DEFINITION OF THE PROBLEM OF
ESTIMATING FISH ABUNDANCE OVER AN AREA FROM
ACOUSTIC LINE-TRANSECT MEASUREMENTS OF DENSITY

by

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ABSTRACT

The problem of acoustic abundance estimation is briefly reviewed. Under proper conditions, fish density can be measured with high accuracy along line transects. Observed variations in fish density consequently reflect biological variations, or inhomogeneity in spatial distribution. The particular problem of estimating fish abundance over an area from line-transect measurements of fish density is defined. Related problems of estimating the variance of the abundance estimate and of mapping the spatial distribution are also defined. A partial list of candidate methods for solving the several problems is given. Among these, the so-called spatial statistical techniques appear to be most promising because of their exploitation of the observed spatial structure.

RESUME DEFINITION DU PROBLEME DE L'ESTIMATION D'ABONDANCE DE POISSONS DANS UNE ZONE A PARTIR DE MESURES ACOUSTIQUES DE DENSITE SUR DES COUPES RECTILIGNES

Le problème de l’estimation acoustique d’abondance est brièvement revu dans cette note. En présence de conditions correctes, la densité de poissons peut être mesurée avec une grande précision le long de coupes rectilignes. Les variations de densité observées sont le reflet de variations biologiques ou de non-homogénéité dans la répartition spatiale. Le problème particulier de l’estimation de l’abondance de poisson dans une zone à partir de mesures de densité sur des lignes est défini. Le problème du calcul de la variance de l’estimation d’abondance et de l’établissement des cartes de distribution est également présenté. Une liste partielle des méthodes possibles pour résoudre les divers problèmes est donnée. Parmi ces méthodes, celles appelées techniques de statistiques spatiales semblent être les plus prometteuses suite à la prise en compte de la structure spatiale des données.
INTRODUCTION

Fish stock estimation is a major occupation of large sectors of the ICES community. Acoustics is one of the basic tools used to quantify fish stocks. It is thus worthy of the closest examination, which is the raison d'être for the perennial meetings of the Fisheries Acoustics Science and Technology Working Group, not to mention impetus behind ICES sponsorship of several international symposia on fisheries acoustics over the past seventeen years.

A particular outstanding problem in acoustic abundance estimation is the statistical combination of line-transect measurements of fish density to estimate abundance over the survey region. This has been recognized by ICES through its Study Group Meeting on the Applicability of Spatial Statistical Techniques to Acoustic Survey Data, held at IFREMER in Brest, 4-6 April 1990. The second, and likely final, meeting of this series may be a workshop planned to address the same topic in 1991.

It is the present aim to contribute to the on-going discussion by defining the problem in terms intelligible to two separate groups: users of the acoustic method and statisticians. It is especially hoped that this second group will bring its insight and tools to bear on a problem of very practical importance.

In the course of presenting the statistical problem at a number of meetings in recent years, some of which were bald attempts to provoke ICES's interest, as at the 1989 Workshop on Spatial Statistical Techniques (Anon. 1989), doubts have been expressed about the acoustic method itself. Some have been well-founded, and these cannot be completely allayed here, but for the purpose of defining the statistical problem they are extraneous. Nonetheless, the ingredients of the acoustic method are described and analyzed, if in summary fashion, as in earlier presentations (Foote 1987, 1989a). The context for defining the statistical problem is thereby established. Some candidate solution methods, involving spatial statistics, are listed and briefly discussed.

METHOD OF ACOUSTIC ABUNDANCE ESTIMATION

A certain minimum of equipment is necessary for performing an acoustic survey. This includes a transducer, which converts an electrical signal to mechanical vibrations and vice versa; a platform to bear this over the survey region; some transmitting and receiving electronics to control pulsing of the transducer and reception of echoes, commonly called an echo sounder; and fish capture gear to identify the species and age or size distribution of observed fish. Additional electronic circuitry or other computing instrumentation is convenient for processing echo signals, while not being absolutely necessary.

Given a biologist's decision about the time and place to survey a particular fish stock, or target species, the transducer is generally carried or towed across the identified region of fish occurrence. The ship's course or track line generally follows or forms a grid-like pattern. This is typically composed of parallel lines or zigzags. These usually aim to cover the total area as evenly as possible within the available time. Other strategies, especially adaptive schemes which place more samples in areas of high density, are also quite common.

While the vessel is sailing, its echo sounding equipment is - or should be - in continuous operation. The transducer pulsing occurs at fixed, finite intervals, typically with a repetition rate of one per second. Given a vessel speed of 10 knots, or 5 m/s, nominal transducer beamwidth of 8 deg, and detection range of at least several hundred meters for individual target fish, the sampling is, to nearly all intents and purposes, essentially continuous.

Echoes are generally indicated by marks on a long strip of paper. If these are drawn across the paper in single lines corresponding to successive pings, in which depth is indicated by the distance from the side or other reference line, an echogram results. This is a visual image of what has been sensed by the acoustic pulse launched by the transducer, whether hull-mounted or towed.

Information is usually extracted from echoes by automatic processing. This was once done principally by analog circuitry; now it is done mainly by digital computer. Objects of the processing may be, for example, the number of individually resolved echoes over each designated sailing interval.
or a cumulative measure for the total number of scatterers, whether resolved or indistinguishably merged. The respective methods are those of echo counting and echo integration (Forbes and Nakken 1972, MacLennan 1990), but others exist too.

By knowing the character of the target fish, thence species and size distribution, values of backscattering cross section, or target strength, and acoustic sampling volume may be assigned. These allow expression of the acoustic quantities in terms of the fish density, for example, number per unit volume or number per unit area along the survey track. If the equipment is calibrated and it performs stably, and if the various procedures described so far are successful, then the distribution of fish over the survey region is characterized by line-transect measurements of density.

These measurements must be interpolated, implicitly or explicitly, to describe the fish quantity over the entire survey region. In many surveys, especially those done on the large scale of marine stocks, the actual area of direct acoustic sampling is small or even miniscule compared to the whole area. In some special, but also important, situations, as in spawning concentrated over a small region, the relative degree of coverage may be quite high.

Following interpolation, the total abundance may be estimated by integrating the area density over the survey region. The result is generally expressed by a small set of numbers, indicating total number or mass of target species, distinguished by size or age group. This complements contour plots of the fish density derived in the interpolation process.

ANALYSIS OF THE METHOD

Many things must evidently happen at the same time in order for the acoustic abundance estimation method to be successful. One requirement is that the overall performance of transducer and echo sounder system be stable. This is generally ensured by conducting calibration exercises at more or less regular intervals throughout the year or at least once during each major survey. Accordingly, the stable, linear, and low-noise operation of the transmitting and receiving electronics in the echo sounder can be verified.

The time-varied-gain function can also be verified in a calibration exercise, and deviations from the desired functional form determined for use in correcting estimates of the acoustic density distribution with depth. Measurement of the sum of source level and receiver voltage response and of the echo integrator scaling factor determines two quantities that are especially useful for long-term monitoring of system stability. The transducer beam pattern, thence equivalent beam angle, may also be measured. While errors may occur throughout the equipment, a calibration exercise is designed to catch these and facilitate their early correction. By means of a standard target, such as copper or tungsten carbide sphere (Foote and MacLennan 1984), and the procedure recommended by ICES (Foote et al. 1987), the system performance can be specified with an accuracy approaching ±0.1 dB.

Errors may also enter the abundance estimation process in the course of signal processing. However, with increasing use of digital technology at steadily earlier points after reception, such errors should be entirely negligible. Naturally, a sufficient sampling rate and sufficient number of quantization levels are necessary, but granted these and the use of widely available processors on the personal-computer or workstation level, signal quality remains essentially unimpaired by the sundry processing operations.

No matter how well the equipment performs, the survey can fail if the equipment is not used to best advantage. Both the survey technique and its time and place of application must be chosen with care. The degree of area coverage is also a crucial factor, but evaluation of its influence on the survey result is non-trivial, although persuasive results can be derived rather simply in some instances (Aglen 1983a, 1989).

Bad conditions can also spoil a survey. Medium absorption and excess absorption due to the presence of extraneous scatterers, such as air bubbles and plankton, can also work against the success of a survey (Dalen and Lyvik 1981), although preliminary work has shown how the negative influence of absorption can be countered (Hall 1989).
Extinction by dense or extended fish schools or layers may also bias measurements of fish density. Such effects are being studied (Olsen 1986, Armstrong et al. 1989, MacLennan et al. 1990), and a simple formula exists for correcting density estimates for extinction (Foote 1990).

Identification of species and age or size composition of target fish is a process that is frequently fraught with uncertainty. Representativity in sampling by trawl, seine, gillnet, or longline, has been a cause célèbre among gear-and-behaviour researchers for some years. As a consequence of a number of studies, for example, those by Engås and Godø (1986, 1989a, b), Godø et al. (1990), Godø and Engås (1990), gear is being improved and attempts are being made to compensate for known selective avoidance effects when sufficient data exist for their quantification. For some commercially important fishes at certain times of the year, the occurrence is sufficiently pure so that little error is incurred due to the physical sampling process itself. Such situations of species purity are exploited whenever possible. Increased use of collateral data on fish occurrence, for example, through monitoring of the hydrographic state or growth of plankton, may improve the identification process in the future.

As implied here, survey planning is also crucial to the success of the acoustic abundance estimation method. The biology of the target fish and possible other species in the same region must always be respected when planning a survey. Specific factors to be considered are the state of concentration or dispersion, the degree of mixing with other species, migration, and, in general, life history of target fish. Ease of registration of the fish, or even the possibility of this, is clearly of paramount importance. If the fish are not accessible to acoustics, as because of the bottom 'dead zone' (Mitson 1983) or near-surface or shallow-water occurrence, then the results must reflect this uncertainty. As with situations of species purity, situations of optimal availability are to be exploited.

Interpretation of the echo record is a quite subjective process. Through this, measures of fish density are allocated to species and age or size group on the basis of the appearance of the echogram, together with such supplementary information as catch data and the salinity-temperature-depth profile. Automatic classification may remove some of the subjectivity from the interpretation process, but this remains a task for the future.

Conversion of acoustic measures of fish density to biological measures, such as number density or biomass density, is also subject to error. In the particular case of the echo integration method, this depends on the aptness of chosen measures of mean fish backscattering cross section and effective equivalent beam angle. These are used, respectively, to determine the quantity of fish and to normalize this to the observation volume. Studies to define the backscattering cross section are extensive, as is evident from the bibliographies in Midttun (1984) and Foote (1988a), but continuing. Studies to specify the effective equivalent beam angle are fewer (Aglén 1983b, Kalikhman and Tesler 1983, Lassen 1986, Ona and Hansen 1986, Ona 1987, Foote 1988b, 1989b), but this is beginning to attract the attention it deserves.

Through these various processes, fish density has been measured along the line transects of the survey grid. But how are these measurements to be interpolated between the transects, and what is the abundance of the fish stock over the survey region? What is often, but fortunately not always, done is to neglect the connectedness, or correlation, of both intra-track and inter-track measurements or estimates of fish density. Thus a problem of unknown magnitude has been incurred in the abundance estimation process. This is the statistical problem defined in the next section.

The overall strength or robustness of the acoustic abundance estimation method can be likened to a chain, as in Fig. 1. The message is that the whole is no stronger than the weakest link.
THE STATISTICAL PROBLEM

The problem has already been defined through the question posed above. It is reformulated here.

It is assumed that the fish stock of interest is distributed over a bounded geographical region and that this can be surveyed acoustically in a time that is short compared to characteristic times of large-scale movement of the fish. It is also assumed that the area fish density is measured, or sampled, without error along line transects that are not necessarily parallel or cover the area in uniform fashion. The problem consists in putting together these line-transect measurements to estimate the total quantity of fish, or average density, over the survey region. It is further desired to describe the variance of this estimate and to map the fish distribution over the region.

The acoustic method in this simplified form results in a set of values \((x,y,z(x,y))\), where \((x,y)\) describes the transects and \(z(x,y)\) denotes the measured density at the point \((x,y)\). As indicated, the following sections will for the most part ignore the possibility of three-dimensional recording of density, measurements of depth of school, temperature, etc. In many cases these factors can be included to reduce the variability in the estimated abundance. Further, the problem of aging of a given stock is ignored, as this introduces an entirely new dimension to the problem.

A proper definition of the quantity which is to be estimated is

\[
I = \int_{\Omega} \int z(x,y) \, dx \, dy
\]

where \(\Omega\) is the region of interest and the "surface" \(z\) is only measured on the transects. The variance of the estimate, \(I^*\), of this quantity also needs to be evaluated. Notice that this variance may not have much relationship with the variation in the surface itself (as estimated, e.g., by an ordinary variance of the \(z\)-values), but is a measure of how close one can expect \(I^*\) to be to \(I\). A useful comparison is the estimation of the total amount in a heap of coal (Shepherd 1986). An analysis, which considers the measurements along cross sections of the heap as being independent measurements, will inevitably obtain a high variance estimate. The reason is not inaccuracies in the measurements, but rather the structure of the coal heap. In fact a method of analysis which initially maps the coal heap very accurately will also obtain a small error in the volume. This should be reflected in the variance estimate.

Most methods that have been suggested for the analysis of line-transect data are in some sense spatial by nature. Notable exceptions are methods which attempt to redefine the entire data collection and analysis in such a fashion as to eliminate any spatial information. The following will be mostly
restricted to methods which in some way incorporate spatial information.

CANDIDATE SOLUTION TECHNIQUES

Classification of potential methods

It must be recognized that the following is by no means a complete enumeration of all possible computational analyses, but it is a fairly comprehensive list of methods that have been suggested for the analysis of line-transect data, incorporating in some way spatial information.

The methods can be initially classified into three groups:

I Methods based on stratification. These methods use averages within squares or regions and perform integration by adding these averages, weighted by the square areas. A "contour map" in this instance usually consists of shading the regions.

II Generalized linear models which are extensions of regression techniques for fitting a model ("response surface") to the density. Integration and contouring is then performed based on the fitted surface.

III Smoothing and interpolation techniques which use some form of averaging to interpolate to points outside the transects, typically onto a grid, to be used for numerical integration and contouring.

The methods within a group can further be classified according to whether and how data are transformed prior to processing and/or according to distributional assumptions (e.g. assuming a Gaussian distribution, logtransforming or taking a nonparametric approach).

In what follows, all references to logarithms will refer to the natural logarithm (unlike the norm in acoustics, where it is usual to let 'log' denote log base 10).

Most methods in the three classes can give not only an estimate of abundance but also a variance estimate. The latter is essential for an evaluation of the quality of the abundance estimate. Unfortunately a number of methods of analysis in actual use do not yield a variance estimate and in fact some methods discard entirely the spatial information and regard the survey as a result of some sort of random sampling scheme. Such a method may well lead to a variance which is much higher than is realistic, when the nature of the patches is considered (cf. the coal example above).

Methods based on stratification

The principal assumption behind stratified analysis is that of homogeneous strata. Thus, the survey area is split up into strata, each of which is assumed to be homogeneous with respect to fish density. The total volume is then estimated by adding strata averages, weighted by area. Possibly some transformations are performed on the raw data.

A proper choice of strata can considerably reduce the variance from that obtained without any stratification.

For later reference, it is useful to note that this estimation procedure can be thought of as first fitting a step function with constant value $z_h$ in strata $h$, and then integrating to compute the volume.

Methods which start out from stratified sampling as in Cochran (1977) assume homogeneity and independence within each stratum in the sense that all the density measurements are statistically independent and are measurements of the same overall mean. Both of these statements are likely to be false, and certainly so when acoustic measurements are made. If a transect cuts across a school of fish, then values near the middle may tend to be higher than those near the edges and this phenomenon violates all assumptions behind the classical analysis associated with stratified sampling. The failure of these assumptions invalidates the variance estimates.
Even ignoring homogeneity and independence, the variances are only useful if the built-in distributional assumptions are (approximately) correct. The highly skewed distributions typically seen in fisheries tend to need some special treatment. Outliers also tend to occur and in some cases the overall average is severely affected by a single value.

Unfortunately the outliers here are usually very important data values, since they tend to indicate where most of the fish are concentrated! This implies that the outliers cannot be simply thrown away, but great care must be taken in order that their weight is not spread incorrectly over a large area.

Finally it should be mentioned that data transformations can easily cause severe problems when an attempt is made to integrate the fitted surface. This applies to all methods which use data transformations and will be treated further below.

Some specific data transformations in the context of stratified analysis include the log-transform (or \( \log(x + c) \)) and the Box-Cox family of power transforms. The latter family has been described in detail by MacLennan and MacKenzie (1988).

**Generalized linear models**

Several authors have suggested using generalized linear models (GLMs) for modeling fish stock abundance. These have been used extensively for modeling trawl survey data, and the same or similar models have been suggested for the analysis of acoustic data (see e.g. Myers and Pepin 1986, Shepherd 1986, Anon. 1990a and Anon. 1990b).

A generalized linear model is specified by describing (1) the connection between a linear predictor and the expected value of a response, and (2) the distribution of the response around that mean. A very useful introduction is given in Aitkin et al. (1989), but other relevant references include Nelder and Wedderburn (1972), McCullagh and Nelder (1983) and McCullagh (1983).

For acoustic (or trawl survey) data, these models can range from the very simple parabolic response,

\[
\ln(z + 1) = \alpha + \beta x + \delta y + \gamma x^2 + \xi y^2 + \eta y,
\]

to the much more complex response, even modeling zero and nonzero values separately. Within this framework it is in fact easy to model fixed-station trawl surveys by analyzing first the probability of a nonzero catch tow, \( p \) (using \( \hat{p} = 0 \) if zero tow, \( \hat{p} = 1 \) if positive), and then the actual number caught, \( y \), in nonzero catch tows. For example, one can take \( \logit(p) \) (defined as \( \log(p / (1-p)) \)) as one linear function of measured parameters and \( \log(y) \) as another linear function. In this setup one might assume a Bernoulli distribution of the 0/1 values and a gamma distribution for the fish counts in nonzero tows. Such models can easily be fit within the GLIM (generalized linear interactive modeling) statistical package, cf. Baker and Nelder (1978).

This class of models has received endorsement for catch-per-unit-effort (cpue) data from the ICES Working Group on Methods of Fish Stock Assessment (Anon 1990a). The models can be thought of as fitting a surface to the measurements, using the well-known techniques of (generalized linear) regression. The term response surface is normally used in this context. After fitting the surface, it can be integrated using any numerical integration technique to obtain the volume, which is the required stock abundance measurement. For acoustic measurements it is quite feasible to use a polynomial model as indicated above, using an arbitrarily high-degree polynomial in \( x \) and \( y \).

Acoustic measurements are usually collected on a fairly fine scale along the transects and large biological variation is often detected. In terms of the response surface, this means that the surface may have large flat sections and occasional, very high, thin peaks. Initial tests using these methods for acoustic data seem to indicate that (at least for some data sets) very high-degree polynomials need to be used since several peaks may be quite well defined in the data due to the large number of measurements. Numerical problems abound, however, when high-degree polynomials are used. Even if orthogonal polynomials are used to eliminate the numerical problems, the surfaces will tend to wander in strange directions at region boundaries or between transects when the degree of the polynomial is
Data transformations may alleviate the fitting problem somewhat, but even a \(\log(x)\) (or \(\log(x+1)\)) transform is often not sufficient.

The primary potential of GLMs (as opposed to simple averages or regressions) lies in the possibility of explicitly specifying the distribution of the response \(z\) around its mean. Thus quite badly behaved distributions can be accommodated as well as a logarithmic link between the measurements and the linear predictor. As an example, it is possible for the analysis of acoustic data to specify a model which includes a log-parabolic functional dependence on location and depth and a gamma density at each point:

\[
\begin{align*}
    z(x,y) \text{ is gamma-distributed with mean } \mu \text{ and variance } \sigma^2, \\
    \ln \mu &= \alpha + \beta x + \delta y + \gamma x^2 + \gamma y^2 + \pi xy + \beta d + \psi d^2, \\
    \sigma^2 &= k \mu^2,
\end{align*}
\]

where the gamma density has been reparametrized with its mean and variance and \(d\) is the depth at location \((x,y)\).

It must be noted that no explicit log-transform is performed. Further, since maximum likelihood is used for estimating the parameters, the estimates satisfy a large number of criteria for optimality. Unfortunately it is often quite hard to specify the appropriate density. In the case of the gamma distribution, the constant \(k\) cannot be easily estimated and the gamma density cannot accommodate zero values.

Further, in linear models (as in most classical statistics), any autocorrelations among the residuals of the fitted model are usually regarded as nuisance parameters.

With the primary interest being the volume under "the surface," it would seem highly desirable to take into account clusters of points which all lie above the surface, by stating that (1) there is spatial autocorrelation present, and (2) since the points in a region are above the surface, the volume should be increased somewhat (potentially reducing the variance of the volume estimate). This philosophy is in direct contrast with the classical approach which tries to eliminate or forget the spatial autocorrelation. On the other hand, this approach is at the heart of methods which treat the surface as a realization of a random process.

### Smoothing and interpolation techniques

A huge body of literature exists on different techniques for smoothing and interpolation. The techniques which have received most attention for estimating and mapping fish stock abundance have their origin in geostatistics, and kriging is already quite widely used.

Other methods have received attention in different fields. Some robust and nonparametric methods have been proposed for handling ill-behaved data, but care must be taken in using them, since such methods may well reduce the effect of outliers so much as to effectively throw away most of the stock in extreme circumstances.

Since kriging and related methods are widely used and seem acceptable for many purposes, emphasis will be placed on these techniques. Appropriate references include Matheron (1963, 1967 and 1969), and Guillard et al (1989) for an application to acoustic data. For a more complete exposition of general smoothing techniques, the reader is referred to Ripley (1981) and to Cleveland (1979) for a more robust smoothing method. Objective analysis is an approach used in meteorology (the primary reference is Gandin 1965, but see also Eddy 1967 and Bleck 1975) and oceanography (see e.g. Bretherton et al 1976) and is quite related to kriging. This method would therefore seem to be a potential competitor to kriging, although seemingly quite untested within the field of mapping and estimating stock abundance. It should be noted that Creutin and Obled (1982) have shown that...
Objective analysis is superior to kriging. However, Hardy (1984) shows that objective analysis (optimal interpolation) is equivalent to universal kriging (defined below), at least in essence, though implementational differences exist.

Intrinsic random functions have also been used for mapping purposes. A relevant reference is David, Crozel and Robb (1986).

More ad-hoc methods abound, and Brown and Shepherd (1978) and Shepherd (1989) give a general smoothing technique, which seems to behave quite reasonably for trawl survey data (cf. Anon. 1990a, where the method was tested). The method is well suited for contouring and integrating, but no variance of the total biomass estimate is automatically produced. Different, also promising, approaches to smoothing are considered by Breiman and Friedman (1985) and by Cleveland and Devlin (1988).

Splines are usually not suitable for use with data as variable (noisy) as those observed when stock abundance is measured. However, Stolyarenko (1988) gives a spline approximating method for estimating the surface. This method has been tested somewhat for acoustic data, but is not in general use.

The basis for kriging is a distributional model of the entire observed surface, \( z(x,y) \). The usual primary assumption is that this surface is a realization of a random process with a constant mean, i.e. \( E[z(x,y)] = \mu \), where \( E \) denotes the expected value and \( \mu \) is a constant. Further, assumptions are made on the correlation between measurements at different locations. The assumption of a constant expected value is usually called the stationarity assumption.

The first step in applying kriging is to obtain a measure of the correlations (or covariances) between the measurements. Let \( c_{ij} \) denote the covariance between measurements \( z_i = z(x_i,y_i) \) and \( z_j = z(x_j,y_j) \), made at points \( (x_i,y_i) \) and \( (x_j,y_j) \). The optimal predictors for \( z \) at a new point \( (x,y) \) depends on the entire matrix \{\( c_{ij} \)\}. To estimate this matrix, some assumptions must be placed on its structure.

It is typically assumed that the covariances only depend on the distances between the locations, i.e. \( c_{ij} = \text{Cov}(z_i,z_j) = C(h) \), where \( h \) is the distance from \( (x_i,y_i) \) and \( (x_j,y_j) \). Unfortunately, kriging advocates do not use covariances (or correlations), but rather the equivalent (but not as easy to interpret) variances of differences, \( \text{Var}[z_i - z_j] = E[(z_i - z_j)^2] \). The usual function considered in geostatistics is the variogram, defined as

\[
\gamma(h) = \frac{1}{2} \text{Var}[z_i - z_j].
\]

Values of \( \gamma(h) \) can be estimated by grouping the \( z \)-values according to distances in location and computing the variance within each group. It is usual practice to plot these values and to fit the function \( \gamma(h) \) to the data. Several methods exist to fit the variogram, but visual inspection of the values is essential since the plot can indicate deviations from the model.

It is thus seen that the basic kriging model and GLMs are two extreme models of the same phenomenon. If we write \( z = \mu + \epsilon \), with \( E[\epsilon] = 0 \), then GLMs typically assume that all of the structural information is in the mean function, \( \mu \), and that \( \epsilon \) is simply random error (independent deviations). The basic kriging model, on the other hand, treats \( \mu \) as a constant and assumes that all of the structural information is in the correlation structure of \( \epsilon \).

Of course there are deviations from this simple approach. For example, GLMs can be fitted with autocorrelated errors, and kriging can be applied after a "trend function" has been removed (the mean function, \( \mu(x,y) \), is commonly called a "trend" or a "drift" among kriging advocates). When the kriging approach is used with an unknown trend, the method is called "universal kriging". This can be done for example by first performing a regression on a variable such as depth, and then performing kriging on the residuals. The fact that there are such in-between approaches illustrates that the true model is between the two extremes, since clearly deviations in a GLM will not be independent, nor will the mean usually be a constant (the latter is typified by repeated surveys over single schools where there is a tendency for big clumps to stay big between repetitions, and also by the "depth effect").
Among the major problems involved in using kriging are the following:

a. separation of structure into $\mu$ and $\varepsilon$,

b. determination of variogram model,

c. choice of (marginal) distribution of $\varepsilon(x,y)$, or choice of data transform.

Finally, a potential practical problem involves the size of the $\{c_{ij}\}$-matrix, since this matrix needs to be inverted during the kriging interpolation process. The matrix has dimensions $N \times N$, where $N$ is the number of measurements, and obviously $N$ can become quite large when an acoustic survey is under consideration.

Several variations of kriging exist (e.g. block kriging which alleviates the problem of too many observations), but since these methods are mostly variations on the basic approach, they are not described here. One particular variation of kriging relaxes the assumption that the variogram depends only on distance. If the variogram is independent of direction, the underlying structure is said to be isotropic. It is possible to compute different variograms, each as a function of distance in a specific direction. Kriging, therefore, can account for specific anisotropies, such as along and across a shelf. This is well summarized by Armstrong (1986).

In some acoustic surveys it becomes quite obvious that the simple kriging model is incorrect, and problem (a) above comes into consideration. This is typified by the depth effect, which can be quite pronounced for some species. In this case the mean function $\mu(x,y)$ depends on (at least) depth and can be modeled as such.

A simple-minded approach might be to perform an regression of $z(x,y)$ on a depth function, using, e.g.,

$$
\mu(x,y) = \alpha + \beta d_{xy} + \delta d_{xy}^2.
$$

If the measurements were independent (and Gaussian), then $\alpha, \beta$ and $\gamma$ should be estimated from

$$
\min_{\alpha, \beta, \gamma} \sum (z(x,y) - \mu(x,y))^2.
$$

Of course the estimates are not independent, but this is one way to start the process. The next step is to compute

$$
W(x,y) = z(x,y) - \hat{\mu}(x,y),
$$

and perform kriging on these new $W$-values. This yields a variogram and smoothed $W$-values for any desired new location. A predicted point on the observed surface is now given by:

$$
\hat{z}(x,y) = \hat{\mu}(x,y) + \hat{W}(x,y).
$$

One point to be made concerning this approach is that the second part of the procedure yields estimates of the covariances which were ignored in the first part. It is possible to iterate, using the covariances in the regression part, rather than using straightforward least squares.

A note on data transformations

The data values observed in acoustic surveys (or trawl surveys) tend to be quite badly behaved in that there are often a few outliers. This is sometimes dealt with by initially transforming the data (using power or log transforms). Thus the datum, $z(x,y)$ is first transformed into, e.g., $w(x,y) = \log(z(x,y))$, and then smoothing or surface-fitting is performed to obtain a full surface $w(x^*,y^*)$ at coordinates $(x^*,y^*)$, where the integration is to be performed.

In the case of a stratified analysis the smoothing step is to compute the strata averages at strata centers corresponding to $w(x^*,y^*)$. The next step is to transform back with the inverse function, e.g.,
\[ z^*(x^*, y^*) = \exp\{ w^*(x^*, y^*) \}. \]

In this way, one obtains the smoothed \( z \)-values on the original scale for each stratum. These smoothed \( z \)-data are then used for integration.

Thus, for the stratified analysis, the approach is essentially to transform the data, fit a surface, transform back and finally integrate. For stratified analysis this is fairly simple, since the function to be integrated is a step function.

In general it must be kept in mind that bias may be introduced during the transform/backtransform process. This is well known with the log-transform, as is the method to correct for it. Unfortunately, the correction factor involves the estimate, \( s^2 \), of variance, and \( s^2 \) is a quantity which is highly variable. Thus the bias correction in the backtransform also gives back variability. Laurec and Perodou (1987) indicate that the bias correction may not be worthwhile due to the added variability that the inverse transform yields.

Further, Myers and Pepin (1987) have found that use of the log transform can be quite dangerous when the true underlying density is (even only slightly) different from lognormal. Thus, though it may often seem reasonable to log-transform, it can be quite dangerous, and using a bias-correction may not be advantageous either.

The following point was driven home by A. Laurec (pers comm). If the raw data are transformed, then the fitted values must be transformed back to the original scale before integration. This is obvious when written down mathematically. The quantity of interest is

\[ I = \int \int z(x, y) \, dx \, dy, \]

but if smoothing/interpolation is based on, e.g., \( w(x, y) = \log(z(x, y)) \), to yield \( w^*(x^*, y^*) \), then

\[ I = \exp\left( \int \int w^*(x, y) \, dx \, dy \right) = \exp\left( \int \int \log[z(x, y)]^* \, dx \, dy \right) \]

bears no obvious relationship at all to \( I \). Note that the integral is simply an integral to find the volume, \( I \), and does not reflect an expected value. Hence the concept of "bias-correction" is not of relevance when comparing \( I \) and \( J \). However it is quite feasible to transform back each point with \( z = \exp(w) \) and compute

\[ I^* = \int \int z^*(x, y) \, dx \, dy = \int \int e^{w^*(x, y)} \, dx \, dy. \]

The optimality fallacy

Many of the methods mentioned above satisfy some optimality criterion. Thus, when normality is assumed, GLMs and kriging are both "optimal" in the sense of unbiasedness and minimum variability. However, the precise definition of these terms and underlying assumptions show that the conditions under which each is the optimal method to use are entirely different.

One point which must be made is that GLMs are only optimal if not only all the distributional assumptions are met, but the model is also known (i.e. all the terms are known). When terms are missing, the definition of optimality in GLMs is somewhat vague. Similarly, the kriging approach tacitly assumes that the variogram is an appropriate summary of the spatial correlations. If this assumption is incorrect, then the optimality statements are likely to be false.

Further, some of the basic assumptions behind kriging and GLMs are almost certainly false when acoustic data are being considered, as mentioned in the previous sections. Similarly, any textbook statement concerning the optimality of classical stratified sampling schemes is also likely to be a fallacy, due to the spatial structure involved when fish species are being considered. Thus it is not at all clear which method will give the "best" results in a sense which is reasonable for the analysis of acoustic data.
At present three classes of methods have been proposed to formally evaluate the different techniques for analysis of line-transect data: simulation, repeated surveys and backwards comparison with Virtual Population Analysis (VPA). All three have good and bad points.

The simulation approach has the tremendous virtue that the correct result is known. The disadvantage, however, lies in the fact that simulated data may not reflect accurately the real world.

Using repeated surveys over a single school of fish has the advantage that repeated estimates, $I^*$, of the stock biomass/index, $I$, becomes possible. This enables the computation of a variance of $I^*$, which can be compared to the estimated variances given by the procedure itself. The drawbacks from this approach are (1) the true size of the school is unknown (so that only the variance can be checked, not the mean), and (2) the number of available repetitions is usually small, which will result in an unreliable variance estimate based on the (few) different $I^*$-values. Potentially this results in little more than mere indications of how well a method estimates variability.

Finally, when a series of annual surveys of a stock is available, it is possible to compare the acoustic estimates with back-calculations based on VPA (cf. Jakobsson 1983). This has been used as an aid in tuning VPA (Halldorsson et al 1986) and gives an estimate of the variance in the acoustic estimates. This approach is probably the ideal one, when the data is available, but it must be noted that it requires (1) that there is a minimum of measurement error and discards in the catch-at-age data (for the VPA to perform reliably), and (2) the raw data are available from the full series of acoustic surveys.

What is expected is that each of the mentioned techniques, if described only generically, has a particular or convenient domain of applicability, which depends on the characteristics of the fish distribution. These may well have to be learned or determined on a case-by-case basis. Nonetheless, the advantages of such tedious work are clear: guidelines can be established for the analysis of line-transect measurements of density. Ultimately, given reliable estimation of abundance, and its variance, and mapping of the resource, improvements in survey design may be expected. These might, for instance, relate the degree of coverage or sampling to the precision of the result, or, given use of prior information about the fish distribution, allow specification of a different, perhaps irregular survey grid to increase precision for the same expenditure of effort.

The problems that are being discussed here with respect to fish also apply to the abundance estimation of krill (Euphausia superba) in the Southern Ocean (Everson 1977). This has been recognized at the Second Meeting of the Working Group on Krill of the Commission for the Conservation of Antarctic Marine Living Resources (CCAMLR), held in Leningrad, 27 August - 3 September 1990, both in discussions and in the “report of meeting”. By extension, the same spatial statistical techniques may also be applied in the acoustic abundance estimation of other zooplankton species.
CONCLUSIONS

The acoustic abundance estimation method may be assumed in many important instances to yield measures of fish density along line transects over the survey region. Statistical combination of these constitutes a well-formulated problem with the following aims:

1. estimation of the total quantity of fish, or average density, over the survey region,
2. specification of the variance of this estimate, and
3. mapping the fish distribution over the region.

Candidate statistical methods for solving the problem make explicit use of the observed spatial structure. The technique and likely degree of success to be achieved for the arbitrary fish stock will undoubtedly depend on both biological factors and the conditions of registration. Notwithstanding lack of a universal solution, valuable information may be derived by use of the best technique for each individual target stock and survey situation. It is hoped that particular statistical techniques may be associated with specific types of fish occurrence or aggregation through the planned Workshop on the Applicability of Spatial Statistical Techniques to Acoustic Survey Data in 1991.

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