Correction of temperatures
and a handy way of making correction
charts for reversing thermometers

BY
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Correction of temperatures and a handy way of making correction charts for reversing thermometers.

This paper deals with how we may easily and rapidly, make a correction chart by means of which we can, in a few seconds, find out the temperature in situ from the observed temperature at the reversing thermometer.

If the temperature of the reversing thermometer \( t \) when read off, is different from the temperature in situ \( T_{is} \), we have to add a correction \( K \) to the observed temperature \( T \) to find \( T_{is} \). The most accurate approximation formula is given by W. Hansen (1. p. 145).

\[
K = \frac{1}{\beta} \left( n + \frac{\varepsilon}{2} \right)
\]

where \( \varepsilon = T - t \) and \( n = T + v_o \). \( v_o \) denotes the volume of the severed Mercury thread below 0°. \( \beta \) denotes the apparent coefficient of expansion for mercury in glass. For those sorts of glass which are used in the reversing thermometers at the present time, we may, without making greater errors than about 1\(^o\)/64, use \( \beta = \frac{1}{6100} \) \( (K, T, t \) and \( v_o \) are given in °Celsius).

The supposition for the legitimacy of formula (1) is that \( T_{is} = T \) when \( t = T_{is} \). Most thermometers, however, are burdened with an index error \( I \), i.e. the scale is not quite true. We then get that \( T_{is} = T + I \) when \( t = T_{is} \). Usually \( I \) varies with \( T \). Instead of formula (1) we then obtain
If \( I = 1^\circ C \) the error will never exceed 0.3% if we write

\[
K = \frac{1}{\beta} - \left( n + \frac{x}{2} \right) \frac{I}{\frac{1}{\beta} - \left( n + \frac{x}{2} \right)} + I
\]

In (3., p. 242) Oscar Sund points out that it is most handy to find the temperature correction graphically. Oscar Sund uses another expression for \( K \) than formula (2) but also by this formula it saves time to calculate \( K \) graphically. For this purpose we may draw a correction chart similar to that given by O. Sund in (3). (\( T \) as abscissa, \( K \) as ordinate, and a separate curve for each integral value of \( t \) (see fig. 2)).

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Original for making of correction chart for any thermometer.

Let us put the first term in formulae (2) equal to \( C \).

\[
C = \frac{1}{\beta} - \left( n + \frac{x}{2} \right)
\]

On transparent millimetre paper we draw \( n \) as abscissa and \( C \) as ordinate. A suitable scale is that \( 1^\circ \) corresponds to 1 cm along the abscissa and \( 0.01^\circ \) to 0.5 cm along the ordinate (fig. 1 about \( \frac{1}{2} \) nat. size).

If \( \alpha \) is kept constant formula (3) defines a curve on the millimetre paper. Curves corresponding to each integral value of \( \alpha \) can be drawn (it is sufficient to draw the curves corresponding to \( 20 \leq \alpha \leq 30 \)) in the following way:

At the back of this publication there is a table which gives a series of values for \( C \) by different values for \( n \) and \( \alpha \) (the most of these values have earlier been calculated and published by A. Schumacher (2., p. 237)). Along the two lines, \( n = n_1 \) and \( n = n_2 \), we plot the \( C \)-values corresponding to each integral value of \( \alpha \). Then we connect points with corresponding \( \alpha \)-values with straight lines (with pencil). The line corresponding to \( \alpha = 25 \) and that to \( \alpha = 28 \) are as examples drawn on fig. 1. These lines approximately coincide with the curves for \( \alpha = con- \)
stant. The numerical value of the error will be greatest by the abscissa 

\[ n = a - \sqrt{(a-n_1)(a-n_2)} \]

where \( a = 6100 - \frac{x}{2} \). By this abscissa the error (correct ordinate — wrong ordinate) is equal to

\[ F_{\text{max}} = \frac{2 \sqrt{(a-n_1)(a-n_2)}}{(a-n_1)(a-n_2)} - \frac{2a}{n_1 + n_2} \]

We see that even if \( |n_1-n_2| = 40^\circ \), \( F_{\text{max}} \) will not exceed 0.0005. Consequently it is sufficient first to plot the \( t \)-values on vertical lines by each 40° and then to connect corresponding \( t \)-values with straight lines. These lines can with sufficient approximation be used as lines for \( t = \) constant.

According to the definition of \( n \) and \( t \)

\[ t = n - v_o - \tau \]

For \( v_o = 100 \) we put \( t = t_{100} \).

\[ t_{100} = n - \tau - 100 \]

By means of formula (5) and the lines for \( \tau = \) constant it is easy to draw smooth curves for \( t_{100} = \) constant on the millimetre paper. Draw these curves in India-ink and rub out curves for \( \tau = \) constant which were drawn with pencil.

Seen apart from the line for \( \tau = -25 \) and for \( \tau = -28 \) which should have been erased, fig. 1 shows how a part of the millimetre paper then looks. The curved lines are the curves for \( t_{100} = \) constant. The value of \( t_{100} \) is plotted at the end of each fifth curve. The values of \( n \) are plotted at the top and the bottom of the paper, and \( C \) at the side (in \( 1/100 \)°).

In the next section we shall show how this drawing can be used as original by making of correction charts.

**Handy method for preparation of correction charts.**

By eliminating \( n - \tau \) in formula (4) by means of formula (5) we obtain

\[ t = 100 - v_o + t_{100} \]

We see that by a thermometer with \( v_o = N \), \( t \) is constant and equal to \( 100 - N + t_{100} \) at a curve where \( t_{100} = \) constant. Hence we see that by the original we may make a correction chart for any thermometer in the following way:
Table over the value (C) of the function \( n^2 - 1 \) by different values of \( n \) and \( r \).

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<th>( n )</th>
<th>50</th>
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<th>70</th>
<th>80</th>
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<td>0.294</td>
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</table>

For negative values of \( r \), \( C \) is also negative but for simplicity the negative signs are omitted in the table.
We find out the \( v_0 \) of the thermometer and the maximum temperature (\( T_{\text{max}} \)) it can show. Then we make a copy of a part of the original. The breadth of the part shall extend from \( n = v_0 - 3 \) to \( n = v_0 + T_{\text{max}} \) and the height from the top to the bottom of the original.

By means of the formulae: \( T = n - v_0 \) and \( t = 100 - v_0 + t_{100} \) we can write on the copy the values of \( T \) and \( t \) which correspond to the \( v_0 \) of the thermometer. In the intersecting points between the line \( C = 0 \) and the lines for \( t = \) constant, \( t = T \).

Then we cut off the parts of the copy where the previous numbers are entered, and a chart for easy calculation of the first term \( (C) \) in formula (2) is ready.

Seen apart from single thermometers which may have a very great \( I \) on account of having been damaged and repaired, the second term in the expression for \( K \) (formula (2)) may be ignored. In order easy to add the last term \( I \) to the \( C \)-values calculated by the chart, we plot \( I \) for different \( T \) on the chart. \( I \) is plotted along the ordinates in the same scale as \( C \) (in \( \frac{1}{100} \) Celsius). Having done this, the correction chart is ready.

Fig. 2 shows as an example the correction chart for thermometer No. 000 with \( v_0 = 70^\circ \) and \( T_{\text{max}} = 19^\circ \). The copy is taken from 67 to 89 at the scale at the top and at the bottom of the original. The numbers are entered in the above-mentioned way, and to illustrate how this is done, the parts of the copy where the previous numbers are entered are not cut off. The red line on the correction chart is the \( \& I \)-line for thermometer No. 000.

By means of a correction chart as described above and a pair of compasses we can easily determine \( K \). First we find the point on the correction chart corresponding to the actual \( T \) and \( t \), and then we add \( I \) by means of the compass and read \( K \) off on the scale at the left-hand margin of the correction chart, e. g. by thermometer No. 000 (fig. 2) \( T = 4^\circ 5, t = -1^\circ, K = 0^\circ 101 \).

On some thermometers \( I \) is, as mentioned above, so great that the second term in the expression for \( K \) (formula (2)) cannot be ignored (e. g. if \( I = 0.3, n = 200 \) and \( r = 20 \) this term is \( 0^\circ 01 \) C). However, in most cases we can write \( I = I_M + i \) where \( I_M \) is a constant and \( i \) does not exceed a value which substituting \( I \) in the second term in formula (2) gives this term a value greater than \( 0^\circ 001 \). Instead of \( T \) we may write \( T_g \) defined by \( T_g = T - I_M \) i. e. \( T_g = n - v_0 - I_M \), on the correction chart. That \( I \)-value which is found by \( T \) is then plotted by that \( T_g \) which has the same value as \( T \). Now we can use this correction chart in the same way as the above-mentioned, treating it as if \( T \) and not \( T_g \) is the abscissa.
Fig. 2. (About 2/3 nat. size).
With the object not getting too great an opening in the compases, we can, instead of plotting $I$ along the ordinates, add $I_M$ to the $C$-values along the ordinates and plot $i$ from the new 0-points at the ordinates in the same way as we plotted $I$ above, because we also then arrive at

$$K = C + I_M + i = C + I$$

REFERENCES.


Fig. 1. (About 1/4 nat. size).