DESIGNING AN IMPROVED TRANSUCER ARRAY GEOMETRY

by

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ABSTRACT

Improvements in a split-beam transducer that is also used for surveying are sought through changes in array geometry. These are assessed through relative changes in the sum of source level and voltage response and the equivalent beam angle. A case is made for increasing the size of the present transducer array and for introducing two operating modes: (1) a central core of elements, with moderate power and relatively wide beam, and (2) the entire array of elements, both core and annulus, with correspondingly enhanced performance.

INTRODUCTION

The Institute has had a particular problem in registering echoes from cod (Gadus morhua) and blue whiting (Micromesistius poutassou) during routine surveys over the past several years. The fish have often been so dispersed and deep-lying that their echoes have been under the detection threshold. This has been confirmed by computation. The resultant estimates of stock size have in some instances been demonstrably too low,
as has been confirmed in subsequent echo integration surveys of the same stocks, when the fish were higher in the water column and more accessible to the acoustic beam of the split-beam transducer.

Use of a towed transducer as an alternative registration strategy has been impracticable because of limitations in towing depth or, equivalently, transducer performance. Even when registration with the towed transducer has been possible, this has often been inconvenient or awkward, owing to the requirement for frequent biological sampling by trawling.

It is the purpose of this work to propose an alternative design for a 38-kHz split-beam transducer, which addresses the two generally contradictory demands of having a wide beam for good acoustic coverage and having a narrow beam for registering weak targets, such as small or deep-lying fish. The solution is an expanded array of elements with two modes of operation: (1) use of a central core of elements, with performance similar to the present SIMRAD split-beam transducer, and (2) use of the full set of elements, with enhanced performance. This is examined for a variety of geometries.

**TRANSDUCER GEOMETRIES**

Three basic geometries of transducer arrays for use at 38 kHz are considered: hexagonal, square and octagonal. Several examples of each are investigated. These are enumerated for convenience.

(1) The basic split-beam transducer design by SIMRAD for use at 38 kHz consists of a hexagonal array of 68 circular elements of 35-mm diameter packed as densely as possible with the following numbers per row: 6,7,8,9,8,9,8,7. (2) The first variant of this is formed by adding a ring of elements along the perimeter. The number of elements per row is thus 7,8,9,10,11,10,11,9,8,7, making 100 in total. (3) The next variant has 140 elements arranged with the following numbers per row: 8,9, 10,11,12,13,14,13,12,11,10,9,8.

The square and octagonal arrays consist of identical square elements that are 30 mm on a side, with nearest-neighbor center-to-center distances of 32 mm, measured along rows and columns. Three elementary examples of the square array are grids with (4) 10×10, (5) 12×12, and (6) 14×14 elements. Octagonal variants of these are formed by lopping off corners, or blunting the arrays, as in the following example: (7) 76 square elements formed by removing six elements from each corner of the 10×10 square array, indicated by the nomenclature: 76=10×10-4×6. Other octagonal array geometries that are considered are: (8) 104=12×12-4×10, (9) 120=12×12-4×6, (10) 136=14×14-4×15, (11) 156=14×14-4×10, and (12) 172=14×14-4×6.

The proposed expanded transducer geometry is considered to have a basic core of elements, with hexagonal, square or octagonal pattern, encompassed by a larger grouping of the same pattern. A permissible design, therefore, is formed by the octagonal arrays with design numbers (9) and (11). Here, the second array is formed by extending each row and column of the first array by exactly one element. This means of
expansion need not be rigorously followed. Thus the combinations (9) and (12), (7) and (9), and (7) and (12), for example, are also allowed.

Two additional core designs are considered. These are meant to share similar measures of source level and receiver voltage response with the basic hexagonal array, design number (1), but have reduced sidelobes. These arrays may be viewed as derivatives of the 10×10 square grid, with the following amplitude weightings:

(13)

0 0 4 8 10 10 8 4 0 0
0 4 8 10 10 10 10 8 4 0
4 8 10 10 10 10 10 8 4 0
8 10 10 10 10 10 10 8 10 0
10 10 10 10 10 10 10 10 10 10
10 10 10 10 10 10 10 10 10 10
8 10 10 10 10 10 10 8 4 0
4 8 10 10 10 10 10 8 4 0
0 4 8 10 10 10 10 8 4 0
0 0 4 8 10 10 8 4 0 0

and

(14)

0 0 7 7 10 10 7 7 0 0
0 7 7 10 10 10 10 7 7 0
7 7 10 10 10 10 10 7 7 0
7 10 10 10 10 10 10 10 7 7
10 10 10 10 10 10 10 10 10 10
10 10 10 10 10 10 10 10 10 10
10 10 10 10 10 10 10 10 10 10
7 10 10 10 10 10 10 10 7 7
7 7 10 10 10 10 10 7 7 0
0 7 7 10 10 10 10 7 7 0
0 0 7 7 10 10 7 7 0 0

The phasing of elements is equal here, as with the other designs. The basic array geometry of design numbers (13) and (14) is thus octagonal, although with the described additional tapering.

PERFORMANCE MEASURES

Four measures of performance are adopted to aid comparisons among the designs. These are the transmitted power, directivity index, source level, and equivalent beam angle. The material and transducing potential of each array element are assumed to be the same for all arrays.
Transmitted power

For a planar transducer of area $S$, uniform phasing, and amplitude-weighting described by the function $w(x,y)$, the transmitted power is proportional to the quantity

$$I = \int_{S} [w(x,y)]^2 \, dx \, dy .$$  \hspace{1cm} (1)

The weighting function is assumed to be scaled to the maximum, cavitation-limited value $w=1$. For a discrete array of $n$ elements with area $s_j$ and amplitude weight $w_j$,

$$I = \sum_{j=1}^{n} w_j^2 \, s_j .$$  \hspace{1cm} (2)

This is conveniently compared to a reference array of $n_0$ identical, maximally weighted elements, each of area $s_0$, hence with

$$I_0 = n_0 \, s_0 .$$  \hspace{1cm} (3)

The relative power is expressed in the logarithmic domain by the quantity

$$\Delta P = 10 \log \frac{I}{I_0} .$$  \hspace{1cm} (4)

Directivity index

By definition (Urick 1975),

$$\text{DI} = 10 \log \frac{4\pi}{\int_{b} d\Omega} ,$$  \hspace{1cm} (5)

where $b$ is either the transmit or receive beam pattern, and $d\Omega$ is an infinitesimal element of solid angle. The integration is performed over the entire solid angle.

The DI of the transmit beam measures the degree of concentration of the transmitted energy in the axial or forward direction. The DI of the receive beam describes the degree of discrimination against isotropic background noise, hence gives a measure of the signal-to-noise ratio in the receiver.

For each of the present arrays, which is used both to transmit and to receive, the two beam patterns are identical. The arrays are assumed to be ideally baffled, hence the beam pattern $b$ is nonvanishing only over the half-space in front of the active surface. Here, its form is that of an $n$-element planar array, namely
where \( b_1(\theta, \phi) \) is the beam pattern of a single element in the direction \((\theta, \phi)\), 
\( k = k(\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \) is the wavevector, and \( r_j = (x_j, y_j, 0) \) describes the position of the \( j \)-th element in the array. The wavenumber \( k = 2\pi/\lambda = 2\pi v/c \), where \( \lambda \) is the wavelength, \( v \) the frequency, and \( c \) the speed of sound. In deriving this expression the frequency is assumed to be constant, and the individual elements identical in form, hence with constant area \( s_j = s \). For a circular element of radius \( a \),

\[
b_1(\theta, \phi) = |2 J_1(ka \sin \theta)/(ka \sin \theta)|^2.
\]  

For a square element of side length \( 2a \),

\[
b_1(\theta, \phi) = \frac{\sin(ka \sin \theta \cos \phi) \sin(ka \sin \theta \sin \phi)}{(ka \sin \theta)^2 \cos \phi \sin \phi}.
\]

**Source level**

This is proportional to the sum of the logarithm of power and the directivity index (Urick 1975). Thus, relative to the described reference array,

\[
\Delta \text{SL} = 10 \log \frac{\sum w_j s_j}{n_0 s_0} + \text{DI} - \text{DI}_0.
\]

Since the material and transducing potential of each array element are assumed constant for all arrays, the voltage response is necessarily constant. Thus the relative source level \( \Delta \text{SL} \) is also the relative sum of source level and voltage response, commonly denoted \( \Delta(\text{SL}+\text{VR}) \). In other words, \( \Delta \text{SL} = \Delta(\text{SL}+\text{VR}) \) for the arrays studied here.

**Equivalent beam angle**

This may be computed in the standard way (Foote 1987), but with allowance made for the use of amplitude-weighting when evaluating the beam pattern. Thus,

\[
\psi = 10 \log \iiint [b(\theta, \phi)]^2 \sin \theta \, d\theta \, d\phi,
\]

where \( b \) is defined in Equation (6).
RESULTS

The results are given in the table. The performance measures are judged relative to the corresponding measures for the current SIMRAD hexagonal transducer array, design number (1), assuming a medium temperature of 5°C and salinity of 35 ppt, for which $c_0=1470$ m/s (Mackenzie (1981)). Thus, $n_0=68$, $s_0=9.62$ cm$^2$, $DI_0=27.79$ dB, and $\Psi_0=-20.24$ dB.

Absolute measures are also given for $DI$ and $\Psi$. These apply at the reference sound speed $c_0$. Values at other values of $c$ can be derived according to the dependence described by Foote (1987). For the present array dimensions and frequency $\nu=38$ kHz, for which $\lambda=4$ cm, both $DI$ and $\Psi$ change with $c$ at nearly constant rate. For example,

$$\Psi(c) = \Psi(c_0) + 20 \log \frac{c}{c_0} , \hspace{1cm} (11)$$

hence,

$$\Psi(c) = \Psi(c_0) + 0.0059(c-c_0) \hspace{1cm} (12)$$

The directivity index has a similar dependence, but with a sign change:

$$DI(c) = DI(c_0) - 20 \log \frac{c}{c_0} , \hspace{1cm} (13)$$

and

$$DI(c) = DI(c_0) - 0.0059(c-c_0) \hspace{1cm} (14)$$

DISCUSSION

Transducer performance generally improves with increasing size or degree of acoustic activity. This is predicted by the several equations; it is observed distinctly in the tabulated computational results. What is to be remarked on particularly is the magnitude of gains that can accompany expansions in the present or similar transducer configurations.

For example, by adding a single ring of elements to the hexagonal transducer array, design number (1), the source level $SL$ increases by 3.4 dB, the directivity index $DI$ increases by 1.7 dB, and the equivalent beam angle $\Psi$ decreases by 1.7 dB, i.e., improves by the same amount. Admittedly, the several gains do not apply in full under general reverberation-limited conditions, but noise-limited conditions, where the gains do apply, are very common in fisheries research.

Addition of a second, supernumerary ring of elements to the basic hexagonal array increases $SL$ and $DI$ by 6.3 and 3.2 dB, respectively, while
<table>
<thead>
<tr>
<th>Transducer design</th>
<th>Basic geometry</th>
<th>$s_n$ (cm$^2$)</th>
<th>$\Sigma w_i^2 s_i$ (cm$^2$)</th>
<th>$\Delta P$ (dB)</th>
<th>$\Delta \text{DI}$ (dB)</th>
<th>$\Delta \text{SL}$ (dB)</th>
<th>$\Psi$ (dB)</th>
<th>$\Delta \Psi$ (dB)</th>
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improving $\gamma$ by 3.1 dB. This describes a cumulative gain in performance of over 9 dB.

Gains in performance with the square or octagonal arrays can be similarly dramatic. For example, the performance of the 10X10 square array exceeds that of the hexagonal array by 2.9 dB in SL, 1.5 dB in DI, and 1.6 dB in $\gamma$. The performance of the 14X14 array exceeds that of the 10X10 by 5.8 dB in SL, 2.9 dB in DI, and 2.9 dB in $\gamma$.

Performance gains are achieved at a cost, of course. This generally includes requirements on space or mounting, in addition to manufacturing cost. However, the value of ten or so decibels in the surveying context can be decisive. In situations of marginal registration with a standard transducer, such an improvement will elevate many echoes over the detection threshold, where they can be integrated or otherwise processed.

A further advantage of increasing the present transducer size and using it in two modes, one with a core of elements with relatively wide beam and the other with the full complement of elements with narrow beam, is realized in target strength studies. A narrower beam will facilitate resolution of single fish, but it will also allow capture and registration of weaker single-fish echoes, allowing better measurement of the distribution of target strengths that characterize fish and their behaviour (Clay and Heist 1984, Foote and Traynor 1988).

A final demonstration of the computations is the damaging effect of amplitude-weighting on transducer performance. This is seen by comparing the performance measures of design numbers (13) and (14) with those for the fully weighted 10x10 array, design number (4). While the several arrays consume essentially the same space when mounted, the performance measures of the full array are about 2.1 dB higher in SL, 0.7 dB higher in DI, and 1.0 dB better in $\gamma$, or nearly 3 dB better with respect to SL and DI. The amplitude-weighting or shading does reduce the sidelobes, but this gain may be offset by the insidious effect of thresholding.

REFERENCES


Foote, K. G. 1987. Dependence of equivalent beam angle on sound speed. ICES C.M./B:2, 6 pp. [mimeo]

