ASSIGNING VALUES OF TARGET STRENGTH AND EQUIVALENT BEAM ANGLE
IN ACOUSTIC SURVEYS OF FISH

by

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ABSTRACT

Contrary to popular opinion, the equivalent beam angle $\Psi$ is not determined solely by the transducer directivity. Rather, like the target strength $TS$, it also depends on the scattering properties of the target fish, but relative to the detection threshold. The connection between the two quantities, $\Psi$ and $TS$, is elucidated. A method of application is illustrated through an example.

RESUME: DETERMINATION DE VALEURS D'INDEX DE REFLEXION ET D'ANGLE EQUIVALENT EN EVALUATION ACOUSTIQUE DE POISSONS

Contrairement à un avis couramment établi, l'angle équivalent $\Psi$ n'est pas déterminé uniquement par la directivité du transducteur. En effet, comme l'index de réflexion $TS$, il dépend aussi des propriétés diffusantes du poisson détecté mais en rapport avec le seuil de détection. La relation entre ces 2 paramètres, $\Psi$ et $TS$, est précisée. Une méthode d'application est décrite grâce à un exemple.

INTRODUCTION

Both fish target strength $TS$ and transducer equivalent beam angle $\Psi$ are essential quantities in echo integration (Forbes and Nakken 1972, MacLennan 1990). It has long been appreciated that the sampling volume $V_S$ depends on the detection threshold. Since $\Psi$ measures the width of $V_S$, it must share the dependences of $V_S$. These are the target range, $TS$, and target orientation.

The several dependences may be apparent from the following arguments. At a sufficient range, the echo from a target on the acoustic axis and in its most favorable orientation, characterized by its maximum $TS$, will be indistinguishable from noise. Here the echo lies under the threshold, and $V_S=0$. If the $TS$ of a target located on-axis and at fixed range is systematically decreased, an echo level will be reached that lies below the threshold, for which $V_S=0$. A directional scatterer at fixed range and on
the acoustic axis may be detectable for one orientation, e.g., that corresponding to the maximum $TS$, but not for another, e.g., that of a null or deep lobe in the scattering pattern. By extension, it is evident that $V_s$ depends on the scatterer orientation distribution.

Given this recognition of the dependences of $V_s$, it may be wondered why $\Psi$ is not accorded similar recognition, but is generally equated with the nominal value $\Psi_0=10 \log \psi_o$,

$$\psi_o = \int b^2 d\Omega \ ,$$  

where $b^2$ is the product of transmit and receive beam patterns of the transducer (Simmonds 1984). It is unnecessary to answer the question, for the present approach shows how $\Psi$, together with $TS$, enters the basic echo integration equation. Estimation of $\Psi$ is described, and its computation is illustrated for the case of northeast Arctic cod (*Gadus morhua*).

**THEORY**

A fundamental quantity in echo integration is the mean volume backscattering coefficient $s_v$. This relates the cumulative backscattering cross section per ping to the accessible or sampled physical volume $V_s$ (Stanton et al. 1987). This is, for a sufficiently large number $n$ of similar scatterers in $V_s$,

$$s_v = \frac{1}{V_s} \sum_{j=1}^{n} \sigma_j$$  

Alternatively,

$$s_v = \frac{1}{4\pi} \overline{\sigma}$$  

where $\rho$ is the number density of scatterers in $V_s$, $\rho=n/V_s$, and $\overline{\sigma}$ is the mean backscattering cross section,

$$\overline{\sigma} = \frac{1}{n} \sum_{j=1}^{n} \sigma_j$$

This quantity may be integrated with respect to both depth $z$ and sailed interval of distance. The result, the area backscattering coefficient, is a measure of the cumulative backscattering cross section per unit surveyed area. Two common expression of this, shown here for a single ping for the sake of simplicity, are the following:

$$s_a = \int_{z_1}^{z_2} s_v dz$$
Both quantities are dimensionless, having units of backscattering area per unit surveyed area. In equation (4a) the two areas are expressed in the same units, e.g., square metres, while in equation (4b) the reference surveyed area is one square nautical mile.

Common elements to the several equations presented here are $\sigma$ and $V_s$. The first of these is largely independent of $V_s$, although in a strict sense it does depend on $V_s$ through the so-called perspectival effect (Foote 1980). For ordinary directional transducers this effect may be incorporated indirectly by increasing the effective standard deviation in tilt angle distribution (Foote 1985).

The sampling volume is not independent of $\sigma$. As shown earlier (Foote 1991), it may be expressed in this general form:

$$V_s = \int H(g\sigma^2 - t) \, dF \, dV,$$  

where $H$ is the Heaviside step function, $H(x) = 0, 1, 1$ as $x$ is respectively less than, equal to, or greater than zero. The argument $g\sigma^2 - t$ compares the echo quantity $g\sigma^2$ to the threshold $t$, where $g$ is a gain or geometric factor. The integration is performed with respect to the distribution function $F$ of scatterer orientations and with respect to the total physical volume, as limited by imposed range, or time, gates and physical boundaries in the sampling medium. The integrand serves to delimit the total volume further by counting only those echo contributions that exceed the threshold.

The equivalent beam angle $\psi$ is a very convenient quantity for use with the echo integration technique. The reason is that $s_A$ values, for example, are often distinguished by depth interval. The basis $s_v$ values are similarly expressed as functions of depth. In both cases, it is most convenient and generally necessary to use a differential measure of $V_s$, namely $\psi$. The connection may be seen from expression of $V_s$ for a thin spherical shell of thickness $\Delta r$:

$$\Delta V_s = \psi r^2 \Delta r,$$

where

$$\psi = \int \int b^2 H(g\sigma^2 - t) \, dF \, d\Omega$$

defines the differential measure, the effective equivalent beam angle (Foote 1991).
In the case of a high signal-to-noise ratio (SNR), with essentially negligible threshold $t$, $H(gb^2\sigma-t)=H(gb^2\sigma)=1$, and $\psi=\psi_o$, as in equation (1). In general, however, $\psi<\psi_o$, and as the threshold is approached, $\psi$ vanishes.

The several quantities underlying the basic echo integration equations, namely $\sigma$ and $V_g$, hence $\psi$ too, are expressed in intensity- or energy-equivalent domains. The logarithmic measure corresponding to $\sigma$ is the target strength,

$$TS = 10 \log \frac{\sigma}{4\pi},$$

where $\sigma$ is customarily expressed in SI units, with reference TS due to a perfectly reflecting sphere of 2-m radius of 0 dB. The logarithmic measure corresponding to $\psi$ is just

$$\psi = 10 \log \psi.$$

**EXAMPLE**

A number of computational examples of $\psi$ have already been given (Foote 1988, 1989, 1991). Here, the example for cod (Foote 1989) is supplemented by specification of TS values.

It is easily appreciated that $\psi$, depending as it does on $\sigma$ and a threshold, in addition to scatterer range and orientation, does not have a unique functional form that can be applied in all circumstances. For similar conditions but different thresholds, however, the form is similar. Its computation here, in addition to being illustrative, may also be useful.

In order to compute $\psi$, a number of parameters must be specified. Apropos of the quantities in equation (6), $b$ is determined for a circular transducer with beamwidth of 8 deg as measured between opposite -3-dB levels. The gain or geometric factor $g$ is associated here with single-fish detection, hence $g=10^{-\alpha r/2r^2}$, where $r$ is the target range and $\alpha$ is the absorption coefficient. The backscattering cross section $\sigma$ is derived from a series of measurements of the tilt angle dependence of target strength of cod at 38 kHz (Nakken and Olsen 1977). A subset of these functions has been selected, namely the functions for the 20 specimens whose length lies between 35 and 55 cm. For the particular sample, the mean length is 42.2 cm. The detection range is assumed to be 400 m for each fish, which might correspond, for example, to electrical-noise-limited operation. Since $\alpha=0.0106$ dB/m at 38 kHz, $t=\sigma_{\text{min}}\sigma_{\text{max}}=5.54 \times 10^{-12}$ cm$^{-2}$. The tilt angle distribution is assumed to be that observed photographically by Olsen (1971) for cod in Lofoten, namely $N(-4.4,16.2)$ deg. The resulting function is shown in Fig. 1.

Computation of $\bar{\sigma}$ for the same orientation distribution is conveniently expressed through the "average" target strength, $TS=10 \log \bar{\sigma}/4\pi$. Based on in situ measurements of gadoids, $TS=20 \log \bar{\lambda} - 67.6$, where $\bar{\lambda}$ is the mean fish length (Foote 1987). Thus for $\bar{\lambda}=42.2$ cm, $TS=-35.1$ dB and $\bar{\sigma}=38.8$ cm$^2$. 
Fig. 1. $\psi_x/\psi_0$ versus range $r$ for detection of isolated cod of mean length 42.2 cm, assuming a maximum detection range of 400 m and other conditions as described in the Example.

DISCUSSION

It is desired to determine the density of fish scatterers from a measurement of $s_v$ or $s_A$. This usually requires knowledge of the fish species and size distribution, barring direct measurement of $T$, which is a comparatively rare event and one which cannot be relied upon in the arbitrary echo integration survey. From the mentioned biological information, an average $T$ can be determined by reference to a standard equation or tabulation. Division of $s_v$ by the corresponding value of $\sigma/(4\pi)$ yields $\rho$ as though there were no threshold effect, or as though the echo measurements were made in the absence of noise. In case there is a threshold effect, the estimated density must be increased by the multiplicative factor $\psi_o/\psi_x$, where the subscript $r$ attached to $\psi$ in the denominator emphasizes its range or depth dependence. The quantity $T$, or $\sigma$, may also depend on depth, but unless specific knowledge exists on this, the usual procedure is to neglect the dependence.

If the density is to be determined from $s_A$, the above procedure may be repeated, but after allowance for the possible non-negligible thickness of the pertinent integration layers.

As the threshold is approached, the size of the correction factor $\psi_o/\psi_x$ becomes increasingly uncertain. At or below the threshold, of course, no estimate of fish density can be given.

The present approach, indicated by the example, relies on a mixture of experiment, theory, and supposition. The fishery researcher and manager would both like to avoid uncertainty at such a critical stage in application of the echo integration technique as that where estimates of acoustic density are converted to estimates of animal number density. Notwithstanding the want of data and knowledge, certain tactics may be employed to reduce the effect of thresholding. One is to lower the transducer to decrease the distance
between transducer and fish, as with a towed body. Another measure may be to use a larger transducer, if not one at another frequency, all to increase the SNR of echoes with respect to the noise field, whether ambient or reverberant.

In the general case it should be clear that determination of \( p \) entails more than a simple application of \( TS \) or \( \overline{S} \). The equivalent beam angle \( \psi \) may also depend on \( \sigma \). Attention to the threshold vis-à-vis scatterer species and size will indicate whether \( \psi \) deviates from its nominal value, in which case adjustment is warranted and may yield a significantly improved estimate of \( p \).

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REFERENCES


