DIGITAL REPRESENTATION OF SPLIT-BEAM-TRANSUDER BEAM PATTERNS

by

Kenneth G. Foote
Institute of Marine Research
5011 Bergen, Norway

ABSTRACT

The farfield beam pattern of each of two different split-beam transducers is computed and represented in the axial region by a finite number of constant levels. The accuracy of representation is determined as a function of the number of levels for a range of medium sound speeds including the nominal design value.

RÉSUMÉ: REPRÉSENTATION DIGITALE DES DIAGRAMMES D'ÉMISSION D'UN TRANSDUCTEUR À FAISCEAU SINDÉ

Le diagramme d'émission en champ lointain de chacun des 2 différents transducteurs à faisceau scindé est calculé puis représenté dans sa partie axiale grâce à un nombre fini de niveaux de constances. La précision de la représentation est fonction du nombre de niveaux dans une gamme de vitesse du son incluant la valeur nominale.

INTRODUCTION

The new split-beam echo sounder developed by SIMRAD Subsea promises to greatly increase our knowledge of in situ target strengths of fish (Foote, Kristensen and Solli 1984, Foote, Aglen and Nakken 1986). Just how accurate will the new estimates be, however?

This question must be a sine qua non of every in situ determination of target strength, no matter what the instrument or technique. Its answer depends on three principal factors: the fish, conditions of measurement, and measuring equipment. To answer the question with respect to just one of these factors, if admitting separate treatment, is a major undertaking.
It is the present, modest aim to determine how a single operation in the split-beam echo sounder measurements limits the accuracy of resulting in situ estimates of target strength. The operation is that of beam-pattern compensation, and the error source is restricted to the digital representation of the beam pattern. More particularly, the accuracy is determined with respect to the number of values or levels used to represent the beam pattern.

TRANSDUCERS

Two split-beam transducers are considered. Both are standard SIMRAD wares.

(1) 38 kHz transducer. This is a hexagonal array of 68 circular elements of 35 mm diameter. The packing is as dense as possible, hence the inter-element distance is also 35 mm. The number of elements per row follows the following sequence: 6, 7, 8, 9, 8, 9, 8, 7, 6. The middle row lacks the end elements of the ordinary hexagonal single-beam transducer.

(2) 70 kHz transducer. This is a square array of 36 circular elements of 17 mm diameter. The inter-element distance along rows and columns is 18 mm.

METHOD OF BEAM-PATTERN COMPENSATION

The method of beam-pattern compensation is the following.

The alongships and athwartships angles, \( \alpha \) and \( \beta \), are measured under assumption that the medium sound speed has the design value, 1472.5 m/s. Each of the angles is quantized by taking the integer part of \( \psi/(4.94/N_q) \), where \( \psi \) represents either \( \alpha \) or \( \beta \) and is expressed in degrees, and \( N_q \) is the number of quantization levels. For the 38 and 70 kHz transducers, \( N_q = 37 \) and 36, respectively. If the resulting number exceeds 37 in the case of the 38 kHz transducer, it is assigned to class 38. In the case of the 70 kHz transducer, the critical number is 36.

The pair of numbers constitutes an address in a table of beam-pattern compensation values. These are determined at the quantized angles by the manufacturer by reference to prior measurement. Individual values are represented digitally by rounding to the nearest whole multiple of 0.375 dB. For those angle pairs for which the two-way beam pattern is less than \(-11.25 \text{ dB}\), the beam-pattern compensation value, in the intensity domain, is equated to zero. This effectively limits the region of acceptable echoes to polar angles lying within about 4.94 deg of the acoustic axis.

MODEL OF BEAM-PATTERN COMPENSATION

In keeping with the aim to determine the accuracy of beam-pattern compensation with respect to the number of levels used to represent the beam pattern, two basic assumptions are made. The first is that, for the design sound speed at least, there is no overall bias in the compensation
process. The second assumption is that the quantized representation of particular compensation values has no adverse effect.

The consequences of the respective assumptions are the following. (1) The compensation value for all angles represented by the same quantized angle pair is determined by averaging the two-way beam pattern over the full set of angles with the same quantized angle pair. (2) The corresponding compensation value is identical with this averaged beam-pattern value, hence without additional quantization, as by the 0.375 dB resolution factor.

METHOD

The fundamental quantity is the farfield beam pattern. For a planar array of n circular elements of radius a this is

\[ b(\theta, \phi) = \frac{2 J_1(ka \sin \theta)}{ka \sin \theta} \sum_{j=1}^{n} \exp(ik \cdot r_j) \]

(1)

where \( k \) is the wavevector and \( r_j \) is the position of the center of the j-th element. If the elements lie in the X-Y plane, then, in Cartesian coordinates,

\[ k = k(\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \]

and

\[ r_j = (x_j, y_j, 0) \]

Because the target strength, or backscattering cross section, of fish is being determined, the transducer is used both in transmission and reception. The significant quantity is thus the two-way beam pattern, \( b_2(\theta, \phi) \). This is expressed logarithmically by \( B(\theta, \phi) \),

\[ B(\theta, \phi) = 20 \log b(\theta, \phi) \]  

(2)

The split-beam systems do not measure \( \theta \) and \( \phi \) directly, rather they measure the alongships and athwartships angles, \( \alpha \) and \( \beta \), respectively. The connection is

\[ \theta = \cos^{-1}(1 + \tan^2 \alpha + \tan^2 \beta)^{-1/2} \]

and

\[ \phi = \tan^{-1}(\tan \beta / \tan \alpha) \]
First problem

In order to determine how well the beam-pattern compensation can be performed, Eq. (2) is evaluated throughout the following sector:

$$\phi \in [0, 90] \text{ deg} \quad \text{and} \quad \theta \in [0, 4.94] \text{ deg}$$

This is divided up into \(N_q^2\) contiguous cells of sides \(\Delta\alpha = \Delta\theta = 4.94/N_q\) (deg).

By virtue of the model assumptions, there is no overall bias imposed by the compensation process. Thus the most interesting effect of the digital representation lies in its distortion of target strengths. The particular measure chosen here to represent this effect is the overall standard deviation of intra-cell beam pattern values from their respective cell averages. The corresponding variance has been computed by summing the individual cell variances over the entire set of \(N_q^2\) cells spanning the defined sector. The variance for a single cell has been computed from a set of 121 values computed over a regular 11x11 grid exactly covering the cell.

The described computations have been performed for each transducer both for the respective system values of \(N_q\) and for \(N_q=2, 4, 8, 16, 32, \text{ and 64.}\)

Second problem

Reference of the beam-pattern compensation values, supplied by the manufacturer, to the design sound speed may also affect the in situ target strength. To determine the magnitude of this effect, some of the computations for the first problem have been represented for two other sound speeds. These are 1448.6 and 1493.4 m/s, which represent the sound speeds in sea water at 0 and 11°C, respectively. The design sound speed, 1472.5 m/s, is taken to apply to sea water at 5.5°C.

In the new computations, however, the deviations in intra-cell values are determined for the low and high sound speeds with respect to the corresponding mean values for the design sound speed. This is done both for the respective intra-cell averages and for their variances. Again, as in the computations for the first problem, the several quantities are determined for a range of values of \(N_q\).

RESULTS

The results of the several computations are combined in Table 1 for the 38 kHz transducer and in Table 2 for the 70 kHz transducer. To simplify the table headings, the sound speeds assumed in the several computations are represented by the corresponding sea water temperatures. Thus the design sound speed of 1472.5 m/s is represented by the temperature 5.5°C. Similarly, the low sound speed of 1448.6 m/s is represented by the temperature 0°C, and that of the high sound speed of 1493.4 m/s by the temperature 11°C.
Table 1. Statistics of errors, in decibels, in beam-pattern compensation for the 38 kHz transducer due to the finiteness in number of levels, \( N_q \). The basic angle resolution is denoted \( \Delta \). By assumption, the overall compensation at the design sound speed, or 5.5°C temperature, is unbiased, hence the mean error there is null. Computations performed at the other temperatures reveal biasing.

<table>
<thead>
<tr>
<th>( N_q )</th>
<th>( \Delta ) (deg)</th>
<th>5.5°C</th>
<th>0°C</th>
<th>11°C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>s.d.</td>
<td>Mean</td>
<td>s.d.</td>
<td>Mean</td>
</tr>
<tr>
<td>2</td>
<td>2.47</td>
<td>-0.29</td>
<td>0.20</td>
<td>0.24</td>
</tr>
<tr>
<td>4</td>
<td>1.24</td>
<td>-0.25</td>
<td>0.16</td>
<td>0.21</td>
</tr>
<tr>
<td>8</td>
<td>0.62</td>
<td>-0.23</td>
<td>0.15</td>
<td>0.19</td>
</tr>
<tr>
<td>16</td>
<td>0.31</td>
<td>-0.21</td>
<td>0.13</td>
<td>0.18</td>
</tr>
<tr>
<td>32</td>
<td>0.15</td>
<td>-0.20</td>
<td>0.13</td>
<td>0.17</td>
</tr>
<tr>
<td>37</td>
<td>0.13</td>
<td>-0.20</td>
<td>0.13</td>
<td>0.17</td>
</tr>
<tr>
<td>64</td>
<td>0.08</td>
<td>-0.20</td>
<td>0.13</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Table 2. Statistics of errors, in decibels, in beam-pattern compensation for the 70 kHz transducer.

<table>
<thead>
<tr>
<th>( N_q )</th>
<th>5.5°C</th>
<th>0°C</th>
<th>11°C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>s.d.</td>
<td>Mean</td>
<td>s.d.</td>
</tr>
<tr>
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<td>0.08</td>
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<tr>
<td>4</td>
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<td>0.09</td>
</tr>
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<td>8</td>
<td>0.62</td>
<td>-0.13</td>
<td>0.09</td>
</tr>
<tr>
<td>16</td>
<td>0.31</td>
<td>-0.13</td>
<td>0.09</td>
</tr>
<tr>
<td>32</td>
<td>0.15</td>
<td>-0.13</td>
<td>0.09</td>
</tr>
<tr>
<td>36</td>
<td>0.14</td>
<td>-0.13</td>
<td>0.09</td>
</tr>
<tr>
<td>64</td>
<td>0.08</td>
<td>-0.13</td>
<td>0.09</td>
</tr>
</tbody>
</table>

DISCUSSION

First problem

A solution is given in the respective columns, in Tables 1 and 2, for the 5.5°C reference temperature corresponding to the design sound speed. The observed tendency of decreasing standard deviation with increasing number of digitization levels, \( N_q \), is plausible. With finer digitization or resolution, the in situ target strength estimates are less affected by performance of the beam-pattern compensation digitally. In the limit that \( N_q \) becomes infinite, the standard deviation should vanish, which would be the case with ideal analog compensation.

Intercomparison of the results for the two transducers shows that those for the 70 kHz transducer are relatively better at small \( N_q \). This is probably a simple consequence of the smaller effective size of the
transducer at 70 kHz, thence flatter or less variable beam pattern where sampled. If this is measured by the ratio of a typical diameter to the acoustic wavelength, then the effective size is about 5.1 for the 70 kHz transducer and 7.2 for the 38 kHz transducer. At the largest investigated Nq, 64, the results are indistinguishable.

At the particular system values of Nq, 37 and 36 for the respective 38 and 70 kHz transducers, the effect of performing the beam-pattern compensation digitally is introduction of a small uncertainty into the target strength measurements. This is estimated to be roughly ±0.1 dB in the mean.

Second problem

For sea water temperatures, or sound speeds, differing from the design condition, the estimated errors are of a different sort. Firstly, a bias is incurred. This is directly related to the dependence of beam shape on sound speed. For narrow beams the beamwidth varies linearly with sound speed, thence with temperature. Thus, for the computed 0°C case, the true beam pattern is narrower than the design or reference beam pattern. Consequently, the compensation is too little, and the resulting target strength values are too low. This is seen through the negative values for the mean. The opposite tendency is seen for the 11°C case.

In both cases, the biases vary only slightly, if at all, with the number of digitization levels, Nq. This is reasonable, for the beamwidth at 0°C, say, is only 1.6% less than that at 5.5°C.

The magnitudes of the standard deviations are slightly smaller than the corresponding means, hence reflect a rather extended, non-Gaussian distribution. It is to be noted that these standard deviations only reflect the degree of variation introduced into the target strength estimates due to assumption of a constant sound speed in performing the beam-pattern compensation.

To determine the total standard deviation as a function of Nq, the several components must be combined, namely the intrinsic error variation and sound-speed-based error variation. The intrinsic error variation varies only very slightly with sound speed, hence the numbers obtained for 5.5°C can be used.

A third problem

Another problem, not addressed quantitatively here, is that associated with the manufacturer's method of assigning the beam-pattern compensation values at the design sound speed or, equivalently, the method of quantizing angles. At present, the beam pattern values apply only for the quantized angles, which are not representative of the applicable set of angle pairs, but only for that pair nearest the acoustic axis. Thus, an overall positive bias is imposed. Because of the number of quantization levels, however, this bias cannot exceed 0.3 dB. Nonetheless, as is evident from the first problem, such a bias is completely avoidable at any sound speed.
Needless to say, perhaps, for precision measurements the bias ought to be eliminated. This may be achieved in practice by the kind of calibration procedure devised by MacLennan and Svellingen (1986), although more fine-grained.

CONCLUSION

The effect of performing beam-pattern compensation with respect to a finite number of levels in the SIMRAD ES split-beam echo sounder has been theoretically investigated. The magnitudes of errors intrinsic to the digitization, both at the design sound speed and at other sound speeds, are consistent with the manufacturer's nominal specification of ±0.375 dB accuracy. For applications requiring higher accuracy, however, adjustment of the results may be necessary to remove biases due to the design assumption of a constant sound speed.

ACKNOWLEDGEMENT

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REFERENCES


MacLennan, D. N., and Svellingen, I. 1986. Simple calibration of a split-beam echo sounder. ICES C.M./B:8, 6 pp. [mimeo].