment level without this constraint. The difference between the desired investment level and the profits from the previous period will be borrowed from- or saved in a bank. Thus we introduce the stock of debt $D$:

\[
\frac{dD}{dt} = I + \gamma K - \Pi = \dot{K} + \gamma K - (Y - wL - rD) \tag{26}
\]

Equation (26) shows that the change in the stock of debt for any given period is the difference between gross investment (net investment plus depreciation) and the residual profits from the previous period, after labor is paid at rate $w$ and the debt stock is financed at interest rate $r$. The stock of debt is not described explicitly in terms of other variables. It is determined implicitly by summing the required financing from all preceding periods.

In the Keen (1995) paper, this equation is denoted "dD/dt = I - \Pi + rD" (p.616), which is obviously not correct. There are two problems: First of all, by definition profits are net of interest payments $rD$, thus they should not be added to debt. If we substitute for profits in the equation, it becomes clear that the way it stands counts the interest payment twice. Secondly, net investment includes replacement of depreciated capital, which must be financed from profits or debt. If we do not add the depreciation to the equation, then the replacement of depreciated capital is free. It is also curious that the capitalist uses earnings before depreciation as a decision rule for investing. On the other hand, given a fixed depreciation rate, it is a simple matter to tweak the investment function to account for it. In any event, the erroneous equation is replaced with the correct one in the version of the model presented in "Commerce, complexity and evolution" (Keen, 1996, p. 96). It is still unclear whether the right or wrong form went into the simulations in the 1995 paper.

The change in debt as a share of output is:
\[ \frac{d}{dt}(\frac{D}{Y}) = \frac{1}{Y^2}(DY - YD) = \frac{(\dot{K} + \gamma K - \Pi)}{Y} - \frac{\dot{Y}}{Y}d \]  

(27)

\[ d = (v - d)(\frac{\dot{K} + \gamma K}{K}) + d \gamma - \pi \]  

(28)

Equation (27) derives the change in the debt to output ratio, separating the debt effect from the output effect. The first term accounts for the effect due to new debt: When it is positive, new debt is incurred; when it is negative, the capitalist pays down debt (saves). The second term accounts for the change in output: If the first term is zero, gross investments equals exactly profits, and the percentage decrease in the debt to output ratio, matches exactly the percentage increase in output (equivalent to the percentage change in capital). This is the case because the debt stock \((D)\) is unchanged, but output \((Y)\) has increased. Equation (28) highlights the change in terms of the gross investment, depreciation and profits. The \((v - d)\) term denotes the equity financed part of capital/output. We see that if gross investment is zero, then the sign on the change to the debt level depends on whether profit share of output is greater than the depreciation rate. It brings out the fact that when profitability and consequently investments are very low, the debt ratio is still increasing at the rate of depreciation. This is important for the stability of the system, which is discussed in more detail below and in the next section.

The interest rate is assumed to be a linearly increasing function of the debt to output level:

\[ r = \varsigma + \varphi \frac{D}{Y} \]  

(29)

This can be motivated by arguing that the probability of default, which is the risk exposure of the bank, increases with the leverage of the investor. In the model, output is always proportional to capital, thus an interest rate proportional to the debt-to-output entails that it is also proportional to the more common measure, debt-to-equity. The simplification is made here that the interest rate on all the debt changes when new debt is issued or old debt retired, which means that the interest rate is floating (variable or adjustable), rather than fixed. This is not an unrealistic assumption, but it is problematic because the capitalist is not taking the effect on old investments into consideration when incurring new debt, which is discussed in more detail in the evaluation section below.

Given this relationship, we can describe explicitly the change in the interest rate as:

\[ \dot{r} = \varphi \dot{d} = \varphi((v - d)(\frac{\dot{K} + \gamma K}{K}) + (d \gamma - \pi)) \]  

(30)

Again we see the connection to the stability of the system: When investment is low, the debt to
output is determined by the difference between depreciation and profits. If the economy enters a state where the depreciation of capital is greater than profits, \( d \) will increase and consequently the interest rate too. The result is that the interest payment burden becomes greater in subsequent periods, bringing net profits down even lower. An extended discussion on stability follows in the next section.

Finally, we can determine the share of the output which is spent on interest payments:

\[
\frac{db}{dt} = \frac{d}{dt} \left( \frac{rD}{Y} \right) = r d + \dot{a} r = \varphi \dot{a} d + \dot{d} r = (r + d \varphi) \dot{d}
\]

(31)

\[
b = (r + d \varphi) \left( (v - d) \frac{\dot{K}}{K} + (d \gamma - \pi) \right)
\]

(32)

Because profit is determined as the residual of output after wages and interest on debt is paid and profitability per unit of capital in turn determines investment levels, the wages and interest burden drives the fluctuations of the system in a cyclical manner:

1. When wages and interest rates are low, profitability is high. High profitability leads to high investments. However, as opposed to the Goodwin model, where capitalists are constrained in the maximum amount they can invest, in the Keen (1995) model the expansion of capital can happen at a much more rapid pace by incurring debt.

2. The high employment rate and increased debt stock increases the wage rate and interest rates, respectively, at the expense profitability. As discussed in the Goodwin model, the capitalists overshoot a sustainable level. In addition to the slow adjustment of wages, interest rates are artificially kept low by the inflated economy. This aspect is discussed in more detail below.

3. Lower profitability induces a reduction in investments. Eventually profitability falls to the point where the capitalist stops incurring additional debt, but the wage bill keeps increasing for a time period which depends on the speed of adjustment (like discussed in the Goodwin model).

4. At this point, low investment combined with depreciation causes the economy to contract. However, while falling wages in isolation would translate into higher profitability, the debt burden becomes greater when output is shrinking. If the debt is not being repaid at a faster rate than the economy is shrinking, the interest rate will increase making the debt burden ever heavier. The crucial question for the stability of the system is whether wages falls (increasing profitability) quicker than interest payments are rising (decreasing profitability).
3.2.3.4 Stability

In terms of stability, the question is whether there are sufficient profits in the contracting stages of the cycle (when profitability and investment is low) to service and/or repay the debt which was incurred in the expansion. In order for profitability to return to levels where investments are lucerative, wages and/or interest rates must come back down.

Wages are an automatic stabilizer for the system, because although it adjusts slowly, it increases when the capital stock is large and falls when the capital stock is small. When investment is high, increasing wages reduce the incentive to invest and when investment is low, falling wages make investment more attractive.

Interest rates, on the other hand, have a destabilizing element to it. Interest rates should rise when investment is high and debt is incurred, but the effect is diminished because the economy is also growing. When profitability and investment is low, the upward pressure on interest rates is amplified because output is falling (from depreciation). This is a result of the $D/Y$ term in determining the interest rate. When the economy is expanding, although debt is incurred, the increased output $Y$ suppresses the overall effect on $d$. Conversely, when the economy is contracting, investment is falling, $Y$ is decreasing and thus $D/Y$ is getting larger.

It is ironic that the debt by itself does not cause instability. When the debt was incurred it was rational for the capitalist to do so, because at that time the profitability was sufficient to service the debt. The possible instability exists because even after the capitalists stop incurring additional debt, wages keep rising (as discussed in the Goodwin model), such that profitability continues to decline, after it reaches the point where it could just barely sustain interest payments. If wages were to adjust immediately, the capitalists would not incur debt that would later become unsustainable.

The question of stability is thus answered by whether the debt burden increases quicker than the falling wages alleviates it, in the contraction stage of the cycle. This depends on the duration and severity of the cycles in profits, wages and interest rates, which really depends on the investment function and the speed of adjustment of wages. If the capitalists and bankers are prudent and cautious, the cycles will be moderate because they will not incur unsustainable amounts of debt in expansions. In contractions there will be more profits than interest payments, allowing the debt to be repaid. On the contrary, if capitalists and bankers are overconfident when profitability is good, they will incur excessive debts which they are unable to settle during contractions.

Further, there is no guarantee that every cycle will be of the same amplitude. The natural question to
ask is then: Will the cycles increase, decrease or remain stable in their amplitude? If the cycles are
categorized by ever increasing amplitude, then eventually the system will collapse once the cycles
become too large. Conversely, if every cycle is less erratic than the last, the economy will converge
to a steady state without cycles. Lastly, if cycles are stable in terms of magnitude, then the economy
will forever be in cycles.

3.2.3.5 Adding a government sector

Beyond saving an economy already in crises, the main purpose of public policy for Minsky was a
preemptive intervention to "establish and enforce a 'good financial society' in which the tendency by
business and bankers to engage in speculative finance is constrained" (Minsky 1977, p.16). If
speculation is constrained, the forces which push for asset price bubbles and crashes would be
severely diminished. Keen modeled such a government by two rules which would govern its
operations. First, by relating government spending to unemployment, so that it will uphold cash
flows when private production is low, or decrease:

\[
\frac{dG}{dt} = G = g[\lambda] \cdot Y
\]  (33)

Second, the government will increase the tax rate when profitability is high so as to temper
investors' euphoric expectations:

\[
\frac{dT}{dt} = T[\pi] \cdot Y
\]  (34)

Further there is not a requirement that the government balances the budget each period. Thus, the
model must be further expanded to allow the government to incur debt if they need to finance more
expenditure than tax receipts are able to.

\[
\frac{dD_g}{dt} = G - T + r \cdot D_g
\]  (35)

The model makes the simplification that government income and expenses will only affect
capitalists. That is, only capitalists pay taxes and all government spending benefit capitalists
directly. Formally the net profit can be defined as:

\[
\Pi_n = \Pi - (T - G) = Y - w \cdot L - r \cdot D_k - (T - G)
\]  (36)

Which means that the change in capitalist debt becomes:

\[
\dot{D}_k = I - \Pi_n = I - \Pi - (T - G)
\]  (37)
Introducing the government complicates the model significantly. For starters, it will expand the number of dimensions of the system from three to six. Further, the government does not directly relate income to expenses. Its income increases (decreases) when profitability is high (low) and expenses increases (decreases) when unemployment is low (high). Although the dynamics of the system entails that these forces will counteract one another to some degree over time, there is no budget restriction which ensures that the government will be able to service or pay down its debt in the long term. This means that although the immediate effect of introducing government is to temper excessive incentives for or against investment, there is no guarantee the debt incurred by the government will not grow beyond all bounds. At high debt levels, the interest rate also increases, which results in the payments becoming an even larger burden. Accumulation of high debt to output ratios has indeed been the case in many western economies over the past sixty years which has been characterized by strict monetary policy and high interest rates to keep inflation under control (Keen 1996, p.25).

3.3 Evaluation

Keen's model certainly captures some key elements of Minsky's financial instability hypothesis. For starters, the financial innovation has led to dynamics in capital markets which cannot ignore the real effects of the availability of money on economic activity. Investors' behaviors in this model are close to how Minsky described; after a downturn, investment and output will be low, but over time high unemployment will bring down wages and make it profitable to invest again. If this occurs investors will leverage up their capital to finance larger investments, not taking into account that profits might fall in the future and thus present problems servicing the debt they have incurred in their euphoria.

On the other hand, the model does not capture the heterogeneity of investors, which plays a central role in Minsky's theory. In the model, all investors behave uniformly and equivalently as if they represented a single agent. In Minsky's financial instability hypothesis, it is a central argument that investors span a wide range from prudent and conservative to Ponzi strategies. It is an important point to note that during boom times liberal lending standards will allow the most optimistic investors to significantly fuel the rising asset prices by leveraging their investments. By bidding up asset prices, this group of investors forces the remaining ones to either take on more leverage or sell their positions. The net result is that they will pull some portion of investors with them. This dynamic is simplified away in Keen's model.
The model also avoids explicitly incorporating the memory and expectations of agents into the pricing of assets. In Minsky's hypothesis, the driving force of the cycle is that both investors and bankers have a memory of the recent past which they extrapolate to future expectations about profitability, which in turn forms the basis for investment decisions and the risk premium demanded by lenders. Following a recession, Minsky imagined that capitalists and bankers are cautious and prudent in their investment and lending decisions. As the expansion progresses, they would then become more optimistic and discount the probability of negative outcomes to a larger extent. As a result, the overshooting in Minsky's theory occurs because rising asset prices yields superprofits to investors over a short time period, incentivizing investments which subsequently fulfills the expectation of further asset price increases. In Keen's adaptation of the Goodwin model, the cycle occurs as a result of real changes in profitability (the slowly adjusting wage rate), rather than the more intangible expectations of investors and bankers.

There are also some loose ends in the model. There is, for example, no explicit modeling of the supply of credits available for lending/borrowing. The only way in which lending is constrained, is through the increasing interest rates from maintaining higher debt to output levels. As workers do not save and capitalists' savings are negative debts, there is no closed system of all the flows within the system. Keen (1995, p. 614) argues that this is a simplification we can live with in order to focus on the dynamic forces which produces cyclical movements in the distribution of output between capitalists, bankers and workers. However, it is not clear how the dynamic behavior would change if such limitations were implemented on the supply of funds. In a currently unpublished paper, Keen (2011) argues that the supply of funds is an irrelevant question for bankers, quoting Alan Holmes paper from 1969 which states: “... In the real world banks extend credit, creating deposits in the process, and look for the reserves later” (p.73).

The open ends of the system described above causes the debt induced breakdowns which Keen's model produces (without government involvement) to be questionable. In the model, the breakdown occurs when profitability is low, such that investment stagnates, and the share of output of the banking sector is growing more rapidly than the wages expense is falling. When the distribution of output between the sectors is disproportionally shifted towards interest payments on the debt stock, profitability never resurfaces to the point where capitalists replace depreciating capital. Eventually, there is no capital or labor employed. Certainly, this would be the demise of the original capitalists who over-indebted themselves to the point where profits could not sustain interest payments, even at very low wages. Even so, it is unlikely that the economy would break down in this way, because the output/money spent on interest payments would not vanish or be used exclusively for
unproductive purposes. Unless bankers (or laborers) hoard money, even when their incomes become large, one should think that they would become capitalists by saving/investing some of their income or increase demand for goods and services, pushing for higher prices and profitability. In the model, the output which is distributed to workers and bankers flows out of the system, regardless of its size, which is highly questionable. It is possible for capitalists to ruin themselves, as a result of the destabilizing effect of debt and interest rates, but it is unlikely that this alone would cause an economy-wide breakdown.

Overall the model makes significant simplifications which abstract it from reality. Given that the model describes a bird's-eye view of the economy, we cannot expect it to capture every detail. This is a trade-off between accuracy and generality of the model.

3.4 Summary

The financial instability hypothesis argues convincingly that there are inherent forces in the existing economic system which generates cycles, which are necessarily stable; longer investment horizons, less speculative investments and stricter lending conditions/practices would go a long way to temper these cycles. The modeling by Keen (1995) captures some central elements of the hypothesis and does confirm that such an economy can produce instability, however it does leave room for further development to more accurately capture the full network of flows and feedbacks.

The question of policy is more straight forward, but also politically unpopular. Once a crisis occurs, however, there is no painless way out of it. There are two ways forward and both involve large concessions and restructuring costs. Manufacturing inflation to sustain incomes is essentially a wealth transfer from borrowers to creditors, because the real value of old debt decreases. This may cause rising interest rates and a loss of confidence as businesses become uncertain of future costs and profits and thus become hesitant to engage in economic activity. We also risk building a new asset price bubble, more severe than the previous one. On the other hand, if we employ the government to expand spending to sustain demand, there are still large restructuring costs as individual sectors of the economy must down-size, labor retrained, assets liquidated and reinvested. The budget deficit must be financed by increased taxes, but not necessarily in the same period.
4 The leverage cycle

4.1 The theory

Geanakoplos (1997; 2003; 2011) argues that the macroeconomic models used by government planners, economists and central banks are missing a crucial element to account for shifts in economic conditions. In particular, he points out that we cannot rely on the interest rate alone to gauge the conditions in financial markets which affect the real economy. While the interest rate certainly is one part of the puzzle, Geanakoplos argues that to fully describe equilibrium in contracts, we must also account for collateral and the terms of default. A shift in the perceived and/or acceptable risk for investments, may not result in a lower interest rate, but rather be expressed through lower collateral requirements on loans. When certain agents gain access to additional leverage (gearing), they will use their increased purchasing power to buy more assets, bidding up prices in the process. Hence, the interest rate may remain unchanged while financial conditions have significantly changed, causing a major impact on the economy. If governments equate a stable interest rate with tranquil economic conditions, they might misjudge the situation considerably. The rest of this section traces the chain reaction set in motion by changes in the risk perception and/or risk aversion of lenders to fluctuations economic activity.

4.1.1 Risk and collateral

A money loan involves two problems for the lender; the opportunity cost and the risk of default, that is, being unable to use the funds until they are repaid and the risk of permanently losing the funds. While the opportunity cost depends on external forces, which neither borrower nor lender can directly affect, the risk of default can be managed by the specifics of the loan contract. To be more specific, the question of who bears the losses in case of a default can be managed. This is the purpose of the collateral. The collateral is a subset of the borrower's assets, which the loan contract gives the lender the legal right to confiscate, if the borrower fails to repay the loan. In particular, the lender has the right to liquidate the asset and use the proceeds to repay the debt.

If the value of the collateral is always sufficient to repay the loan, the lender is not exposed to any risk of losses even if the borrower cannot repay him in the regular way. The problem is that the value of the collateral is not fixed over time in most cases. Businesses usually provide their assets as collateral -- individuals their homes. The value of a firm's assets are usually stable as long as the business is operating normally, but if the firm encounters financial distress, the value of their assets will generally fall. For individuals the most common example is a mortgage loan, where the house
itself is the collateral. Again, property values are stable or slowly rising "in normal times", but they are susceptible to sudden, steep declines which can quickly eradicate the value. If the value of the collateral falls below the outstanding principle on the loan, lenders have lost the "guarantee" of being repaid and become exposed to the risk of losses.

4.1.2 Margins and volatility

The critical question is thus: When the value of the collateral varies over time, how much risk is each of the two parties (borrower and lender) exposed to? From the lenders perspective, the answer depends on how quickly the value of the collateral can fall below the outstanding principal on the loan. If he can confiscate and liquidate the collateral before this occurs, then he will not incur any losses. There are three factors which contribute to this:

The first is the difference between the value of the collateral and the outstanding principal on the loan, which is called the margin (and is equivalent to the equity financed part of the investment). The margin provides a buffer for the lender, because any price decline in the value of the collateral will reduce the margin before causing losses to him. As a result, the larger the margin, the lower the risk the lender is exposed to. The second component is the future volatility of the value of the collateral. In particular, the lender is worried about the probability and size of a downward shift in the value of the collateral before he can liquidate it. The third component is the speed with which the collateral can be confiscated and liquidated. The longer this window of time is, the larger the potential decline in the value of the asset before the lender can intervene to prevent further losses. While the delay between a breach of contract and confiscation (beyond the practically possible minimum) can be negotiated in the contract, the liquidation may be subject to varying market conditions. For example, the agents cannot know in advance how long it will take to sell a house, thus the price the market is willing to pay for it might fall in the time before it gets sold.

Because the last two components are largely out of the agents' control, the lender must use the margin to offset the probability that the lender will incur losses. This is done by determining a threshold value for the margin (margin requirement), which if crossed will trigger a margin call. When the borrower receives a margin call he must either restore the margin to above the threshold value or surrender the collateral for liquidation. The margin requirement is set such that the risk of losses, after confiscating and liquidating the collateral, are acceptable for the lender. It should be clear that if the volatility is very low, the threshold can be correspondingly low, while ensuring that the probability of losses are negligible and conversely if volatility is high, the lender must demand a high margin requirement to ensure the same level of security. The conclusion is that as long as
lenders do not change their willingness to take on risk, the margin requirements they demand will be highly responsive to changes in the estimated volatility of the collateral value.

As a side note before we move on to the next section; while increasing collateral requirements makes the loan safer, it also reduces the volume that the lender can collect interest from. In particular for banks in a competitive market, operating with stricter or more prudent requirements than their competitors will likely lead to lost market shares. On the other hand, lowering margin requirements below a "safe" threshold equates to taking on risk. Hence there is a tradeoff between safety and competitiveness (risk and reward), which the bank/lender must balance. The variable willingness to take on additional risk in order to remain competitive is not critical to establishing the connection between margin requirements and economic fluctuations, but it is noteworthy that it may contribute to an excessively high supply of credit at times.

4.1.3 Heterogeneous agents and prices

The previous section established the connection between volatility and margin requirements. This one argues for the connection between margin requirements and market prices. In order to establish this connection, Geanakoplos (2003; 2010) argues that agents have different expectations about the future cash flows of assets (their value). The future is uncertain and depends on a large number of variables, thus it is not unreasonable that agents have subjective and diverse opinions about what will occur, based upon their individual knowledge and/or skills, leading to estimations of cash flows (asset values) which differ from those of other agents. Although there are a number of reasons that this assumption might be fulfilled, the theory uses the proxy of optimism for simplicity. An agent who values an asset high relative to other agents is called optimistic.

If we accept the premise of heterogeneous agents, then we can arrange them according to their optimism (valuation of an asset). Furthermore, we can safely assume that a risk neutral investor who believes that the asset is worth more than it is trading for, will want buy it. In his opinion this constitutes an arbitrage opportunity. An agent in this position will henceforth be referred to as a "natural buyer". Conversely, an agent holding the asset will want to sell it if he is offered more than he believes it is worth. In order secure more assets, a natural buyer will have to offer more than the least optimistic owner of the asset believes that it is worth. With every purchase, he (and all other buyers) will face an increasingly optimistic seller in the next round, thus the price increases. Theoretically, the most optimistic agent will want to buy the entire stock of that asset, because he is willing to offer more than the second most optimistic agent demands to sell it. However, the volume that he, or any other natural buyer, can purchase will be limited by the amount of their own funds
they are willing and able to invest and the amount they can borrow from lenders. The consequence is that their leveraged purchasing power depends on the margin requirement. For example, if a buyer has $10,000 to invest, and the margin requirement that a lender or the bank demands of him is 20%, then he can borrow $40,000. The total which he can use to buy the asset is then $50,000. If the margin requirement is only 5%, then he can borrow $190,000, and buy $200,000 worth of the asset. As argued above, the more he can purchase, the higher the price will become. Hence the margin requirement demanded by lenders affect the equilibrium prices.

The assumption of risk neutrality makes prices particularly sensitive to margin requirements, but it is not necessary to choose this extreme scenario in order to generate the effect. As long as the natural buyers demand more assets than they are willing and able to spend out of pocket, their ability to act and the resulting equilibrium price will depend on the availability of credit.

### 4.1.4 Cycles

Based upon the dependence of prices on the expected volatility, Geanakoplos states that cycles of asset price bubbles and busts occur because the perceived or expected volatility tends to be understated when markets are tranquil or steadily growing, resulting in high leverage and inflated asset prices. As long as confidence amongst investors and lenders is high, inflated prices can prevail. Hence if confidence is shaken by some significant indication that a major decline is more likely than previously anticipated, then the fear of that scenario turn into higher volatility in calculations, higher margins and lower prices. It is an important observation to make that the fear of a crash can cause it. Even if this fear does not prevail, but there is more uncertainty in both directions, such that the probability of both extraordinarily good and extraordinarily bad results are higher, the result is the same. The reason is that the creditors are only affected by the potential downside, but do not stand to gain from the upside. Thus their expected return and consequently the amount they are willing to lend decrease when the volatility increases and vice versa.

The severity of the crash is amplified by the level of leverage prevailing when it starts. It is well known from finance that leverage multiplies the investors' returns, both gains and losses. When a negative shift in the price of an asset held by a leveraged investor occurs, his equity (and the margin) declines by his leverage times the price movement. Lenders will make a margin call once the investor falls below the threshold margin. If the market consists of heavily leveraged investors, a small price shift can cause a large number of margin calls, forcing investors to sell their positions. The flood of sellers in the market cause a further price decline on top of the original one, bringing even more investors below their margin requirement.
The crash can also be described in terms of a decreased purchasing power of the most optimistic agents. A negative shift reduces their purchasing power by the multiple corresponding to their leverage. When they can no longer afford to hold as many assets at the prevailing margin requirements, the asset must then be held by some of the less optimistic agents and result in a lower equilibrium price.

4.2 The model

4.2.1 A two period model

Geanakoplos (2010) starts with a simple example to illustrate how the equilibrium price is determined by heterogeneous agents. In this example there are two periods. The initial or current state 0 and a later period in which either state U or state D will be realized. Let $S_t$ denote the state of the world which is realized at time t. Then, $S_0 \in [0]$ and $S_1 \in [U, D]$. There are two goods in this world; a consumption good ($C$) and an investment good ($Y$). Y will yield $C$ if $S_1 = U$ and $0.2 C$ if $S_1 = D$. The aggregate supply of both $Y$ and $C$ are scaled to 1 and both are further infinitely divisible and endowed uniformly among all agents in the initial state.

Assumptions:

A1. There is a large number of agents who are uniformly distributed on the interval [0,1] in their estimated probability that $S_1 = U$ occurs. Put another way, agent $h \in [0,1]$ believes that $P(S_1 = U) = h$. E.g. the most optimistic agent ($h = 1$) thinks there is a 100% chance of the $Y$ delivering $C$ and 0% chance that it delivers $0.2 C$. Conversely, the least optimistic agent ($h = 0$) thinks there is a 0% chance of the $Y$ delivering $C$ and a 100% that it yields $0.2 C$. The agents are distributed uniformly between these two extremes.

A2. Agents are risk neutral; they are indifferent between a certain yield and a gamble with the same expected value.

A3. Agents try to maximize total consumption and are insatiable in consumption in either period.

A4. Agents are indifferent toward consuming in the early period or the later period. They only care about their total consumption.

A5. All trades in one state are made simultaneously in a uniform price auction.
An agent \( h \) will determine the expected value of the asset as:

\[
E^h(Y) = h(1) + (1-h)0.2
\]  

If the market price \( p \) (exchange rate of \( C \) per \( Y \)) turns out to be higher than \( E^h(Y) \), then the agent will stand to gain (in his subjective opinion) from selling (trading \( Y \) for \( C \)) and conversely if the price is lower he stands to gain from buying (trading \( C \) for \( Y \)). Thus there will be a marginal agent \( h=b \) such that everyone who is more optimistic than him (buyers) will value the \( Y \) higher than it is trading for and consequently want to trade all their \( C \) for \( Y \) and conversely everyone less optimistic (sellers) will trade all their \( Y \) for \( C \), as they value it less. The equilibrium price will be equal to the valuation of this marginal buyer:

\[
E^b(Y) = b(1) + (1-b)0.2 = p
\]  

The buyers (\( E^b(Y) > p \)) will have a total of \((1 - b)\) units of \( C \) to trade and the sellers (\( E^h(Y) < p \)) have \( b \) units of \( Y \) to offer. Thus the exchange rate (price) must be:

\[
p = \frac{1-b}{b}
\]  

Solving for \( h \) and \( p \):

\[
b + (1-b)0.2 = \frac{1-b}{b}
\]

\[
0.8b^2 + 1.2b - 1 = 0
\]

\[
b = 0.596
\]

\[
p = E^b(Y) = 0.596(1) + (1-0.596)(0.2) = 0.677
\]
A graph of the supply and demand at a range of prices is presented below (a full schedule is provided in the appendix). The axes are chosen in the traditional manner, although it is inaccurate to say that either price or quantity is determined independently of the other:

Illustration 4.2: Supply-Demand for two-period model without borrowing

This example illustrates how the price is determined in equilibrium. In the model, trading occurs by a uniform price auction, where every asset is traded at the price at which there are sufficient buyers to purchase the whole supply (Krishna 2002, p.169). This is equivalent to the price where supply equals demand. At every price there is a number of agents who demand (want to buy) and all the remaining agents want to sell. For every increment of the price, some agents turn from buyers to sellers and vice versa. The market clears at the price where the combined purchasing power of all the buyers is sufficient to buy the assets offered. An interesting and highly relevant property of this market mechanism, which is highlighted in the next section, is that only the valuation of the most optimistic agents determines the price. If the most optimistic agents could obtain more funds through lending (or endowment for that matter), then a smaller group would hold the complete supply of $Y$ and drive the price up further, but they are credit constrained so their influence over the price is limited. Further, it should be noted that the uniform price auction suppresses the effect, because in order to buy an additional asset, the buyer group has to pay more for every asset. If they were able to price discriminate such that they would trade each asset at the value that each individual seller demanded, then the average price would be significantly lower and enable them to purchase more assets with the same funds, causing the equilibrium price to become even higher.
In this example, agents are distributed on a continuum, but if their valuations had been discrete, then the price would necessarily be negotiated to a price between the two agents who were the closest to the market clearing price. In terms of an Edgeworth-Bowley box (Bowley 1924), all of the prices between these two agents valuations lie in the core, which neither coalition (buyers or sellers) will block. But when we assume a continuous distribution of agents, the solution is unique.

### 4.2.2 Introducing borrowing

To illustrate the effect of borrowing on prices hinted towards above, let us first assume that lenders demand full repayment in the worst case scenario. Even in the state where \( Y \) turns out to deliver 0.2, borrowers must be able to repay loans completely. All agents expected values remain unchanged, but the budget has changed because the natural buyers now have access to additional funds. In the worst case scenario, the full supply of \( Y \) delivers 0.2 \( C \), thus the maximum that can be borrowed across all buyers is exactly 0.2 \( C \). If this scenario occurs, those who bought will then be left with nothing after they repay their debts and sellers will hold the whole supply of both \( C \) and \( Y \).

Will optimistic agents borrow the maximum amount that they can? The answer is yes at 0% interest, because they are risk neutral and in their view the asset is underpriced. Why would the pessimistic agents agree to lend to the optimists? Firstly, there is no risk as the collateral is sufficient to repay the loan even in the worst case scenario and by assumption they are indifferent between consuming in the first or the second period so they are really indifferent between making the loan and not doing so. Secondly, it will turn out that borrowers will only ask to borrow \( 0.2/0.686 = 29.15\% \) of each lenders \( C \) to fund their purchases, thus the competitive interest rate is 0% with no alternative use for the consumption good.

The price is again determined by the total amount of \( C \) available to buyers, divided by the amount of \( Y \) they want to purchase, which must be equal to the valuation of the marginal buyer who is indifferent toward buying or selling.

\[
\frac{(1-b)+0.2}{b} = b + (1-b)0.2 \\
\Rightarrow b = 0.686 \\
p = 0.686 + (1-0.686)0.2 = 0.749
\]

Now that the optimistic agents have access to credit, a smaller group of agents are able to buy the entire stock of the assets, because the natural buyers have increased purchasing power. Every buyer (except the marginal one) is still credit constrained and if they could borrow more with the asset as
collateral at no interest, they would still do so, but the example illustrates that when the optimistic agents have access to more funds, a smaller group will hold all the assets and will bid up the price. This time 31.4% of the agents hold the complete supply of \( Y \), compared to 40.4% before. After trading, each buyer then holds \( 1 / 0.314 = 3.185 \) times the assets they started with and owe \( 0.2C / 0.314 = 0.637 \) in \( S_0 \). If \( S_1 = D \) the 3.185 \( Y \) delivers exactly the 0.637 \( C \) needed to settle the debt, leaving the borrower with nothing.

The equilibrium price has also increased from 0.677 before to 0.749. One way to think of this is that the buyers had to purchase the "last unit of \( Y \)" from a more optimistic seller. Again we notice that it makes no difference that there are some sellers at the bottom of the distribution which value the asset at a price that is far below the market price. These agents' views are not relevant beyond the fact that they are located below the market price. All that matters is the purchasing power of the most optimistic buyers.

In the previous two examples, the collateral level was arbitrarily set to illustrate the fact that the equilibrium price and the size of the group of asset holders depends vitally on the access to borrowing to leverage purchases, but there is no guarantee that markets will decide upon these terms for loan contracts or that everyone will trade the same contract. The question is then; what contracts will be traded and what are the terms of those contracts? The most optimistic agents still find that the asset is underpriced according to their valuation, so they would really like to buy more if they could, given that the benefit does not outweigh the costs. Additional borrowing would mean that they would default on some part of their debt in \( S_1 = D \), but it is not unreasonable that the lenders will allow this if there was a premium interest rate paid to take on this risk.

It seems paradoxical that an agent's risk preference appears to depend on whether he is buying or selling, particularly because the same agent might become a buyer and borrower or seller and lender depending on the distribution of optimism and wealth among the other agents. On the one hand, if an agent turns out to be a buyer (asset is underpriced in their view), he is risk neutral, willing to gear up purchases until an adverse movement will completely eliminate his equity. On the contrary, if the same agent is a seller, because the other agents are more optimistic than him, then he will be perfectly risk adverse, unwilling to lend a single dollar unless it is guaranteed to be repaid. The answer to this riddle lies in the debt structure which both buyer and lender will agree to.

Geanakoplos (2010) shows that loans backed by less a full guarantee in \( S_1 = D \) will be priced at interest rates which vary inversely proportional to the level of collateral, but that these are not
traded in this example as they denote an inefficient allocation for both borrowers and lenders. Intuitively, the lenders would essentially be betting against the belief that they hold (lower probability of state U) by allowing additional leverage which only pays off if the economy goes to state U and incurring a loss in state D, when the borrower defaults. The buyer faces a similar problem: if the economy goes to state D, then his losses are limited to the collateral (he defaults). If the economy goes to state U he has more of the asset, but earns less per unit of the asset as a result of the interest he now has to pay. At the interest rates which the lender will accept, this tradeoff is not lucrative for the optimist who thinks that the state U is more likely to occur. As a result, the only debt structure which both borrower and lender will agree to is the one which pays the same in either future state. Thus, the apparent disparity of risk preferences of agents is not an inconsistency, but merely the result of every agent behaving rationally and utility maximizing.

4.2.3 A three period model

To illustrate how prices can crash without a crash in fundamentals, Geanakoplos (2003; 2010) considers a three period model of a very similar type to the one previously considered. Agents and goods are distributed uniformly as before, but information about the investment good is revealed in two increments. This time it will take two consecutive "negative" movements for the 0.2 outcome to occur, all other sequences deliver 1. Agents believe that the probability of an upward movement in each period is $h$ and that the outcomes are independently and identically distributed in both periods. Some additional notation is required: Let $p_s$ denote the price in state $s$. Next, $a$ and $b$ represents the marginal buyer in $S_0$ and $S_1 = D$, respectively. In the third period, the possible states are contingent on what state was realized in the second period. The possible states in the third time-period are denoted: $S_2 \in \{UU, UD, DU, DD\}$.

Illustration 4.3: A three-period model
Again, agents can incur debt as long as they are able to fully repay in the worst case scenario. The noticeable difference from the previous example is that there will be short term contracts which expire after a single period has elapsed. In particular, the debt incurred in \( S_0 \) expires in \( S_1 \), before the \( Y \) yields any \( C \). Although the assumption is made here that only debt which can be fully repaid is traded here, Geanakoplos (2003;2010) shows that it is also the only contract that is traded in equilibrium even if we allow the agents full freedom to trade any debt contract. The 2-period contract is not traded because it is riskier and thus allow for less leverage.

Further, the asset \( Y \) has become more attractive to all agents (except \( h=1 \) and \( h=0 \)), as it takes two adverse movements for the asset to deliver the lesser result. Every agent now believes that the 0.2 result occurs with probability \((1-h)^2 < (1-h) \). Agent \( h \) now calculates the expected value of the asset to:

\[
E^h[Y] = h^2(1) + h(1-h)(1) + (1-h)h(1) + (1-h)^2(0.2) = 1 - 0.8(1-h)^2
\]

(45)

Agents will behave like they do in the two-period example: They will buy as long as the price is below their valuation and borrow to purchase further unless the interest rate discourages them from doing so. All the agents who are more optimistic than \( a \) (the marginal buyers in \( S_0 \)), will collectively buy the complete stock of \( Y \) on margin. Lenders will demand sufficient collateral to settle all debts even in the worst case scenario, thus the amount which can be borrowed depends on the price the asset will command in \( S_1 = D \). If \( S_1 = D \), then those who bought in \( S_0 \) will need to surrender all their assets to the creditors to repay the loans. The remaining \( a \) agents will then hold all the assets and consumption goods. If the supply of 1 asset and 1 consumption good is held by \( a \) agents, they each hold \((1/a)\) of both goods. There will then be a new marginal buyer \( b \) with a pool of \((1/a)(a-b)\) funds with which to buy all the remaining \((1/a)b\) assets. With the full stock of assets, they can then guarantee repayment of a debt of 0.2, like before.

Those are all the pieces needed to establish the price in \( S_1 = D \):

\[
p_D = \frac{\frac{1}{a}(a-b) + 0.2}{\frac{1}{a}b} = \frac{1.2a-b}{b}
\]

(46)

The numerator states the total amount of funds available to the natural buyers in \( S_1 = D \). The buyers constitute the interval \([a,b]\); those above \( a \) are bankrupt and those below \( b \) think the asset is overpriced. They each hold \(1/a\) of the consumption good \( C \) with which to trade and can borrow a
total of 0.2 \( C \), using the asset as collateral. The denominator states that each of the \( b \) sellers hold \( 1/a \) of the asset.

(46) can be rewritten as:

\[
a = \frac{b(1 - p_D)}{1.2}
\]  

(47)

We know from before that this must be equal to the valuation by the marginal buyer \( b \) in state \( D \):

\[
p_D = b + (1 - b) 0.2
\]  

(48)

Using the expression for the price in \( S_1 = D \), the price in \( S_0 \) can be stated. Because it will be possible to borrow \( p_0 \) against the asset, the price is determined as follows:

\[
p_0 = \frac{(1 - a) + p_D}{a}
\]

(49)

Further we note that agent \( a \) must be indifferent between buying on margin and lending and selling (speculating to become a buyer in state \( D \) when the price is very low in his view):

\[
(a) \left[ \frac{1}{p_0 - p_D} (1 - p_D) + (1 - a)(0) \right] = (a)(1) + (1 - a) a \left( \frac{1 - 0.2}{p_D - 0.2} \right) + (1 - a)^2 (0)
\]

(50)

Equation (50) warrants some explanation. It states his expected return per unit he spends on either of two strategies: On the left, buying assets in \( S_0 \), and on the right, saving in \( S_0 \) (actually lending, but it is irrelevant for him as there is no risk or return) and buying if \( S_1 = D \). For each strategy there are three unique scenarios to consider. First if \( S_1 = U \), second if \( S_1 = D \) and \( S_2 = DU \) and third if \( S_1 = D \) and \( S_2 = DD \). Because agent \( a \) goes bankrupt if he chooses to invest at \( S_0 \) and \( S_1 = D \), the last two scenarios are the same under that strategy.

The left hand side explains that by investing in \( S_0 \), agent \( a \) can buy \((p_0 - p_D)\) units of \( Y \) for each \( C \) he is endowed with. For each unit of \( Y \) he purchases, he will borrow \( p_D \) (the maximum that lenders will allow) and contribute the balance \((p_0 - p_D)\) himself. If \( S_1 = U \), which the agent believes will occur with probability \( a \), he is left with \((1 - p_D)\) per unit purchased, after the loan is repaid. If \( S_1 = D \), the agent is left with nothing after the loan is repaid.
The right hand side explains agent $a$’s valuation of the strategy to sell in the first period and buy on margin in the second, if \( S_1 = D \). If \( S_1 = U \), which he believes will occur with probability $a$, a unit of $C$ is worth simply $C$. If \( S_1 = D \), he will buy assets, paying down \( (p_D - 0.2) \) on each and borrowing the balance. If \( S_2 = DU \), he will be left with \( (1 - 0.2) \) per unit of $Y$, with probability \( (1 - a) \) and if \( S_2 = DD \), he is left with nothing. Substituting the expression for $p_D$ from equation (48) yields the above equation.

With the set of equations established above, a trial and error procedure can find the values which satisfy all constraints at:

$$p_0 = 0.95, p_D = 0.69$$  \hspace{1cm} (51)

$$a = 0.87, b = 0.61$$  \hspace{1cm} (52)

We notice that the price falls in state $D$ without a crash in the fundamentals as a reaction to the news that a bad scenario has become more likely. As Geanakoplos (2010) points out, the price falls more than any individual investor expects it to. Part of this fall is due to the increased volatility, but another significant part is due to the elimination of the most optimistic investors and the lower leverage which lenders will tolerate in the second period. In state $0$, the buyers take on leverage of $0.95/0.26 = 3.6$, whereas in state $D$ buyers can only get $0.69/0.49 = 1.4$, because when the volatility of the fundamentals increase, lenders will not agree to leverage purchases to the same extent.

It is also important to notice that the marginal buyer $a$ thinks that the asset is underpriced (he values it at approximately $0.985$), but still opts to sell (save) his funds in order to buy on margin if state $D$ occurs. A range of agents will do this, "pulling" the price down in state $0$. The implications of this are discussed towards the end of the next section.

### 4.3 Evaluation

Geanakoplos explains convincingly how the market clearing mechanisms produce different prices at varying debt structures. It provides important insights onto how given the heterogeneity of agents combined with a loosening of collateral requirements results in the price being set by a smaller group of investors. The leverage cycle further gives plausible evidence toward the relationship between expected volatility and margin requirements, causing significant effects on prices before and possibly without there ever being an actual shock to fundamentals.

The theory also suggests answers to observed phenomenon which a rational expectations
framework does not without resorting to psychological and/or behavioral irrationality. In the model, agents are simply utility maximizing given their unique assumptions and information set. Heterogeneous agents play the central role, and although a uniform distribution is employed to simplify the examples, any continuous distribution would operate by the same mechanisms.

Although the model demonstrates well how prices are influenced by collateral requirements, it does not create cycles without some further assumptions about the changing views of market participants. In the verbal exposition of his theory, Geanakoplos describes positive feedback in the way markets reach the new equilibrium after some news increases volatility forcing agents to deleverage:

All three elements feed back on each other; the redistribution of wealth from optimists to pessimists further erodes prices, causing more losses for optimists, and steeper price declines, which rational lenders anticipate, leading then to demand more collateral.

(Geanakoplos 2011, p.4)

However, in the model the price movements are simply amplified by the level of indebtedness which can be adopted; the feedback is not modeled. This binary mechanism is not procyclical, it merely amplifies the shock in the two possible states of the world, without affecting future events. When agents claim prices fall "more than their view of the fundamentals warranted", this is exactly the result which they anticipated. It important to note is that this is an implicit assumption of the model. Without calculating the price fall, they could not set the leverage requirement in the initial state. In the three period model, some agents actually used the anticipated price fall to speculate in investing in the middle period. It is also misleading to argue as if it was a surprise that "[high leverage] will cause a crash even before there has been a crash in the fundamentals and even if there is no subsequent crash in the fundamentals." Every agent knew that this would happen, because this is how they set the leverage requirement; that is the arrangement they agreed to. The result is that there are no cycles beyond the amplification of the result of "good" or "bad" news.

The high desired leverage amplifies shocks, to which there are associated costs, but does not create endogenous cycles without further assumptions about how agents behave or the economy in subsequent periods. For example, one could argue that "good news" redistributes wealth towards optimists, who might hold on to their optimism and want to buy more, driving prices further up in subsequent periods. Another example is when production is added to the economy, as Geanakoplos (2010, p. 44) discusses, but does not model. Asset price fluctuations cause difficulty in predicting and possibly errors in correctly estimating future profitability, thus opening for over- and
underproduction.

In the three step model, some agents find it optimal to bet against the market, because they find that the asset will be heavily underpriced after a crash and thus it is more profitable than buying it outright even if the asset is underpriced, in their view, in the original state. This line of argumentation has consequences which would be interesting to pursue. For example, it is easy to see that the appearance of such investors in a highly leveraged market can break a spiral of rising prices and leverage and cause. There are a number of immediate consequences of the leverage cycle which indicates endogenously generated cycles; it is therefore surprising that these arguments are not pursued.

A possible short-coming with strict market clearing requirements is that it does not accurately describe less efficient markets, such as the labor market, where prices are some degree of sticky and/or some resources will not be offered/employed at the market clearing price. Though the focus here is on certain assets that are always for sale, it does make the theory less applicable elsewhere. This is a trade-off we have to live with, when a model is specific and accurate, there is a good chance it will not at the same time be applicable everywhere.

4.4 Summary

The leverage cycle theory highlights the effect on prices of changes in the expectations that lenders hold regarding the collateral. In particular, it points out that even though expected values remain unchanged, increased uncertainty about the range of future values or prices has a significant effect. This is because creditors have an unlimited downside, but a limited upside. The connection between margins and prices is that margins limits the purchasing power of the most optimistic agents and hence their influence over prices. The consequence of these two observations is that the interest rate does not give the full account of market conditions, because there are really two channels through which a change in the equilibrium of financial markets can be expressed; either through interest rates or through margin requirements. A government body which seeks to stabilize economic fluctuations by only manipulating the interest rate will fail to notice and counteract the effects of changes in the margin requirements.
5 Comparing the financial instability hypothesis to the leverage cycle

It is perhaps a leap to say that the theories proposed by Minsky (1977) and Geanakoplos (2003, 2010) are the same, but central underlying assumptions and arguments are shared. Both argue for a relationship between leverage and asset prices, such that an expansion or contraction of credit amplify and/or cause corresponding fluctuations in asset prices and investment activity. Although employed with certain differences, the connection between leverage and prices is established as a result of introducing heterogeneous agents. It is a significant finding that heterogeneous agents can affect the real economy, because it contradicts the popular rational expectations theory. While the results and conclusions of the two theories are similar, the two associated models discussed in this thesis are indeed quite different: While the dynamic model produced by Keen (1995) focuses on the continuous feedback between investment and prices to generate endogenous cycles, the Geanakoplos (2010) model focuses on the price determination in equilibrium given changes to the value of the collateral.

5.1 Feedback and cycles

In both theories, economic fluctuations are caused and/or reinforced by conditions in the credit market. In the Leverage Cycle (LC), a period of low volatility allows investors to stretch collateral further, because lenders perceive that less can go wrong over a short period of time. Prices soar as optimistic investors' can obtain more credit and crashes when volatility increases, causing lenders to reduce the purchasing power of investors through higher margins. In the Financial Instability Hypothesis (FIH), a profitable period makes both investors and banks more optimistic, resulting in agents incurring more debt, fueling even higher prices. Both theories agree that high leverage makes agents more vulnerable to shocks and that an initial reversal of trends will trigger an avalanche of falling prices and forced deleveraging. However, while the feedback is a central element in the Keen model, Geanakoplos does not model the feedback explicitly, but focuses instead on how the price is determined and moves in different states.

In terms of cycles, the LC model generates amplification of shocks, not cycles. The FIH, on the other hand, considers smooth transitions through time with endogenous positive feedback which generates cycles. In the LC there are only two states: The "normal" state, in which high levels of leverage is undertaken by risk neutral agents and a "crash" state, in which optimists go bankrupt, resulting in deleveraging a drastically lower prices. Optimistic agents are knowingly betting all their wealth on a positive future asset price, fully aware that they will go bankrupt if they are wrong. This
is not a procyclical dynamic, but rather an amplification of shocks. Geanakoplos (2010, p. 3) notes that if agents do extrapolate trends (as the FIH argues), then the cycles will be much worse, but stresses that this is not necessary for excessive price movements and does not go on to model another type of feedback like he describes in the introduction. In the FIH there is a continuous positive feedback between market conditions and future investments and speculation, such that a state of over-leveraging develops endogenously as agents extrapolate a series of repeatedly improving past results into ever more optimistic expectations (or negative results into negative expectations).

5.2 Leverage and debt structures

When it comes to the endogenous borrowing conditions there are differences in argumentation, but these are not necessarily contradictory. Minsky argued for and Keen modeled agents whose perceptions change with time. Both lenders and borrowers are affected by the memory of the recent past. When the past has been profitable, they expect the future to be profitable and thus they will both accept an increased level of leverage. Geanakoplos derives explicitly that in periods of low volatility, agents perceive investments to be safer, thus more debt can be incurred against the same assets, which fits well with the description Minsky outlined.

Both theories argue that a highly leveraged and optimistic agent is rewarded when the economy goes well, thus allowing him to take on more debt and make the economy more volatile, as a smaller shock can wipe out the equity of investors. Put another way, good market conditions gradually redistributes wealth toward the optimistic agents whose increased purchasing power allows them to incur more debt and increase market volatility. This is merely implied by Geanakoplos' LC, but explored in more detail by Minsky. In the Keen (1995) model, heterogeneity is not modeled explicitly, but there is a direct positive feedback between changes in investment and production.

5.3 Agents

While the agents described are similar in the two theories, there are differences. Primarily, how agents are affected by other investors' behaviors. The LC describes a heterogeneous set of agents who are risk neutral utility maximizers, given the possibilities they are facing. Even so, it does not describe why agents hold the beliefs that they do at any given time or discuss whether perceptions change endogenously with market conditions, e.g. perceived stage of the business cycle the
economy is in. The time component does play a role, on the other hand, as different kinds of news generate uncertainty among investors, translating into volatility of results which affects borrowing conditions and thus prices. The agents in the LC are also indifferent to the behavior of other agents. They will merely acknowledge the market price and determine whether to buy or sell depending on their own preferences.

The FIH also emphasizes heterogeneous agents, but also describes the behavior of agents as affected by market conditions. In the FIH, increased investment feeds back into rising prices, making firms more profitable. When profits are high, agents want to invest further making their behavior change significantly depending on current trends. Another key point is that conservative investors are forced to deal with market conditions set by speculating or "Ponzi" investors. As the aggressive speculators crowd out both investments and the money supply, previously solid firms are forced to deal with higher interest rates and rising costs making their own operations riskier and less profitable.

5.4 Price determination

This section asks the question; where do the two theories complement one another and what ideas are contradictory? Let us first consider the way margin requirements affect prices. The LC shows explicitly how the purchasing power of the optimistic agents increase as a result of lower margin requirements. The FIH employs a similar argument, although on the macroeconomic scale. There it is argued that new debt "...results in the financing of either some additional demand for capital and financial assets or of more investment. This results in a higher price of both assets..." (Minsky 1986, p. 178). These arguments are sufficiently similar that it can hardly be argued that they contradict one another.

The next connection to consider is how agents determine margin requirements. It is clear that once expectations are formed, lenders are interested in setting margins such that they minimize the chances that they incur losses. Thus the real question is how agents form their expectations in the first place. In the LC assumes that agents have knowledge about the range of future values that the collateral can take and that they are able to predict other agents actions in each contingency. Thus they can calculate the equilibrium price that the asset will command in every possible state. With this information, they can set margin requirements such that the collateral is sufficient to settle the debt even in the worst case scenario. In the FIH, lenders are really trying to achieve the same result, but it is argued that agents are unable to predict the value of the collateral in the future. They must
instead resort to a trial and error method of setting margins; if losses are below expectations, they are emboldened and will try lower margin requirements in subsequent periods and conversely if losses are high, they become more prudent and set margins higher for new loans. Neither of these two clear cut assumptions are very realistic in every scenario, but each hold a part of the truth about how lenders determine margin requirements based on the risk they are exposing themselves to. Over a short time horizon, it is plausible that agents can predict with some accuracy what will occur. Geanakoplos also finds that agents prefer the short term loans, because the less damage can be done in a short time and thus more leverage can be employed over that window of time. However, many loans must be made over longer time horizons. For example business loans, which are the main focus of Minsky's analysis, generally have long durations. For these loans it is clear that it is difficult or impossible for the lender to accurately predict the value of the company's assets over the entire course of the contract.

On the last point, the two theories diverge resulting in different dynamics. In FIH, extrapolating recent results cause a positive feedback between expectations, margins and prices. However the probability that the trend reverses increases the further away from equilibrium the system gets. The result is endogenous cycles of booms and busts. In the LC, shocks are absorbed at every time interval. This results not in a cycle, but rather an amplification of good and bad news, as they arrive; leverage and prices increasing when good news arrives and falling when bad news arrive. Because both theories provide important insights, given different assumptions about behavior, it is more accurate to say that the two theories complement each other than it is to say that they contradict one another.

5.5 Theory and policy

Despite their differences in theoretical background, analytical approach and modeling apparatus, the policies which the authors prescribe to minimize fluctuations and associated costs coincide with one another. While it is still true that there are certain assumptions and mechanisms which are shared, it is still a testimony to the robustness of their results that such different analyses end up with the same recommendation. In the end, they both argue that governments should intervene before a crisis unfolds to ensure that agents are not allowed to incur excessive debt. Minsky is vague about the specific measures to be undertaken, but does state that "in order to do better than hitherto, we have to establish and enforce a 'good financial society' in which the tendency by business and bankers to engage in speculative finance is constrained" (Minsky 1977, p.16). Geanakoplos is more specific as he traces the problem to the excessively stretched collateral, making individual investments more
sensitive to shocks, increasing covariance and systemic risk and consequently causing agents to become more susceptible to default. He then argues that by managing not only the interest rate, but also collateral, prices would not soar as high to begin with and investments will be more robust against shocks so that the chance of margin calls on loans (leading to further sales and price depression) are smaller.

5.6 Summary

The leverage cycle and the financial instability hypothesis both highlight important mechanisms in the economy. There are differences in focus, but not direct contradictions in understanding how cycles are generated. Where the LC focuses on price determination in equilibrium and the connection between volatility, leverage and prices, the FIH describe in greater detail the positive feedback effect between expectations, investment decisions, prices and agents' wealth. While the underlying assumptions are close to identical, the methodology and modeling are far apart. While the LC focuses on microeconomic aspects, modeled by a game theory approach to clear all markets, the FIH is angled towards macroeconomic explanations and applies dynamic modeling to account for the interaction of different forces in the economy. In the end, both theories argue for the curtailing of excessively leveraged speculation. Two theories complement one another well because the LC is narrow and accurate, while the FIH is broad and general. Each portrays a different angle on the topic, both calling attention to the same issue, namely the close connection between the financial sector and the real economy and the problems which arise from excessive speculation.

The next section discusses whether we can see the mechanisms described in the two preceding models in real world settings and then moves on to see whether the policies implemented are in line with the recommendations provided by these theories.

6 Discussion

_The long run is a misleading guide to current affairs. In the long run we are all dead._
_Economists set themselves too easy, too useless a task if in tempestuous seasons they can only tell us that when the storm is past the ocean is flat again._ - Keynes (1923)

It has been argued in the preceding sections that there are, on the microeconomic and the macroeconomic level, forces which reinforce shocks and potentially drive the economy away from
a theoretical equilibrium. This contradicts the prevalent efficient market hypothesis, which argues that such shocks will be absorbed efficiently (quickly) and a new stable equilibrium reached (Fama 1970). In particular, the connection between financial markets and the real economy entails incentives which produce price bubbles at excessive debt levels which are very likely to produce subsequent crashes. It may be the case that if one abstracts from these fluctuations and/or cycles it could be observed that the economy is characterized by a slow, steady growth path over long time periods. Nonetheless, as Keynes points out in the quote above, it is little comfort to know that markets will eventually work things out in the long run, if there are excessive gains and losses in the short run. The natural question is thus, do we see price/debt bubbles and positive feedback effects in the real world?

6.1 Procyclical interaction between credit and economic activity

6.1.1 Leverage inflated asset prices

There is ample support in the literature for a significant correlation between credit and property prices in most western countries during the last two decades (Borio, Furfine & Löwe, 2001; Hilbers et al, 2001; Hutchison & McDill, 1999). Goodhart and Hoffman (2003) finds the same result and adds that credit growth is also leading consumer price indexes, but that it is harder to establish a direct connection between credit and stock prices. Semmler & Bernard (2011, p. 3) points to three specific examples of credit fueled boom-bust cycles in asset prices: the emerging markets crisis in the 1990s, the dot com bubble around the turn of the millennium and the U.S. housing bubble in 2007.

For a long time the discussion has been over whether or not credit is leading, interpreted as causing, changes in other macroeconomic effects. However, this misses the point of positive feedback, because in a dynamic system there is not a single cause and effect, there is constant interplay between variables. If there is feedback between credit and asset prices, it is of little importance which variable moves first, because after an initial shock to either, the effect will bounce back and forth. Adrian and Shin (2009) found empirical evidence that firms, and in particular investment banks, adjust their balance sheets in response to price movements, in order to target some leverage level. This behavior is pro-cyclical, as an increase in prices (which lowers leverage on assets), leads to more debt-financed purchases to restore the target leverage level, which in turn pushes prices up even further. In a later report, they also found that firms target leverage level was strongly influenced by a desire to maintain the Value-at-Risk (VaR) to equity ratios (Adrian & Shin, 2010). Again this behavior is also pro-cyclical as firms are forced to deleverage when markets are strained,
causing even further price depression.

### 6.1.2 Interest payments and solvency

One area where the inherent instability of the economy has been particularly visible recently is in the interest rates charged on Greek government bonds. The Greek government debt-crisis emerged in early 2010 as a result of the rising fear that the Greek government would default on its rapidly growing debt. As a result of a combination of falling economic growth and large budget deficits before and during the global financial crisis, the Greek debt-to-GDP ratio rose from around 100% in 2005 to over 140% in 2010 (Eurostat, 2012). At that point creditors began to consider it a significant possibility that the Greek government might default on their debt and began demanding extraordinarily high interest rates on bonds to compensate them for taking on that risk. Throughout 2010, interest rates on newly issued bonds rose from 4.72% to 13.75% (Bank of Greece, 2012). While this may be bad enough in isolation, the increased interest rates are in fact increasing the probability that the feared default will occur. The fear of default causes the interest rates to go up and the increased interest rates give the creditors more reason to fear a default.

Once the debt level of the nation reached a sufficiently high level, combined with uncertain future incomes, investors became more cautious of lending new funds and demanded higher interest rates. Even if the Greek government were to balance their budget, such that the debt stock would not increase, they would still have to issue new debt at the market interest rates in order to roll over existing bonds as they mature. Their previous budgets were sustainable when they made them, given the prevailing market conditions at the time. However, when interest rates rose beyond what they expected, their expenses shot upwards. The consequence of higher expenses is that their ability to service existing debt and run fiscal surpluses are significantly weakened, giving creditors additional reason to fear a default. It is somewhat ironic that without the fear, Greece's financial problems would not be nearly as dramatic. Nevertheless, once "the boat capsizes", as Fisher explained, the indebted nation's finances do not return to equilibrium, but continues to propel itself away from it.

### 6.2 Theory based recommendations and actual practice

The U.S. sub-prime mortgage market crash and subsequent global financial crisis is a fantastic example of the mechanisms described by the FIH and the LC. A rising trend in housing prices led to a low perceived risk attached to mortgage loans. Loose lending practices meant buyers were able to bid up prices even further, amplifying the initial trend in prices. Geanakoplos (2011, p. 8) reported
that the most leveraged half of all loans were issued at a down payment of 13% in 2000 compared to 2.7% in 2006 (a dollar of collateral could borrow $7.7 in 2000 and $37 in 2006). Over the same time period, housing prices rose by 90%. Between 2006 and 2008, margins rose back to over 15% for the same loans and housing prices came crashing down. The falling prices and defaults which followed caused a dramatic fall in investments, employment and lending practices which froze global credit markets.

In comparing the policy recommended by the FIH and the LC with actual responses there are two policy questions to deal with. The first relates to the short run when the crash has just happened: What should be done to minimize the duration and costs of the recession? The second is looking at the longer perspective: How, if possible, to avoid or reduce the probability of another serious crisis in the future?

On the first question, both theories hold that to prevent or stop a destructive downward spiral in prices and economic activity, central authorities should expand credit (restore leverage) to sustain or stop the free fall of prices and incomes. This is apparently also the policy which the U.S. government and the Federal Reserve Bank (FRB) are pursuing, at least to some extent. The FRB increased the supply of money until the federal funds rate reached zero and proceeded to increase it further by purchasing other assets from banks (quantitative easing). While the FRB tripled the monetary base between April 2008 and December 2011, M2 only increased by about 25% in the same period. This means that the financial institutions now have quite large excess reserves and the policy has reduced effectiveness because markets have been unwilling increase activity even though funds can be obtained at a minimal cost (FRED, 2012a; 2012b; 2012c). This expansion is in line with the policy recommended; however, Minsky warned that although increasing the money supply can prevent a looming depression: "because the Federal Reserve intervention has protected various financial markets, the recovery can soon lead to a resumption of an inflationary boom" (Minsky 1977, p. 15). The excess reserves give financial institutions the possibility to expand the money supply significantly. However, in the current climate there is insufficient demand to make it happen.

Geanakoplos (2010) also addresses the moral hazard which results from underwater loans. He explains that for example home owners with a mortgage that exceeds the house value are unlikely to invest in maintaining the structures which results in additional losses for the lenders, as eviction can take as long as 18 months. His prescription is to write down the principle on loans to encourage lenders to keep the house and avoid additional losses. The "Home Affordable Refinance Program" is in line with Geanakoplos recommendation, though it only targets borrowers who are stuck with over-market interest rates, not those who are already underwater and most likely to default.
Another response has been certain bailouts and government takeovers of central actors who were facing bankruptcy. In March 2008, the Federal Reserve purchased $30 billion of Bear Stearns' assets which JPMorgan Chase, who were to merge with Bear Stearns, were unwilling to buy (Federal Reserve, 2012a). In the fall of the same year the Federal reserve also created an extraordinary line of credit for the American International Group (AIG) of up to $85 billion and also purchased approximately $50 billion worth of assets from the company (Federal Reserve, 2012b). This intervention sought to prevent the escalating panic of the type which Greece is now facing. The companies would still be forced to write down the value of their assets, but as long as they retain access to credit at "normal" interest rates, their chances are much better than if they were to forced to meet the market demands for interest rates. Geanakoplos (2010) arguments for writing down principals in times of crisis, noting that creditors are much more likely to be able to collect a written down debt which allows the individual or company to continue operating than they are if the company were to face an uncoordinated bankruptcy. However, while bailouts may be the best course of action in an individual case, it is a dangerous practice as it signals to investors that risky investments could be payed for by government intervention. In fact, many economists argue that much of the reason why asset prices were inflated and many risky contracts were written in the first place was that belief among investors that the government would intervene and pay for their losses if the bets went wrong (Bernstein, 2009; Stiglitz, 2010; Trumbull, 2009). This creates an upward bias on investments' expected payoffs and could incentivize agents to create another bubble. Thus while a countercyclical intervention might be the right approach in theory, it gives rise to additional unstable dynamics in the economy.

The broader question is how to change the financial environment long term to avoid excessive fluctuations and bubbles in the first place, rather than letting them happen and implement policies of damage control after the fact. To curtail the excessive cycles, the FIH and LC suggest active, preemptive management and regulation of credit and leverage so that agents' opportunity to engage in speculation are curtailed. Both agree that regulation can produce the desired results in the short term, but notes that because the agents are seeking to engage in speculative financing, they will attempt to invent new ways and/or asset classes to circumvent existing limitations on their activities. On this background, Minsky argues that "any regulatory or intervention system will lose effectiveness over time." (1986). Geanakoplos argues the same way, but his argument centers on collateral. He argues that because agents want to create virtually unlimited debt contracts, but collateral to guarantee repayment is scarce, agents constantly seek to stretch the little collateral which exists in creative ways. He points to the recent crisis in which a variety of derivative contracts on mortgages and other assets (MBSs, CDOs) were invented and sold in mass quantities.
The main response of western governments to the recent crisis has been to mandate higher capital requirements or equivalently higher margins, lower leverage and less gearing. In the U.S., broad changes to the oversight and regulation of financial institutions was implemented with the "Dodd-Frank Wall Street Reform and Consumer Protection Act" (Dodd-Frank). The reform aims specifically to monitor and manage risks to the financial system, through the establishment of the Financial Stability Oversight Council, to prevent the need for using tax payers money for bailouts of system critical firms and to protect investors by legislating transparency and accountability for financial firms (Brost & Oblack, 2012). In Europe, the countermeasures have largely aimed toward strengthening individual financial institutions' ability to withstand shocks from internal problems and counterparty risks which were a prevalent problem in the recent crisis. The purpose is to prevent the possibility that the failure of individual firms spill over to a large number of firms causing credit markets to freeze. By legislating minimum capital ratios, it also sets a boundary on speculation for central actors in the financial system (BIS, 2010a; BIS, 2010b).

Both Minsky and Geanakoplos finds that financial institutions are intrinsically motivated to invent new ways to stretch collateral to issue more debt. Thus the problem for governments is that while the regulations that they put in place will likely produce the desired results for a while, but as long as there are agents who constantly seek ways to circumvent these regulations, the rules must be constantly updated to remain relevant.

### 6.3 Problematizing theoretically desirable policies

While certain policies and institutional structural changes might be technically desirable in the long run, there are inherent forces in our economy and our current political systems which will make certain policies ineffective, politically undesirable or both.

#### 6.3.1 Popular resistance

Large entities, like nations, are especially unlikely to default on debt, because their future incomes are unlikely to deteriorate significantly. Nevertheless, there is still a threshold for the amount of debt they can take on before creditors will begin to consider it to be a real possibility that the borrower will default on repayments. Like Greece, several other European countries, including but not limited to, Spain, Portugal, Italy and Ireland, have run large deficits immediately before and/or during the financial crisis. Combined with low economic growth, these deficits have led to a significant increase in the debt burden on these nations’ finances. The result is that creditors are starting to fear a default, calling for cuts in government spending in order to ensure that there are
sufficient incomes to meet their debt obligations. At the same time, several of the same countries are also plagued by unemployment and low economic activity. While cutting spending in order to achieve a budget surplus might be desirable to reduce the debt burden, it will also increase an already problematically high unemployment and suppress aggregate demand as a result of the reduced income of the laid off workers.

In nations or unions where both the fiscal authorities and monetary authorities are closely tied together, the debt can, of course, be monetized; i.e. repaid by extending a loan from the central bank to the government. This is problematic in the European Union, where the interests of the affected nations are not completely aligned and the central authorities have limited influence over each individual nation's fiscal decisions. In this situation, governments with problematic debt burdens are left with the choice between two evils; spending, which will increase the already worrying debt, and 'austerity', which will amplify the problems of unemployment and low activity.

Even after deciding which strategy is desirable, either one is likely to be unpopular with a significant part of the voters in the country, making it politically unviable to implement. Governments who attempt to implement a policy which is unpopular will likely be replaced at the next election. Recent elections in Greece and France have shown that their population will resist 'austerity measures' and elect a political leadership who do not seek to curtail growing debt by cutting social programs (Barkin, 2012; Granitsas, Bouras & Stevis, 2012; Heyer, 2012). Similarly, last year, elections in Spain, Portugal and Ireland replaced governments which failed to handle their respective nation's finances and unemployment problems in a satisfactory manner (Tremlett, 2011; Woolls, 2011). In Italy, Berlusconi was forced to resign, in similar circumstances to the previous Greek Prime Minister Papandreou, over the soaring interest rates on government bonds. Elections in Italy next year (2013) will reveal the populations' judgment over the austerity measures now undertaken by the interim technocrat government. The lesson from these observations is that even if economists or politicians have the 'right answer' to the problems, it may not be possible for them to implement the remedy.

**6.3.2 Disaster myopia**

The wave of legislation we are now seeing aim towards, among other goals, to increase the capital buffer of firms such that they will be better able to withstand shocks without defaulting and preventing such shocks from spilling over firms' counterparties and/or other sectors in the economy. However, in markets with strong competition such buffers make firms less profitable in 'normal' times. To remain competitive in such markets, firms must always be on the very edge of
profitability. Thus, in the rare event of a crash, the most aggressive firms do not have the necessary buffer to remain solvent in the downturn.

A good example of this problem is found in the banking sector, where the rate of default of loans is stable while the economy functions 'normally', but rises sharply in the rare event of a crisis. The bank would optimally account for this low-probability event by increasing the interest rate in normal times to build up a buffer. In a highly competitive banking sector, however, it is tempting for firms bitterly fighting for market share to discount the possibility entirely (or simply gamble that it will not happen in the near future). If one or more banks undertake such a strategy, they will likely be able to price products cheaper than their competitors and gain market share as buyers flock to the cheapest alternative. In turn, other banks who were previously more prudent, might consider changing their strategy to avoid losing more market share. It is particularly likely that such a scenario might unfold, when the 'large shock' occurs very rarely. It becomes a difficult dilemma for banks whether to lower their prices and remain competitive at an unsustainable rate or to build the necessary buffer, but lose business to lower priced competitors.

Guttentag and Herring (1984) modeled this dilemma and argued that this sort of "herd behavior" is a result of an availability bias, i.e. that agents underestimate the probability of events that have not occurred recently. The bias occurs because humans tend to remember more vividly their more recent experiences and assign those more weight when making decisions than the experiences of the long gone past. Borio et. al. (2001) argues that the problems stem from the problems with assessing risk over longer periods of time and the misalignment between incentives and financial stability for the individual bank.

While the constant striving of firms to get an advantage on their competitors and yield the maximum return to investors cause them to deliver goods and services to end users at a low cost, it also incentivizes them to take on risky projects and employ debt rather than equity to finance investments. The price of these activities is sharply increased volatility of returns to the point where insolvencies and defaults are highly likely in the event of a negative shock. The result is a procyclical force in the economy, because it tends to reward the most adventurous agents when the economy is doing well, and punishes both investors and creditors harshly in recessions. The conclusion which can be drawn from this is that even if we regulate collateral levels and other risky activities, the agents in the economy will still be intrinsically motivated to take positions which does not handle unexpected shocks well. Thus it is highly likely that agents will eventually invent new products, contracts and entities which allow them circumvent current regulations. It will therefore not be sufficient to create policies and regulation which can successfully handle these
issues today and consider the case closed. On the contrary, economic conditions and financial activities must be continuously monitored, and policies and regulation adapted to changing practices in order to remain effective.

At the same time, it should be acknowledged that while there is popular and political willingness to implement such regulation today, with the consequences of failing regulation right in front of us, it is likely to wane over time. Even if regulation is implemented, if there is subsequently a long period of steady economic growth, it might eventually appear unnecessary, causing the above mentioned forces to successfully lobby for their removal. The dismal prediction is thus that these problems are indeed very likely to resurface in the future, even if we solve them today.

7 Outlook and future research

The research by both Geanakoplos and Minsky-Keen provides important insights about how the current economic situation and the functioning of the economy in general. Both avenues are so fresh that much remains to be done. In the case of Geanakoplos' leverage cycle, it provides important insights with respect to price determination within a narrow set of assumptions; thus it will be interesting to see how well the theory performs beyond his examples and whether it can be incorporated in a meaningful way into a broader model of a modern economy. On the other hand, Keen's model of Minsky's theory employs a bird's-eye view, which highlights the problems which arises as a result of agents who discount the significance of past experiences heavily, instead attributing greater importance to the most recent ones. However, the model makes a number of simplifications which abstracts it from reality. Further work here can hopefully improve the model to bring it closer to a realistic description of the economy. Both avenues are promising and illuminate important problems with respect to the impact financial markets, in their current form, have on the economy. Both theories and their models has a long way to go, but that the foundation has been laid, paving the way for subsequent research to gain a better understanding of how the economy operates and how to best deal with the problems which arise from economic fluctuations.

8 Concluding remarks

It has been the goal of this thesis to highlight the important connections between financial markets and the real economy, which has played an important role in the recent economic calamity which is still unfolding in the United States, Europe and around the globe. It is argued that the financial
dimension of economics constitutes a potentially procyclical and/or destabilizing force which incentivizes individuals, firms and nations under specific circumstances to incur a level of debt, which appeared sustainable under the tranquil economic conditions that prevailed when it was undertaken, but leaves them vulnerable and sensitive to unexpected negative shocks. For individuals and firms these incentives are likely to result in the creation of asset price bubbles and subsequent collapses. When nations incur excessive debt, the consequences are potentially catastrophic.

Further, it is argued that the existing economic understanding and policy tools, which in the past were successful at keeping economic cycles within acceptable bounds, are no longer sufficiently effective as a result of developments and innovations in financial markets. Two separate and independently developed models have been presented which both show that given the existing economic climate, such economic turbulence as we have seen lately are in fact an inherent characteristic of the system, as opposed to an anomaly. The consequence is that we cannot discount the possibility of similar or worse fluctuations in the future.

To better direct policy, we require a better understanding of the changes which has taken place in financial markets in the last century and how it affects economic stability. The economic literature has often sought to discover laws to govern economics analogous to the laws of nature, which remain constant over time. Admittedly, the basics hold, et ceteri paribus, like Alfred Marshall taught us; however, because the overarching goals of macroeconomics depend on a complex and constantly evolving set of conditions, it is unlikely that we can create rules and institutions which will remain relevant for all time. To remain effective, a regulatory regime must adapt to developments and innovations which renders old rules ineffective and thus obsolete.
9 Reference list


Overstone, S.J.L. (1837). Reflections Suggested by a Perusal of Mr. J. Horsley Palmers' pamphlet on the Causes and Consequences of the Pressure on the Money Market.


10 Appendix

Table 1: Leverage cycle schedule.

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